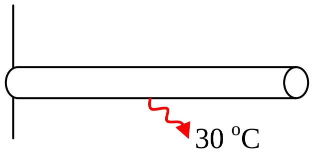
Problem Sheet 08

Problem 1: Heat conduction in a Metal Rod (re-visited)

Consider the problem of metal rod heated at one end and exposed to atmosphere so that it loses heat to the surroundings. This system was seen in an earlier problem set.



With appropriate derivation, the model for this system can be represented as:

$$\frac{d^2T}{dz^2} = 0.1(T - 30) \quad \text{subject to} \quad T_{(z=0)} = 100; T_{(z=5)} = 30$$

Discretize the above equation in n equal sized intervals using the standard central difference formula. Write down the resulting equations and solve them for $n = \{2, 4, 10, 20\}$. Verify how the results change with increasing number of intervals.

Problem 2: Shooting Method

Solve the previous problem using Shooting Method. Use RK-4 method for solving the ODEs and bisection rule to generate new guesses for the initial condition, $T'_{z=0}$.

Problem 3: Nonlinear Model of Heat Transfer

In the previous problem, a *linear* model for heat transfer in a solid rod was solved. Now, we will repeat the problem for the case of nonlinearity in the ODE. The model for nonlinear thermal conductivity becomes:

$$\frac{d^2T}{dz^2} = 3\left(\left(\frac{T}{30}\right)^{1.25} - 1\right) \quad \text{subject to} \quad T_{(z=0)} = 100; T_{(z=5)} = 30$$

Solve this problem using method of your choice.

Problem 4:

The dimensionless balance on moles or reactant A in a fixed bed reactor with axial dispersion is:

$$0 = \frac{1}{\text{Pe}}\frac{d^2C}{dz^2} - \frac{dC}{dz} - \text{Da}\, C$$

In the above expression, Pe is the Peclet number, Da is the Damkohler number, C is the dimensionless concentration and z is the dimensionless axial distance (so that $0 \leq z \leq 1$ is the domain of interest). The boundary conditions for this system are:

$$z = 0: \quad \frac{dC}{dz} = \mathrm{Pe}(C - 1)$$

$$z = 1: \quad \frac{dC}{dz} = 0$$

Solve the above equation using Finite Difference method for $h = 0.1$. Pay careful attention to the two boundary conditions, especially the one at $z = 0$. Use Pe = 10^3 and Da = 1.

For the same value of Pe, repeat for Da = 0.1 and 10. Comment on the results obtained.