Mathematics for Chemistry: Assignment 4

June 15, 2017

1. Consider the set of linear equations:

$$\begin{aligned} ax_1 - x_2 &= 0 \\ x_1 + ax_2 + x_3 &= 0 \\ x_2 + ax_3 + x_4 &= 0 \\ x_3 + ax_4 &= 0 \end{aligned}$$

The condition for a nontrivial solution of this equation to exist is
(a) $a^4 = 3a^2$
(b) $a^4 = -a^2 + 3$
(c) $a^4 = a^2 + 1$
(d) $a^4 = a^2 - 1$
Answer (c)

2. Consider the system of linear equations below:

$$\begin{aligned} 3x + 2y + z &= 7 \\ 4x + y + 3z &= 11 \\ x - 6y - 2z &= 0 \\ 4x + 21y + 10z &= 18 \end{aligned}$$

These equations are:

(a) sufficient and consistent.
(b) not consistent.
(c) consistent but not sufficient.
(d) sufficient but not consistent.

Answer (a)

3. The determinant of the matrix

$$\begin{pmatrix} 1 & b & 0 & b \\ b & 1 & b & 0 \\ 0 & b & 1 & b \\ b & 0 & b & 1 \end{pmatrix}$$

is equal to
(a) 1 (b) -1 (c) $1 + 2b$ (d) $1 - 4b^2$
Answer (d)

4. Consider the matrix:

$$\begin{pmatrix} 1 & b & 0 & b \\ b & 1 & b & 0 \\ 0 & b & 1 & b \\ b & 0 & b & 1 \end{pmatrix}$$

Given that $b < 0$, the largest eigenvalue is
(a) 1 (b) -1 (c) 1 + 2b (d) 1 - 2b
Answer (d)

5. Amongst Symmetric, Hermitian and Unitary matrices, which are the ones that will always have real eigenvalues ?
   (a) Symmetric and Hermitian
   (b) Symmetric only
   (c) Hermitian only
   (d) Hermitian and Unitary

   Answer (a)

6. Consider the matrix $\frac{1}{\sqrt{1+2b^2}}$

$$\begin{pmatrix} 1 & b & 0 & b \\ -b & 1 & b & 0 \\ 0 & -b & 1 & b \\ -b & 0 & -b & 1 \end{pmatrix}$$

   The above matrix is
   (a) symmetric but not orthogonal
   (b) orthogonal but not symmetric
   (c) both symmetric and orthogonal
   (d) neither symmetric nor orthogonal

   Answer (b)

7. Consider the matrix:

$$\begin{pmatrix} 1 & b & 0 & b \\ b & 1 & b & 0 \\ 0 & b & 1 & b \\ b & 0 & b & 1 \end{pmatrix}$$

   The inverse of this matrix is equal to

   (a)
$$\begin{pmatrix} 1 & b & 0 & b \\ b & 1 & b & 0 \\ 0 & b & 1 & b \\ b & 0 & b & 1 \end{pmatrix}$$

   (b)
$$\begin{pmatrix} 1 & b & 0 & b \\ -b & 1 & b & 0 \\ 0 & -b & 1 & b \\ b & 0 & -b & 1 \end{pmatrix}$$

   (c) $\frac{1}{1+2b^2}$
$$\begin{pmatrix} 1 & b & 0 & b \\ -b & 1 & b & 0 \\ 0 & -b & 1 & b \\ -b & 0 & -b & 1 \end{pmatrix}$$

   (d) $\frac{1}{1+2b^2}$
$$\begin{pmatrix} 1 & b & 0 & b \\ 1/b & 1 & b & 0 \\ 0 & 1/b & 1 & b \\ 1/b & 0 & 1/b & 1 \end{pmatrix}$$

   Answer (c)

8. Consider the following set of equations:

$$\begin{aligned} 4x + 3y + z &= 4 \\ x - 4y + 2z &= 10 \\ 3x + 7y - z &= -5 \end{aligned}$$

The set of equations above has
(a) no solution
(b) a unique solution
(c) multiple (but finite) number of solutions
(d) infinite number of solutions
Answer (a)

9. Consider a vector in 3D (-1,2,5). When this vector is rotated by 30^o about the Z-axis, the resulting vector is closest to (a) (0.87,1.23,0.5) (b) (-1.87,1.23,5) (c)(-0.5,0.87,2.5) (d) (3.1,1.5,2.6)

Answer (b)

10. Consider a vector in 3D (-1,-1,-1). When this vector is rotated by 45^o about the Z-axis, followed by 45^o about the X-axis, the resulting vector is closest to (a) (-0.71,-0.71,-1.41) (b) (+0.71,0.71,-1.41) (c)(0,-0.29,-1.71) (d) (0,0.29,1.71)

Answer (c)