Mathematics for Chemistry : Quiz 2

June 11, 2017

1. Of the following collection of objects, the one that DOES NOT represent a Real Vector Space is

   (a) Collection of all functions $f(x)$ of a single variable x where $a < x < b$

   (b) Collection of all functions of a single variable x, such that $\int_a^b f(x)^2 dx \; < \; \infty$

   (c) Collection of all functions of a single variable x, such that $\int_a^b f(x)^2 dx \; = \; 1$

   (d) Collection of all functions of a single variable x of the form $f(x) = c_1 x + c_2 x^2$ were c_1, c_2 are arbitrary real numbers

   Answer (c)

2. Of the following collection of objects, the one that forms a 2-dimensional Real Vector Space is

   (a) The set of all 2X2 matrices whose elements are real numbers from $-\infty$ to $+\infty$

   (b) The set of all complex numbers of the form $a + ib$ where a and b are real numbers between $-\infty$ and $+\infty$

   (c) The set of all pairs of integers from $-\infty$ to $+\infty$

   (d) The set of all polynomials of the form $a_0 + a_1 x + a_2 x^2$, where a_1, a_2, a_3 are real numbers

   Answer (b)

3. Of the following collection of objects along with the definition of the product, the one that forms a 2-dimensional Real Inner Product Space is

   (a) The set of all complex numbers; the product of two complex numbers $c_1 = a_1 + ib_1$ and $c_2 = a_2 + ib_2$ being defined as $(c_1, c_2) = |c_1 * c_2|$.

   (b) The set of all 2X2 matrices whose elements are real numbers from $-\infty$ to $+\infty$; the product being the usual matrix product.

   (c) The set of all functions of a single variable $f(x)$, such that $a < x < b$; the product of $f_1(x)$ and $f_2(x)$ being defined as $\int_a^b f_1(x) f_2(x) dx$.

   (d) The set of all polynomials of x of the form $a + bx + cx^2$, where a, b, c are real numbers and the product of two polynomials $c_1(x) = a_1 + b_1 x + c_1 x^2$ and $c_2(x) = a_2 + b_2 x + c_2 x^2$ being defined as $(c_1, c_2) = a_1 a_2 + b_1 b_2 + c_1 c_2$.

   Answer (a)

4. A real inner product of two arbitrary vectors a and b is denoted by (a, b). We present three conditions below
   (A) $(a, a) > 0$ for all a
   (B) $(a, b) = (b, a)$ for all a and b
   (C) $(a, b)^2 \leq (a, a)(b, b)$
   The conditions that need to be satisfied for (a, b) to be a valid definition of the innner product are

(a) A,B and C
(b) B and C, but not A
(c) A and C, but not B
(d) A and B, but not C

Answer (b)

5. The INCORRECT statment about basis vectors in a vector space is

   (a) The number of basis vectors is equal to the dimensionality of the space
   (b) Any two basis vectors are linearly independent
   (c) For every vector space, there is exactly one choice of the set of basis vectors
   (d) Any vector in the vector space can be expressed as a linear combination of basis vectors

   Answer (c)

6. Below we show various sets of vectors in appropriate dimensions (think in terms of cartesian coordinates).
   (A) (3,4) and (1,3)
   (B) (4,3,1), (2,4,5), (3,1,2) and (1,$\sqrt{2}$, - 5)
   (C) (1,0,0), (0,2,1), (0,2,-1)

   The linearly independent sets of vectors amongst the choices above are:

   (a) A,B and C
   (b) A and B but not C
   (c) A and C but not B
   (d) B and C but not A

   Answer (c)

7. Consider the following sets of functions of a single variable x
   (A) x, x^2 and x^3
   (B) $\sin(x)$, $\cos(x)$ and $\tan(x)$
   (C) $\sin(x)$, $\cos(x)$ and $\sin(2x)$
   The linearly independent sets of vectors amongst the choices above are:

   (a) A,B and C
   (b) A and B but not C
   (c) A and C but not B
   (d) B and C but not A

   Answer (a)

8. The force on a particle at the point (1,-1,$\sqrt{2}$) due to the potential

$$V(x,y,z) = \frac{1}{2}\left(x^2+y^2+z^2\right) - \frac{1}{\sqrt{x^2+y^2+z^2}}$$

   is equal to

   (a) -1.125 $\sqrt{2}\,\hat{\imath}$ - 1.125 $\hat{\jmath}$ +1.125 $\sqrt{2}\hat{k}$
   (b) 1.125 $\hat{\imath}$ - 1.125 $\hat{\jmath}$ +1.125 $\sqrt{2}\hat{k}$
   (c) -1.125 $\hat{\imath}$ + 1.125 $\hat{\jmath}$ + $\sqrt{2}\hat{k}$
   (d) -1.125 $\hat{\imath}$ + 1.125 $\hat{\jmath}$ -1.125 $\sqrt{2}\hat{k}$

   Answer (d)

9. Consider a scalar field $\rho(x,y) = 3e^{-(x^2+y^2)/4}$ and a vector field given by $\vec{v}(x,y) = 4y\hat{i} + 4x\hat{j}$. The divergence of the field $\rho(x,y)\vec{v}(x,y)$ is given by

   (a) $12xy$

   (b) $e^{-(x^2+y^2)/4}$

   (c) $12xye^{-(x^2+y^2)/4}$

   (d) $-6(x^2y^2)e^{-(x^2+y^2)/4}$

   Answer (c)

10. Consider a vector field given by $\vec{v} = yz\hat{i} + xz\hat{j} + xy\hat{i}$. The curl of this field at the point (1,-1,0) is equal to

   (a) 0

   (b) $\hat{i} - \hat{j} + \hat{k}$

   (c) $\hat{i} + \hat{j} + \hat{k}$

   (d) $-\hat{i} + \hat{j} - \hat{k}$

   Answer (a)