Exercise 1

Show that the Jacobian of the inverse transformation from polar to cartesian is $1/\rho = 1/\sqrt{x^2+y^2}$

Exercise 2

Evaluate $\int\int xy\,dx\,dy$ where the region of integration is the part of the area between circles of radii 1 and 2 that lies in the first quadrant.

(Ans. $15/8$)

Exercise 3

Evaluate the Gaussian integral $I = \int_0^\infty e^{-x^2} dx$

[Hint : The integration cannot be done using cartesian coordinates but is relatively easy using polar coordinates and properties of definite integrals. By changing the dummy variable x to y, one can write $I = \int_0^\infty e^{-y^2} dy$, so that we can write

$$I^2 = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\,dy$$

Transform this to polar. Range of integration for ρ is from 0 to ∞ and that of θ is from 0 to $\pi/2$ (why ?)]

[Answer : $\sqrt{\pi}/2$]

Exercise 4

Find the cylindrical coordinate of the point $3\hat{\imath} + 4\hat{\jmath} + \hat{k}$.

[Hint : Determine ρ and $\tan\theta$ using above transformation]

(Ans. $5\hat{\rho}+\tan^{-1}(4/3)\hat{\theta}+\hat{k}$)

Exercise 5

A particle moves along a spherical helix. its position coordinate at time t is given by

$$x=\frac{\cos t}{\sqrt{1+t^2}},\quad y=\frac{\sin t}{\sqrt{1+t^2}},\quad z=\frac{t}{\sqrt{1+t^2}}$$

Express the equation of the path in spherical coordinates.

(Ans. $r=1,\ \cos\theta=t/\sqrt{1+t^2}\ \ \phi(t)=t$)

Exercise 6

Using direct integration find the volume of the first octant bounded by a sphere $x^2+y^2+z^2=9$

Exercise 7

Using direct integration find the volume of the first octant bounded by a sphere $x^2+y^2+z^2=9$