

Lecture-8

6.3.9 Chebyshev Array Design

In this case, the main goal is to approximate the desired AF with a Chebyshev polynomial such that

- The side-lobe level meets the requirements, and
- The main beam width is as small as possible.

An array of N elements has an AF approximated with a Chebyshev polynomial of order m , which is always $m = N-1$.

where $N = 2M$, if N is even; and $N = 2M+1$, if N is odd.

Statement: Design a broadside Dolph-Tschebyscheff array of N elements with spacing d between the elements. The sidelobes are R_0 dB below the maximum of the major lobe. Find the excitation co-efficients and form the array factor.

Procedure:

1. Select the appropriate AF for the total number of elements N .

$$(AF)_{2M} = \sum_{n=1}^M a_n \cos\left[\frac{(2n-1)}{2} kd \cos \theta\right] \quad N = 2M$$

$$(AF)_{2M+1} = \sum_{n=1}^{M+1} a_n \cos[(n-1)kd \cos \theta] \quad N = 2M+1$$

2. Replace each $\cos(mu)$ term in the array factor by its appropriate expansion in terms of powers of $\cos(u)$.

3. Determine the point $z = z_0$ such that $T_m = R_0$ (Voltage ratio). Find z_0 such that

$$R_0 = T_{N-1}(z_0) = \cosh\left[(N-1)\cosh^{-1} z_0\right]$$

$$z_0 = \cosh\left[\frac{\cosh^{-1} R_0}{N-1}\right]$$

4. Substitute $\cos(u) = \frac{z}{z_0}$ in the array factor of step 2. This substitution normalizes the array factor sidelobes to a peak value of unity.

5. Equate the array factor of step 4 to $T_{N-1}(z)$ and determine the array coefficients.

Example-6: Design a Dolph–Chebyshev array (broadside) of $N = 10$ elements with a major-to-minor lobe ration of $R_0=26$ dB. Find the excitation coefficients.

Solution:

The order of the Chebyshev polynomial is $m = N-1 = 9$. The AF for an even-number array is:

$$AF_{2M} = \sum_{n=1}^5 a_n \cos[(2n-1)u]$$

$$u = \frac{\pi d}{\lambda} \cos \theta, \quad M = 5.$$

Step 1: The array factor for 10 element array :

$$AF_{10} = a_1 \cos u + a_2 \cos 3u + a_3 \cos 5u + a_4 \cos 7u + a_5 \cos 9u$$

Expand the $\cos(mu)$ terms of powers of $\cos u$:

$$\cos 3u = 4\cos^3 u - 3\cos u,$$

$$\cos 5u = 16\cos^5 u - 20\cos^3 u + 5\cos u,$$

$$\cos 7u = 64\cos^7 u - 112\cos^5 u + 56\cos^3 u - 7\cos u,$$

$$\cos 9u = 256\cos^9 u - 576\cos^7 u + 432\cos^5 u - 120\cos^3 u + 9\cos u$$

Step 2: Determine z_0 :

$$R_0 = 26 \text{ dB}$$

$$\Rightarrow R_0 = 10^{26/20} \approx 20$$

$$\Rightarrow T_9(z_0) = 20$$

$$\cosh[9\cosh^{-1}(z_0)] = 20,$$

$$z_0 = \cosh 0.41 \Rightarrow z_0 = 1.08515$$

Step 3: Express the AF from Step 1 in terms of $\cos u = z / z_0$

$$\begin{aligned} AF_{10} &= \frac{z}{z_0} (a_1 - 3a_2 + 5a_3 - 7a_4 + 9a_5) \\ &\quad + \frac{z^3}{z_0^3} (4a_2 - 20a_3 + 56a_4 - 120a_5) \\ &\quad + \frac{z^5}{z_0^5} (16a_3 - 112a_4 + 432a_5) \\ &\quad + \frac{z^7}{z_0^7} (64a_4 - 576a_5) \\ &\quad + \frac{z^9}{z_0^9} (256a_5) = \\ &= \underbrace{9z - 120z^3 + 432z^5 - 576z^7 + 256z^9}_{T_9(z)} \end{aligned}$$

Step 4: Finding the coefficients by matching the power terms:

$$256a_5 = 256z_0^9 \Rightarrow a_5 = 2.0860$$

$$64a_4 - 576a_5 = -576z_0^7 \Rightarrow a_4 = 2.8308$$

$$16a_3 - 112a_4 + 432a_5 = 432z_0^5 \Rightarrow a_3 = 4.1184$$

$$4a_2 - 20a_3 + 56a_4 - 120a_5 = -120z_0^7 \Rightarrow a_2 = 5.2073$$

$$a_1 - 3a_2 + 5a_3 - 7a_4 + 9a_5 = 9z_0^9 \Rightarrow a_1 = 5.8377$$

Normalize coefficients with respect to edge element ($N = 5$):

$$a_5 = 1; \quad a_4 = 1.357; \quad a_3 = 1.974; \quad a_2 = 2.496; \quad a_1 = 2.789$$