Combustion in Air-breathing Aero Engines
Assignment No. 9

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This assignment contains 8 multiple choice questions with 4 possible answers to each. Only one of the choice is correct and so select the choice that best answers the question. Correct choice rewards you with 1 point for each question. Wrong answers will reward you with 0 points (no negative marking). The questionnaire contains both numerical and concept-based questions. All the best!!!

Q1: Estimate the stretch rate for a spherically contracting flame. Let R denote the instantaneous radius of the spherical flame.

1. $\frac{1}{R}\frac{dR}{dt}$
2. $-\frac{2}{R}\frac{dR}{dt}$
3. $\frac{2}{R}\frac{dR}{dt}$
4. $-\frac{1}{R}\frac{dR}{dt}$

Ans: Stretch rate $\kappa = \frac{1}{A}\frac{dA}{dt}$ For, a spherical flame $A = 4\pi R^2$, where R is the instantaneous radius at time t. Therefore, we can write

$$\begin{aligned} \kappa &= \frac{1}{4\pi R^2}\frac{d\left(4\pi R^2\right)}{dt} \\ &= \frac{2}{R}\frac{dR}{dt} \end{aligned}$$

Since, the flame is contracting its area reduces with time and its stretch rate is negative. Therefore, $\kappa = -\frac{2}{R}\frac{dR}{dt}$. Therefor, the correct answer is 2.

Q2: A turbulent V-shaped flame is stabilized in the wake of circular rod placed in the middle of a duct of square cross-section 25mm × 25mm. Assume that the flame goes all the way to the duct wall. The flow through the duct is 0.03kg/s. The unburned mixture is at 310K at 1atm with a molecular weight of 29.6kg/kmol. The turbulent burning velocity is 5m/s. Estimate the length of the flame from tip to the duct wall.

1. 150mm
2. 208mm
3. 104mm
4. 52mm

Ans: Given, $S_T = 5$m/s; $\dot{m} = 0.03$kg/s. To find the mean area $\overline{A}$ using $S_T = \frac{\dot{m}}{\rho_u \overline{A}}$, we need to find the ρ_u.

$$\begin{aligned} \rho_u &= \frac{P}{RT_u} \\ &= \frac{101325 \times 29.6}{8314 \times 310} \\ &= 1.164 kg/m^3 \end{aligned} \qquad (1)$$

Substituting, ρ_u in the expression above, we get $\overline{A} = 5.2 \times 10^{-3}\text{m}^2$. The length is calculate below:

$$\begin{aligned} \overline{A} &= 2 \times depth \times L \\ L &= \frac{5.2 \times 10^3}{2 \times 25} \\ L &= 104mm \end{aligned} \tag{2}$$

Therefore, the correct choice is 3.

Q3: For the flame mentioned in question 2, estimate the included angle at the tip.

1. 16^o
2. 8^o
3. 6.9^o
4. 13.8^o

Ans: Using geometry, half of included angle $\alpha = \arcsin\left(\frac{25/2}{104}\right)$. $\alpha = 6.9^o$. Hence, the correct choice is 4, because the included angle $= 2\alpha$.

Q4: Consider a can combustor of diameter 0.3m. The mean velocity in the combustor is 100m/s. The fuel used is has following parameters $S_L = 88.4$cm/s and flame thickness $\delta_L = 0.039$mm with the mean molecular weight of 28.8kg/kmol. The pressure inside the combustor is 15atm and the relative turbulence intensity w.r.t mean is 10%. The integral length scale is 1/10th of the can diameter. The temperature $T = 2000$K. The dynamic viscosity $\mu = 6.89 \times 10^{-5}$kg/m-s. Estimate the turbulent Reynolds number and the Damköhler number in the can combustor.

1. 4354 and 6.8
2. 4354 and 68
3. 11451 and 6.8
4. 11451 and 68

Ans: First we calculate $Da = \tau_o/\tau_c$.

$$\begin{aligned} \tau_o &= \frac{l_o}{u_o} \\ &= \frac{D/10}{0.1 \times \overline{u}} \\ &= \frac{0.3/10}{0.1 \times 100} \\ &= 3ms \\ \tau_c &= \frac{\delta_L}{S_L} \\ &= \frac{0.039 \times 10^{-3}}{88.4 \times 10^{-2}} \\ &= 0.044ms \end{aligned}$$

Therefore, $Da = 68$. Now, the turbulent Reynolds number is $Re_o = \rho u_o l_o/\mu$

$$\begin{aligned} Re_o &= \frac{(15 \times 101325 \times 28.8/(8314 \times 2000))\,0.3/10 \times 0.1 \times 100}{6.89 \times 10^{-5}} \\ &= 11451 \end{aligned}$$

Therefore, the correct choice is 4.

Q5: For the question 4, estimate the Karlovitz number Ka and the regime of combustion.

1. 0.64; corrugated flamelet regime
2. 1.6; thin reaction zone regime

3. 0.015; corrugated flamelet regime

4. 1.6; corrugated flamelet regime

Ans: We know, $Ka = \tau_c/\tau_\eta$, where $\tau_\eta = \frac{\eta}{u_\eta}$. Therefore,

$$\begin{aligned}
\eta &= l_o \times Re_o^{-3/4} \\
&= \frac{0.3}{10} \times (11451)^{-3/4} \\
&= 2.7 \times 10^{-5} m \\
u_\eta &= u_o \times Re_o^{-1/4} \\
&= 0.1 \times 100 \times (11451)^{-1/4} \\
&= 0.97 m/s \\
\tau_\eta &= 0.028 ms
\end{aligned} \tag{3}$$

Therefore, the $Ka = 1.6$. Since, the $Ka > 1$, this implies that turbulent eddies of the size of η may penetrate into the pre-heat zone. Since $Da > 1$, the large eddies cannot penetrate. Thus, the flame is thin-reaction zone regime. Therefore, the correct answer is 2.

Q6: When the flame is in thin reaction zone, which of the following conditions are met.

(a) Reaction zone remains unperturbed

(b) Eddies enter into the pre-heat zone

(c) Flame thickness usually increases

(d) The transport properties remain same as laminar

Select the choice that best answers your choices

1. a. only
2. a. and b.
3. a., b., and c.
4. All of the above

Ans: The correct choice is 3.

Q7: Consider a circle with initial radius R_o. The circle expands with a self-propagation speed which is given by $A + B/\kappa$, where κ is the local curvature. A and B are constants with consistent units. What is the equation of the circle at time t.

1. $\frac{1}{A}\left[B\left(1 - exp(-At)\right) + AR_o exp(-At)\right]$
2. $\frac{1}{B}\left[A\left(1 - exp(-Bt)\right) + BR_o exp(-Bt)\right]$
3. $R_o exp(-Bt)$
4. cannot be determined

Ans: This problem can be solved using G-equation. Since, the surface is a circle its equation in polar co-ordinates can be written as:

$$G(r, \theta, t) = r$$

The initial condition is $G(r, \theta, 0) = R_o$ and $|\nabla G| = 1$. The G-equation will reduce to

$$\frac{\partial G}{\partial t} = w$$

Since, $G(r, \theta, t) = r$, therefore $\partial G/\partial t = \partial r/\partial t$, and since, $r = r(t)$ due to symmetry, $\partial r/\partial t = dr/dt$. Hence, the equation for rate of change of radius can be written as:

$$\begin{aligned}
\frac{dr}{dt} &= w \\
&= A + B/\kappa \\
&= A + Br
\end{aligned}$$

Since, curvature for a circle is inverse of its radius $\kappa = 1/r$ and the above equation can be integrate to give the following result.

$$R(t) = \frac{1}{B}\left[A - (A - BR_o)\, exp(-Bt)\right]$$

The correct choice is 2.

Q8: A premixed propane-air mixture emerges from a round nozzle with a uniform velocity of 75cm/s. The laminar flame speed of the propane-air mixture is 35cm/s. A flame is lit at the nozzle exit. What is the cone angle of the flame.

1. 27.8^o
2. 62.2^o
3. 30^o
4. 60^o

Ans: $\alpha_u = \arcsin\left(\frac{S_L}{u_u}\right)$. Therefore, $\alpha_u = \arcsin\left(\frac{35}{75}\right)$. Therefore, $\alpha = 27.8^o$ and the correct choice is 1.