# Problem Set 1: SOLUTIONS

1. Calculate average $\bar{X}$ and standard deviation $\sigma$

$$\bar{X} = \frac{10.13 + 10.44 + 10.81 + 10.50 + 10.72}{5} = 10.52$$

$$\sigma^2 = \frac{(10.52 - 10.13)^2 + (10.52 - 10.44)^2 + (10.81 - 10.52)^2 + (10.52 - 10.50)^2 + (10.52 - 10.72)^2}{4}$$

$$\sigma = 0.27 \quad \Rightarrow \quad \frac{\sigma}{\sqrt{N}} = 0.12$$

Reported value = $10.52 \pm 0.12$

Report as $10.5$

2. Successive values keep shifting to the right (to greater values). Hence error is SYSTEMATIC

3. Average and Standard Deviation can only be used for RANDOM errors.

4. Normalization condition $\int_{-\infty}^{+\infty} p(x)\, dx = 1$

$$\Rightarrow A \int_{-1}^{2} e^{-|x|} = 1 \qquad e^{-|x|} = e^{x} \text{ for } x < 0$$
$$= e^{-x} \text{ for } x > 0$$

$$\therefore \quad A \int_{-1}^{0} e^{x} dx + A \int_{0}^{2} e^{-x} dx = 1$$

$$\therefore \quad A = 0.67$$

5. $p(x,y) = A \sin^2(\pi x)\sin^2(2\pi y)$ for $0 \le x < 2$, $0 \le y < 2$

$= 0$ otherwise

$$\int_0^2\int_0^2 p(x,y)\,dx\,dy = 1$$

$$\Rightarrow A \int_0^2 \sin^2 \pi x \, dx \int_0^2 \sin^2 2\pi y \, dy = 1$$

$$\Rightarrow A \left[\frac{x - \frac{\sin 2\pi x}{2\pi}}{2}\right]_0^2 \left[\frac{y - \frac{\sin 4\pi y}{4\pi}}{2}\right]_0^2 = 1$$

$$\Rightarrow A \times 1 \times 1 = 1 \Rightarrow A = 1$$

6. $p(x) = A \sin^2 2\pi x$ $\quad 0 \le x < 1$

$= 0$ otherwise

Normalizing, we get $A = 2$

$$\langle x \rangle = \int_0^1 x\,p(x)\,dx = \int_0^1 2x \sin^2 2\pi x \, dx$$

$$= 2\int_0^1 x \cdot \left(\frac{1 - \cos 4\pi x}{2}\right) dx$$

$$= \int_0^1 x\,dx - \int_0^1 x \cos 4\pi x \, dx$$

$$= \frac{1}{2} - \left[x\frac{\sin 4\pi x}{4\pi}\right]_0^1 + \int_0^1 \frac{\sin 4\pi x}{4\pi} dx$$

$$= \frac{1}{2}$$

7. $p(x) = A \sin^2 \pi x$ $\quad -1 \le x < 1$

$= 0$ otherwise

Similar to Problem 5, we find $A = 1$

$$\langle x^2 \rangle = \int_{-1}^1 x^2 \sin^2 \pi x \, dx = \int_{-1}^1 \frac{x^2}{2} dx - \int_{-1}^1 \frac{x^2 \cos(2\pi x)}{2} dx$$

$$= \frac{1}{3} - \left[\frac{x^2 \sin(2\pi x)}{4\pi}\right]_{-1}^1 + \int_{-1}^1 \frac{2x \sin 2\pi x}{4\pi} dx$$

$$\langle x^2 \rangle = \frac{1}{3} + 0 + \left[\frac{-x\cos 2\pi x}{4\pi^2}\right]_{-1}^{1} + \int_{-1}^{1} \frac{\cos(2\pi x)}{4\pi^2}dx$$

$$= \frac{1}{3} - \frac{1}{2\pi^2} + 0$$

$$\simeq 0.28$$

8. $\langle u \rangle = \sqrt{\frac{8RT}{\pi M}} = 200 \Rightarrow \frac{2RT}{M} = \frac{40000 \times \pi}{4}$

$$\frac{2RT}{M} = 31400$$

$M \rightarrow$ in kg

$$P(u) \propto u^2 e^{-\frac{m}{2k_BT} \cdot u^2}$$

$$\propto u^2 e^{-\frac{M}{2RT} u^2}$$

$$\propto u^2 e^{-\frac{u^2}{31400}}$$

9. For a typical Gaussian Distribution

$$p(x) \propto e^{-x^2/2\sigma^2}$$

$$\Rightarrow \sigma^2 = 4 \Rightarrow \sigma = 2$$

10. $p(x) = A e^{-x^2/32}$

$$\int_{-\infty}^{+\infty} p(x)\,dx = 1 \Rightarrow A \int_{-\infty}^{+\infty} e^{-x^2/32}\,dx = 1$$

$$A \cdot \sqrt{\frac{\pi}{1/32}} = 1$$

$$A = \frac{1}{\sqrt{32\pi}} \simeq 0.10$$