ASSIGNMENT 2 : SOLUTIONS

1 Set of all functions $f(x)$ s.t. $\int_a^b f(x)^2 \, dx = 1$ does not satisfy axioms of vector space since $f_2(x) = c\, f(x)$ does not belong to the space for any $c \neq 1$. All other sets correspond to real vector spaces

2 Set (a) is a 4D vector space, Set (c) is not a vector space, Set (d) is a 3D vector Space.
Set (b) is a 2D vector space

3 For set (a), we have $(c_1, c_2) = |c_1^* c_2|$

Consider $(c_1 + \lambda c_2, c_3) = |(c_1 + \lambda c_2)^* c_3|$

$$= | c_1^* c_3 + \lambda c_2^* c_3 |$$

But this is not equal to $|c_1^* c_3| + \lambda |c_2^* c_3|$ in general

$\therefore \quad (c_1 + \lambda c_2, c_3) \neq (c_1, c_3) + \lambda (c_2, c_3)$

This is not a real inner product space.

Set (b) is a 4D real inner product space, set (c) is an infinite dimensional inner product space.
Set (d) is a 2D inner product space

4 Condition (A) is not satisfied for $a = 0$.

5 There can be many choices for basis vectors for a space.

6 (B) contains 4 3D vectors and they have to be dependent.

$\boxed{7}$ All sets of vectors are linearly independent as can be easily verified. For example, we cannot write

$$x^3 = ax + bx^2 \quad \text{for arbitrary } x.$$

Similarly for the other cases.

$\boxed{8}$

$$\vec{F} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

$$-\frac{\partial V}{\partial x} = -x - \frac{1 \times 2x}{2(x^2+y^2+z^2)^{3/2}} = -x\left(1 + \frac{1}{r^3}\right)$$

$$r = \sqrt{1^2+1^2+\sqrt{2}^2} = 2$$

$$-\frac{\partial V}{\partial x} = -x \times 1.125 \; ; \; -\frac{\partial V}{\partial y} = -y \times 1.125 \; ; \; -\frac{\partial V}{\partial z} = -z \times 1.125$$

$$\vec{F} = -1.125\,\hat{i} + 1.125\,\hat{j} - 1.125\sqrt{2}\,\hat{k}$$

$\boxed{9}$

$$\rho\vec{v} = 12\, e^{-\frac{x^2+y^2}{4}} (y\,\hat{i} + x\,\hat{j})$$

$$\vec{\nabla}\cdot\rho\vec{v} = \frac{\partial}{\partial x}\left[12y\, e^{-\frac{x^2+y^2}{4}}\right] + \frac{\partial}{\partial y}\left[12x\, e^{-\frac{x^2+y^2}{4}}\right]$$

$$= -12\,xy\, e^{-(x^2+y^2)/4}$$

$\boxed{10}$

$$\vec{\nabla}\times\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & xz & xy \end{vmatrix} = \hat{i}\left[\frac{\partial}{\partial z}(xz) - \frac{\partial}{\partial y}(xy)\right] + \ldots$$

Similar terms for $\hat{j}$ & $\hat{k}$

$$= \hat{i}\,[x - x] + \hat{j}\,[y - y] + \hat{k}\,[z - z]$$

$$= 0$$