

# Combustion in Air-breathing Aero Engines

## Assignment No. 5

July 20, 2017

**Release Date:**

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This assignment contains 8 multiple choice questions with 4 possible answers to each. Only one of the choice is correct and so select the choice that best answers the question. Correct choice rewards you with 1 point for each question. Wrong answers will reward you with 0 points (no negative marking). The questionnaire contains both numerical and concept-based questions. All the best!!!

Q1: Using the Rankine-Hugoniot relations, estimate the drop in pressure across a standard premixed flame. Express your answer with the help of following symbols:  $\rho$  for density;  $P$  is pressure;  $Ma$  is the Mach number;  $\gamma$  is the specific heat capacity ratio. Subscripts  $u$  and  $b$  denote the corresponding parameters in the unburnt or burnt side of the premixed flame.

1.  $\frac{\gamma}{2} Ma_u P_u \left( \frac{\rho_b}{\rho_u} - 1 \right)$
2.  $\frac{\gamma}{2} Ma_u^2 P_u \left( \frac{\rho_b}{\rho_u} - 1 \right)$
3.  $\frac{\gamma}{2} Ma_u^2 P_u \left( \frac{\rho_u}{\rho_b} - 1 \right)$
4.  $\frac{\gamma}{2} Ma_u P_u \left( \frac{\rho_u}{\rho_b} - 1 \right)$

Ans: From Rankine-Hugoniot relations,

$$\begin{aligned} P_u + \frac{1}{2} \rho_u u_u^2 &= P_b + \frac{1}{2} \rho_b u_b^2 \\ P_u - P_b &= \frac{1}{2} \left( \frac{\rho_b^2 u_b^2}{\rho_b} - \frac{\rho_u^2 u_u^2}{\rho_u} \right) \end{aligned}$$

From continuity equation,  $\rho_u u_u = \rho_b u_b$

$$\begin{aligned} P_u - P_b &= \frac{1}{2} \rho_u^2 u_u^2 \left( \frac{1}{\rho_b} - \frac{1}{\rho_u} \right) \\ &= \frac{1}{2} \rho_u u_u^2 \left( \frac{\rho_u}{\gamma P_u} \right) \gamma P_u \left( \frac{1}{\rho_b} - \frac{1}{\rho_u} \right) \end{aligned}$$

Hence,

$$\Delta P = \frac{\gamma}{2} (Ma_u)^2 \left( \frac{\rho_u}{\rho_b} - 1 \right) \quad (1)$$

Therefore,, the correct choice is 3.

Q2: Following your result in question 1 and considering that  $\frac{\rho_u}{\rho_b}$  is nearly 7 for a typical hydrocarbon-air mixture with  $\gamma = 1.4$ , estimate the relative drop  $\frac{\Delta P}{P_u}$  in pressure as a function of  $Ma_u$

1.  $0.12 Ma_u^2$
2.  $4.2 Ma_u^2$

3.  $4.2Ma_u$
4.  $0.12Ma_u$

Ans: Using Eq. (1), i.e., the expression of pressure drop derived previously,

$$\frac{\Delta P}{P_u} = \frac{1.4}{2} Ma_u^2 (7 - 1)$$

$$\frac{\Delta P}{P_u} = 4.2 Ma_u^2$$

The correct answer is 2.

Q3: Consider three flames: Flame A, Flame B, and Flame C, with Lewis number  $Le < 1$ ,  $Le = 1$ , and  $Le > 1$ , respectively. If the flame structure is being analyzed at the reaction-sheet level and  $l_Y$  and  $l_T$  denote the thicknesses of the flame based on mass-fraction  $Y$  of fuel and temperature  $T$ , then which of the following option is correct.

1. Flame A:  $l_Y > l_T$ ; Flame B:  $l_Y = l_T$ ; Flame C:  $l_Y < l_T$
2. Flame A:  $l_Y < l_T$ ; Flame B:  $l_Y = l_T$ ; Flame C:  $l_Y > l_T$
3. Flame A:  $l_Y = l_T$ ; Flame B:  $l_Y = l_T$ ; Flame C:  $l_Y = l_T$
4. Thickness cannot be estimated using given data

Ans: Lewis number  $Le = \frac{\alpha}{D}$  is a measure of thermal diffusivity to mass diffusivity. To estimate the thickness you can make use of simple scaling arguments, like,  $l_Y \sim \frac{D}{S_L}$  and  $l_T \sim \frac{\alpha}{S_L}$ . Here,  $S_L$  is the planar laminar flame speed. The characteristic thickness is directly proportional to the diffusivity. Therefore, for  $Le < 1 \Rightarrow \alpha < D$  and  $l_T < l_Y$ . Similarly, for  $Le = 1$ ,  $l_Y = l_T$  and for  $Le > 1$ , we have  $l_T > l_Y$ . Thus, the correct choice is 1.

Q4: Consider the transport equation of temperature for 1-D standard premixed flame as discussed in the lectures. Let PHZ be used to denote pre-heat zone and RZ be used to denote reaction zone. Which of the following conditions due to think holds correct?

1. In PHZ, convection and diffusion balance each other; in RZ diffusion and reaction balance each other
2. In PHZ, convection and reaction balance each other; in RZ diffusion and reaction balance each other
3. In PHZ, convection, diffusion, and reaction all are important; in RZ diffusion and reaction balance each other
4. In PHZ, convection and diffusion balance each other; in RZ all three are important

Ans: In PHZ, diffusion and convection are important and balance each other, while in RZ reaction and diffusion balance each other. Therefore, the correct choice is 1.

Q5: Following your result in question 4. and using scaling arguments, estimate the flame thickness for a standard premixed flame analyzed to the reaction-sheet level.

1.  $\frac{\lambda}{f^o c_P}$
2.  $\frac{\lambda}{(f^o)^2 c_P}$
3.  $\frac{f^o c_P}{\lambda}$
4.  $\frac{(f^o)^2 c_P}{\lambda}$

Ans: The 1-D transport equation for temperature in a standard premixed flame is as given below:

$$f^o c_p \frac{dT}{dx} - \lambda \frac{d^2 T}{dx^2} = q_c w$$

In the reaction-sheet level, only the PHZ is resolved and therefore, the PHZ thickness is the flame thickness. Since, in the PHZ the convection and the diffusion balance, therefore, for PHZ we can write the following

$$f^o c_p \frac{dT}{dx} = \lambda \frac{d^2 T}{dx^2}$$

Using, scaling arguments and let  $T_b$  and  $T_u$  represent the temperature on burnt and unburnt sides

$$f^o c_p \left( \frac{T_b - T_u}{l_F} \right) = \lambda \left( \frac{T_b - T_u}{l_F^2} \right)$$

$$l_F = \frac{\lambda}{f^o c_p}$$

The correct answer is 1.

Q6: Let  $u_x$  be the velocity along  $X$ . The dilatation rate in a standard premixed flame is given by:

1.  $-\frac{u_x}{T} \frac{dT}{dx}$
2.  $\frac{u_x}{T} \frac{dT}{dx}$
3.  $-u_x \rho \frac{d\rho}{dx}$
4.  $u_x \rho \frac{d\rho}{dx}$

Ans: The dilatation rate is given by  $\nabla \cdot \mathbf{u}$ . The standard premixed flame is steady and one-dimensional, therefore,  $\nabla \cdot \mathbf{u} = \frac{du_x}{dx}$  from continuity, we have:

$$\nabla \cdot (\rho u) = 0$$

$$\frac{d(\rho u_x)}{dx} = 0$$

$$\frac{du_x}{dx} = -\frac{u_x}{\rho} \frac{d\rho}{dx}$$

Using equation of state  $P = \rho RT$ , we get  $\frac{1}{\rho} \frac{d\rho}{dx} = -\frac{P}{RT^2} \frac{dT}{dx}$ . Here, we've made use of the isobaric (or low Mach no. assumption) in consonance with the assumption of the standard premixed flame. Therefore,

$$\frac{du_x}{dx} = -\frac{u_x}{P} RT \left( -\frac{P}{RT^2} \right) \frac{dT}{dx}$$

$$\frac{du_x}{dx} = \frac{u_x}{T} \frac{dT}{dx}$$

Thus, the correct answer is 2.

Q7: Which of the following best approximates the order of flame speed for detonation waves and deflagration waves, respectively?

1.  $\mathcal{O}(10^3)$ cm/s;  $\mathcal{O}(1 - 10^2)$ m/s
2.  $\mathcal{O}(10^3)$ cm/s;  $\mathcal{O}(1 - 10^2)$ cm/s
3.  $\mathcal{O}(10^3)$ m/s;  $\mathcal{O}(1 - 10^2)$ m/s
4.  $\mathcal{O}(10^3)$ m/s;  $\mathcal{O}(1 - 10^2)$ cm/s

Ans: The correct answer is 4.

Q8: Which of the following occur?

1. Strong deflagration and strong detonations
2. Strong deflagration and weak detonations
3. Weak deflagration and strong detonations
4. Weak deflagration and weak detonations

Ans: The correct answer is 3.