Integrated Circuit Operational Amplifiers

Analog Integrated Circuit Design
A video course under the NPTEL

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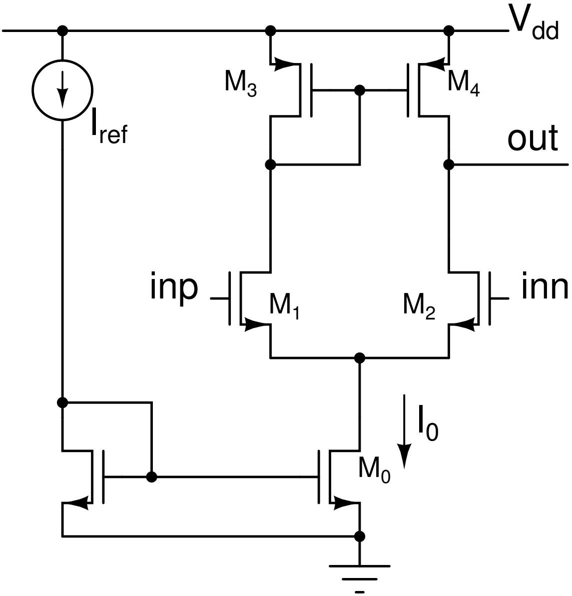
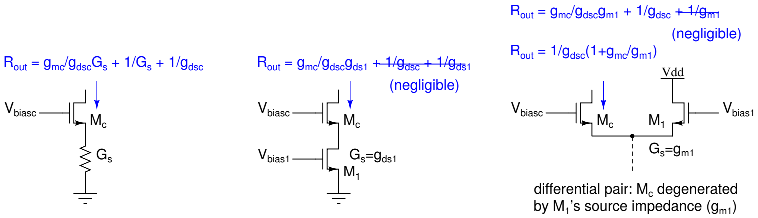
National Programme on Technology Enhanced Learning

Differential pair opamp



Cascode output resistance


$R_{out} = g_{mc}/g_{dsc}G_s + 1/G_s + 1/g_{dsc}$

$R_{out} = g_{mc}/g_{dsc}g_{ds1}$ $\cancel{+ 1/g_{dsc} + 1/g_{ds1}}$ (negligible)

$R_{out} = g_{mc}/g_{dsc}g_{m1} + 1/g_{dsc}$ $\cancel{+ 1/g_{m1}}$ (negligible)

$R_{out} = 1/g_{dsc}(1+g_{mc}/g_{m1})$

differential pair: M_c degenerated by M_1's source impedance (g_{m1})

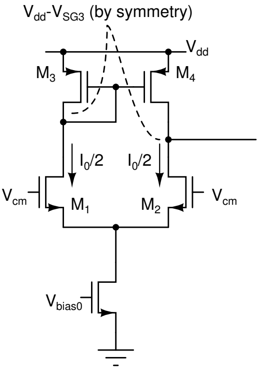
- Output resistance looking into one side of the differential pair is $2/g_{ds1}$ ($g_{m1} = g_{mc}$ in the figure)

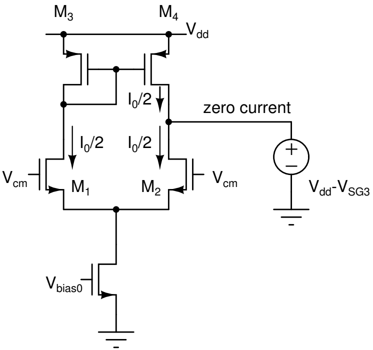
Opamp: dc small signal analysis

- Bias values in black
- Incremental values in red
- Impedances in blue

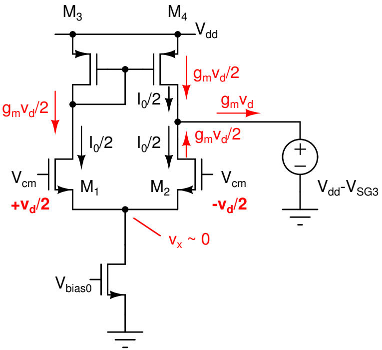
Total quantity = Bias + increment

Differential pair: Quiescent condition

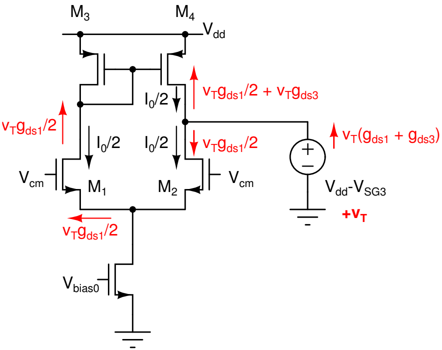




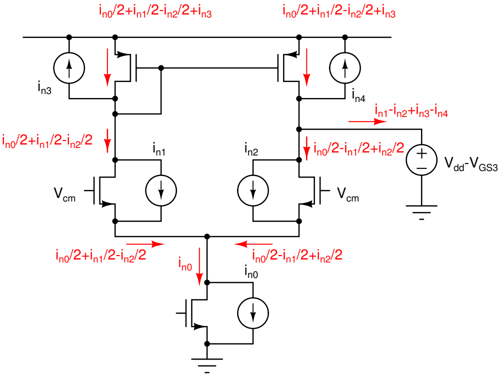
Differential pair: Transconductance



Differential pair: Output conductance



Differential pair: Noise

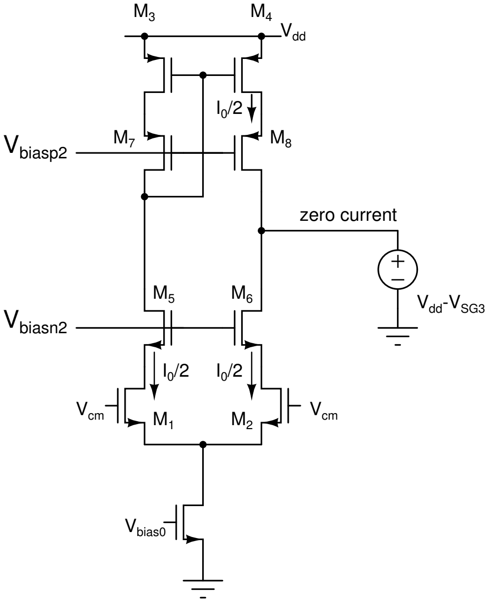


- Carry out small signal linear analysis with one noise source at a time
- Add up the results at the output (current in this case)
- Add up corresponding spectral densities
- Divide by gain squared to get input referred noise

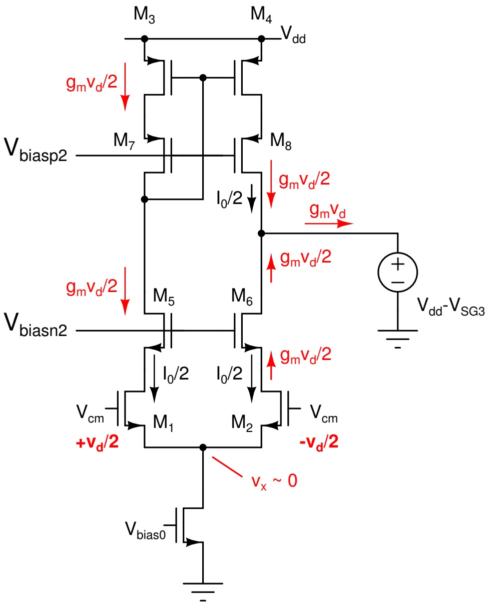
Differential pair opamp

G_m	g_{m1}
G_{out}	$g_{ds1} + g_{ds3}$
A_o	$g_{m1}/(g_{ds1} + g_{ds3})$
A_{cm}	$g_{ds0}/2g_{m3}$
C_i	$C_{gs1}/2$
ω_u	g_{m1}/C_L
p_k, z_k	$p_2 = -g_{m3}/(C_{db1} + C_{db3} + 2C_{gs3});\ z_1 = 2p_2$
S_{vi}	$16kT/3g_{m1}\,(1 + g_{m3}/g_{m1})$
σ^2_{Vos}	$\sigma^2_{VT1} + (g_{m3}/g_{m1})^2\sigma^2_{VT3}$
V_{cm}	$\geq V_{T1} + V_{DSAT1} + V_{DSAT0}$ $\leq V_{dd} - V_{DSAT3} - V_{T3} + V_{T1}$
V_{out}	$\geq V_{cm} - V_{T1}$ $\leq V_{dd} - V_{DSAT3}$
SR	$\pm I_0/C_L$
I_{supply}	$I_0 + I_{ref}$

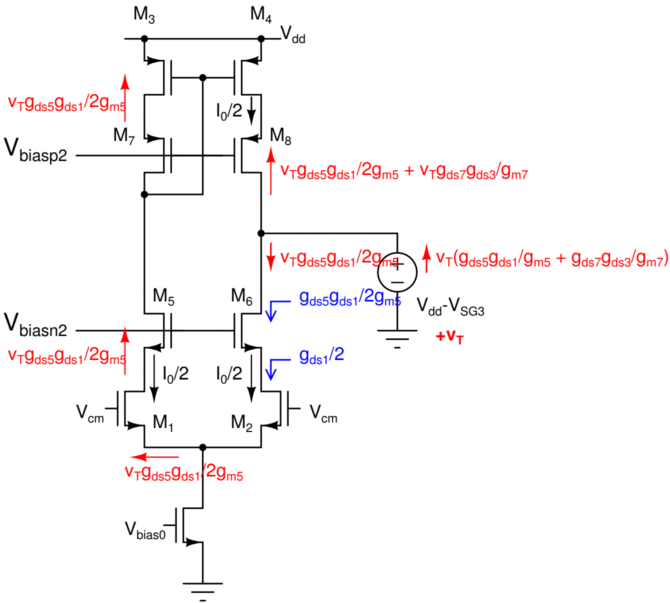
Telescopic cascode: Quiescent condition



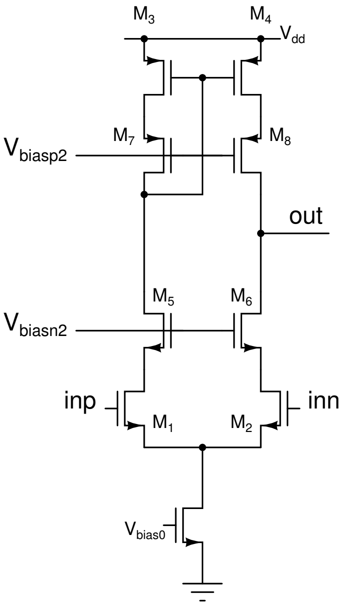
Telescopic cascode: Transconductance



Telescopic cascode: Output conductance



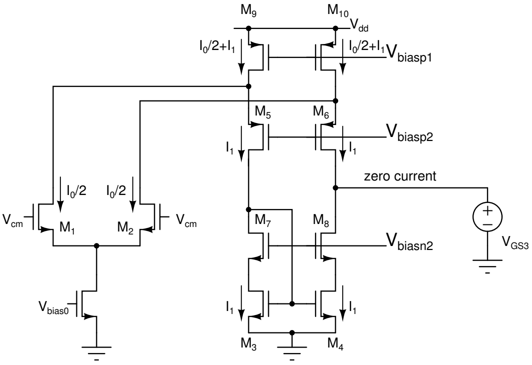
Telescopic cascode opamp



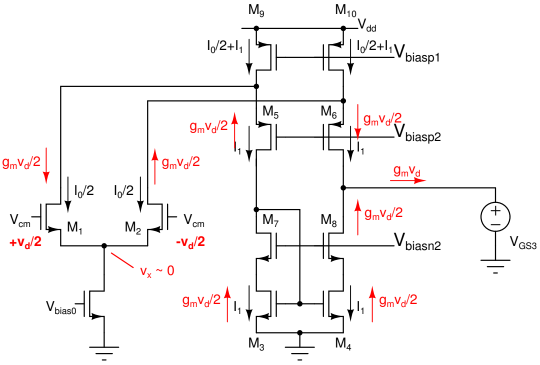
Telescopic cascode opamp

G_m	g_{m1}
G_{out}	$g_{ds1}g_{ds5}/g_{m5} + g_{ds3}g_{ds7}/g_{m7}$
A_o	$g_{m1}/(g_{ds1}g_{ds5}/g_{m5} + g_{ds3}g_{ds7}/g_{m7})$
A_{cm}	$g_{ds0}/2g_{m3}$
C_i	$C_{gs1}/2$
ω_u	g_{m1}/C_L
p_k, z_k	$p_2 = -g_{m3}/(C_{db1} + C_{db3} + 2C_{gs3})$ $p_3 = -g_{m5}/C_{p5}$ $p_4 = -g_{m7}/C_{p7}$ $p_{2,4}$ appear for one half and cause mirrror zeros
S_{vi}	$16kT/3g_{m1}\left(1 + g_{m3}/g_{m1}\right)$
σ^2_{Vos}	$\sigma^2_{VT1} + (g_{m3}/g_{m1})^2\sigma^2_{VT3}$
V_{out}	$\geq V_{biasn1} - V_{T5}$ $\leq V_{biasp1} + V_{T7}$
SR	$\pm I_0/C_L$
I_{supply}	$I_0 + I_{ref}$

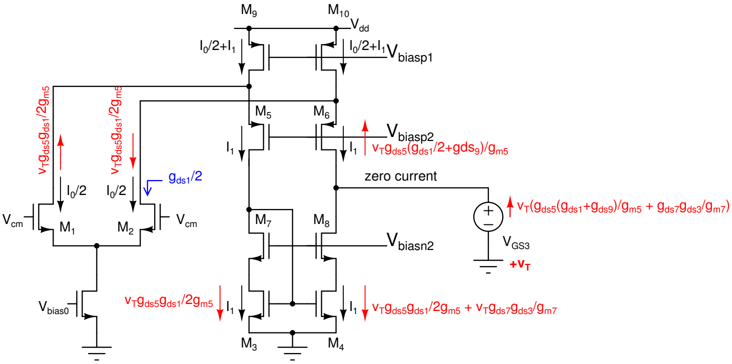
Folded cascode: Quiescent condition



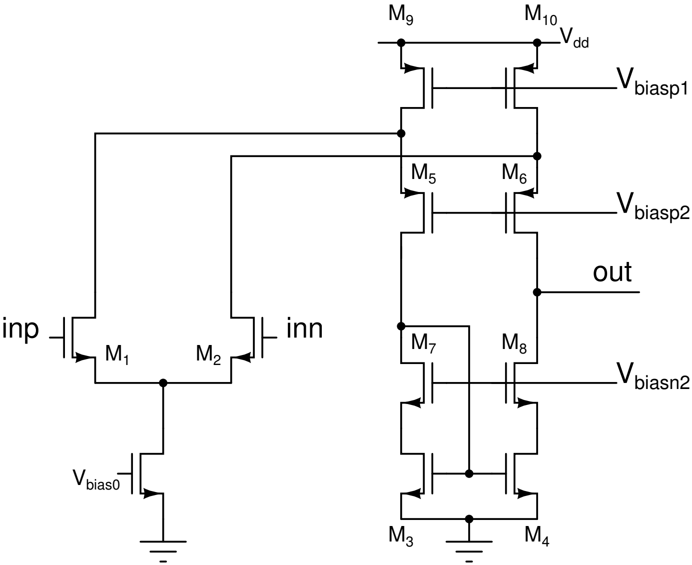
Folded cascode: Transconductance



Folded cascode: Output conductance



Folded cascode opamp



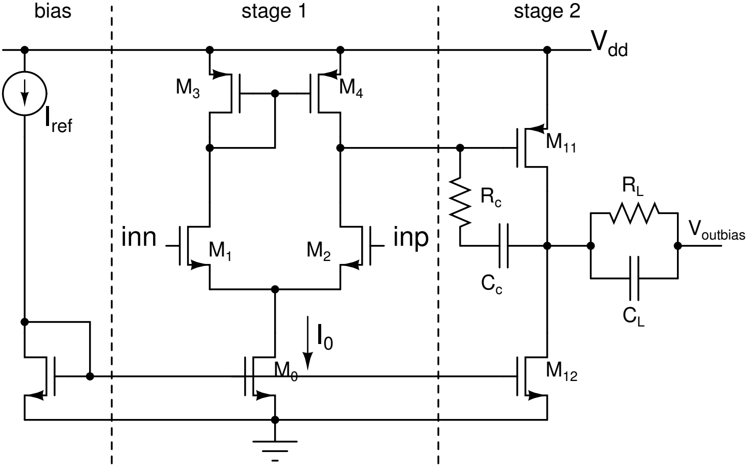
Folded cascode opamp

G_m	g_{m1}
G_{out}	$(g_{ds1} + g_{ds9})g_{ds5}/g_{m5} + g_{ds3}g_{ds7}/g_{m7}$
A_o	$g_{m1}/((g_{ds1} + g_{ds9})g_{ds5}/g_{m5} + g_{ds3}g_{ds7}/g_{m7})$
A_{cm}	$g_{ds0}/2g_{m3}$
C_i	$C_{gs1}/2$
ω_u	g_{m1}/C_L
p_k, z_k	$p_2 = -g_{m3}/(C_{db1} + C_{db3} + 2C_{gs3})$ $p_3 = -g_{m5}/C_{p5}$ $p_4 = -g_{m7}/C_{p7}$ $p_{2,4}$ appear for one half and cause mirrror zeros
S_{vi}	$16kT/3g_{m1}\,(1 + g_{m3}/g_{m1} + g_{m9}/g_{m1})$
σ^2_{Vos}	$\sigma^2_{VT1} + (g_{m3}/g_{m1})^2\sigma^2_{VT3} + (g_{m9}/g_{m1})^2\sigma^2_{VT9}$
V_{out}	$\geq V_{biasn1} - V_{T5}$ $\leq V_{biasp1} + V_{T7}$
SR	$\pm\min\{I_0, I_1\}/C_L$
I_{supply}	$I_0 + I_1 + I_{ref}$

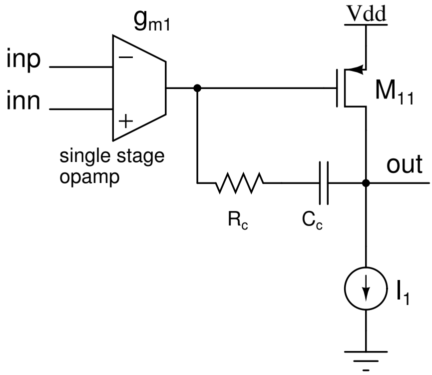
Body effect

- All nMOS bulk terminals to ground
- All pMOS bulk terminals to V_{dd}
- A_{cm} has an additional factor $g_{m1}/(g_{m1} + g_{mb1})$
- $g_{m5} + g_{mb5}$ instead of g_{m5} in cascode opamp results
- $g_{m7} + g_{mb7}$ instead of g_{m7} in cascode opamp results

Two stage opamp



Two stage opamp



- First stage can be Differential pair, Telescopic cascode, or Folded cascode; Ideal g_{m1} assumed in the analysis
- Second stage: Common source amplifier
- Frequency response is the product of frequency responses of the first stage g_m and a common source amplifier driven from a current source

Common source amplifier: Frequency response

$$\frac{V_o(s)}{V_d(s)} = \left(\frac{g_{m1}g_{m11}}{G_1 G_L}\right)\frac{sC_c(R_c - 1/g_{m11}) + 1}{a_3 s^3 + a_2 s^2 + a_1 s + 1}$$

$$a_3 = \frac{R_c C_1 C_L C_c}{G_1 G_L}$$

$$a_2 = \frac{C_1 C_c + C_c C_L + C_L C_1 + R_c C_c(G_1 C_L + C_1 G_L)}{G_1 G_L}$$

$$a_1 = \frac{C_c(g_{m11} + G_1 + G_L + G_1 G_L R_c) + C_1 G_L + G_1 C_L}{G_1 G_L}$$

- G_1: Total conductive load at the input
- G_L: Total conductive load at the output
- C_1: Total capacitive load at the input
- C_L: Total capacitive load at the output

Common source amplifier: Poles and zeros

$$
\begin{aligned}
p_1 &\approx -\frac{G_1}{C_c(\frac{g_{m11}}{G_L}+1+\frac{G_1}{G_L}+G_1R_c)+C_1(1+\frac{G_1}{G_L})}\\
p_2 &\approx -\frac{g_{m11}\frac{C_c}{C_1+C_c}+G_L+G_1\frac{C_c+C_L}{C_1+C_c}+G1G_LR_c\frac{C_c}{C_1+C_c}}{\frac{C_1C_c}{C_1+C_c}+C_L+\frac{R_cC_c(G_1C_L+G_LC_1)}{C_c+C_L}}\\
p_3 &\approx -\left(\frac{1}{R_c}\left(\frac{1}{C_L}+\frac{1}{C_c}+\frac{1}{C_1}\right)+\frac{G_1}{C_1}+\frac{G_L}{C_L}\right)\\
z_1 &= \frac{1}{(1/g_{m11}-R_c)C_c}
\end{aligned}
$$

Unity gain frequency

$$
\omega_u \approx \frac{g_{m1}}{C_c\left(1+\frac{G_L}{g_{m11}}+\frac{G_1}{g_{m11}}+\frac{G_1G_LR_c}{g_{m11}}\right)+C_1\left(\frac{G_L}{g_{m11}}+\frac{G_1}{g_{m11}}\right)}
$$

Common source amplifier: Frequency response

- Pole splitting using compensation capacitor C_c
  - p_1 moves to a lower frequency
  - p_2 moves to a higher frequency (For large C_c, $p_2 = g_{m11}/C_L$)
- Zero cancelling resistor R_c moves z_1 towards the left half s plane and results in a third pole p_3
  - z_1 can be moved to ∞ with $R_c = 1/g_{m11}$
  - z_1 can be moved to cancel p_2 with $R_c > 1/g_{m11}$ (needs to be verified against process variations)
  - Third pole p_3 at a high frequency
- Poles and zeros from the first stage will appear in the frequency response—$Y_{m1}(s)$ instead of g_{m1} in V_o/V_i above
  - Mirror pole and zero
  - Poles due to cascode amplifiers

Compensation cap sizing

$$
\begin{aligned}
p_2 &\approx -\frac{g_{m11}\frac{C_c}{C_1+C_c}}{\frac{C_1C_c}{C_1+C_C}+C_L}\\
\omega_u &\approx \frac{g_{m1}}{C_c}
\end{aligned}
$$

Phase margin (Ignoring $p_3, z_1, \ldots$)

$$
\begin{aligned}
\phi_M &= \tan^{-1}\frac{|p_2|}{\omega_u}\\
\frac{|p_2|}{\omega_u} &= \tan\phi_M\\
\frac{g_{m11}}{g_{m1}}\left(\frac{C_c}{C_L}\right)^2 &= \frac{C_c}{C_L}\left(1+\frac{C_1}{C_L}\right)\tan\phi_M+\frac{C_1}{C_L}\tan\phi_M
\end{aligned}
$$

- For a given ϕ_M, solve the quadratic to obtain C_c/C_L
- If C_1 is very small, $p_2 \approx -g_{m2}/C_L$; further simplifies calculations

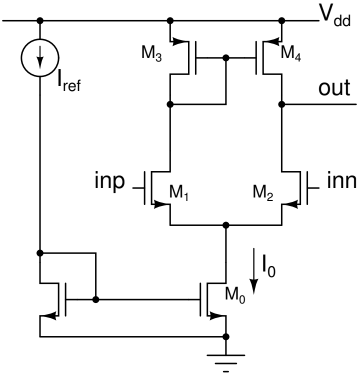
Two stage opamp

A_o	$g_{m1}g_{m11}/(g_{ds1}+g_{ds3})(g_{ds11}+g_{ds12})$
A_{cm}	$g_{ds0}g_{m11}/2g_{m3}(g_{ds11}+g_{ds12})$
C_i	$C_{gs1}/2$
ω_u	g_{m1}/C_c
p_k, z_k	See previous pages
S_{vi}	$\approx 16kT/3g_{m1}\left(1+g_{m3}/g_{m1}\right)$
σ^2_{Vos}	$\approx \sigma^2_{VT1}+(g_{m3}/g_{m1})^2\sigma^2_{VT3}$
V_{cm}	$\geq V_{T1}+V_{DSAT1}+V_{DSAT0}$ $\leq V_{dd}-V_{DSAT3}-V_{T3}+V_{T1}$
V_{out}	$\geq V_{DSAT12}$ $\leq V_{dd}-V_{DSAT11}$
SR+	I_0/C_c
SR-	$\min\{I_0/C_c, I_1/(C_L+C_c)\}$
I_{supply}	$I_0+I_1+I_{ref}$

Opamp comparison

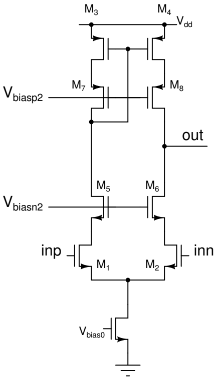
	Differential	Telescopic	Folded	Two
	pair	cascode	cascode	stage
Gain	$-$	$++$	$+$	$++$
Noise	$=$	$=$	high	$=$
Offset	$=$	$=$	high	$=$
Swing	$-$	$-$	$+$	$++$
Speed	$++$	$+$	$-$	$+$

Differential pair



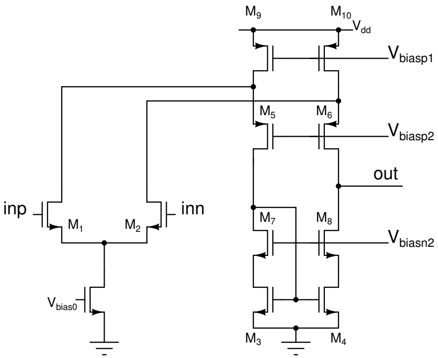
- Low accuracy (low gain) applications
- Voltage follower (capacitive load)
- Voltage follower with source follower (resistive load)
- In bias stabilization loops (effectively two stages in feedback)

Telescopic cascode



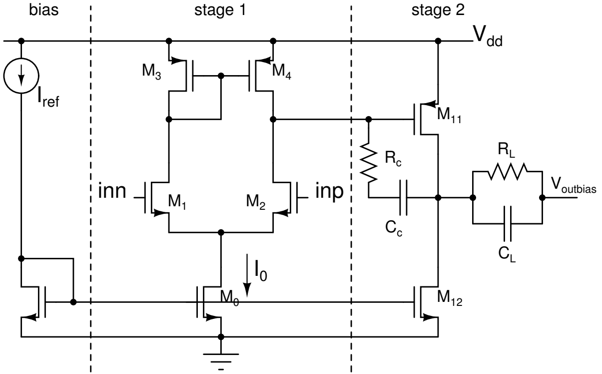
- Low swing circuits
- Switched capacitor circuits
  - Capacitive load
  - Different input and output common mode voltages
- First stage of a two stage opamp
  - Only way to get high gain in fine line processes

Folded cascode



- Higher swing circuits
- Higher noise and offset
- Lower speed than telescopic cascode
  - Low frequency pole at the drain of the input pair
- Switched capacitor circuits (Capacitive load)
- First stage of a two stage class AB opamp

Two stage opamp

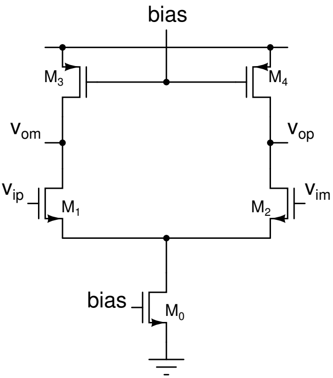


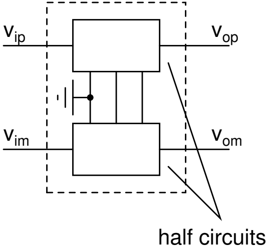
- Highest possible swing
- Resistive loads
- Capacitive loads at high speed
- “Standard” opamp: Miller compensated two stage opamp
- Class AB opamp: Always two (or more) stages

Opamps: pMOS versus nMOS input stage

- nMOS input stage
    - Higher g_m for the same current
    - Suitable for large bandwidths
    - Higher flicker noise (usually)
- pMOS input stage
    - Lower g_m for the same current
    - Lower flicker noise (usually)
    - Suitable for low noise low frequency applications

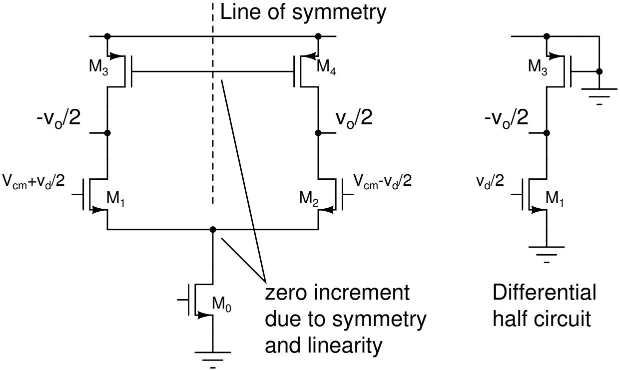
Fully differential circuits





- Two identical half circuits with some common nodes
- Two arms of the differential input applied to each half
- Two arms of the differential output taken from each half

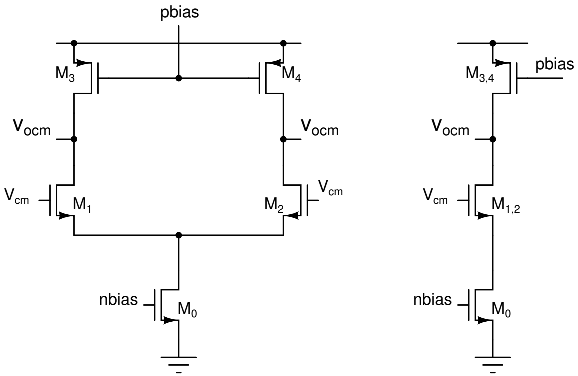
Differential half circuit



Symmetrical linear (or small signal linear) circuit under fully differential (antisymmmetric) excitation

- Nodes along the line of symmetry at 0 V (symmetry, linearity)
- Analyze only the half circuit to find the transfer function

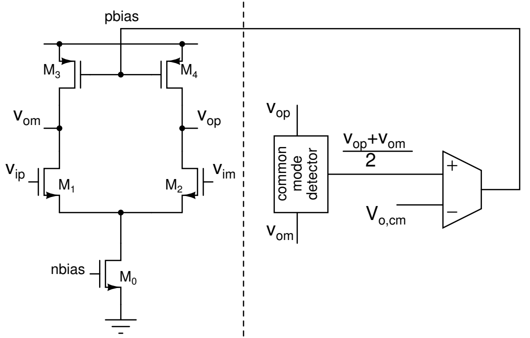
Common mode half circuit



Symmetrical circuit (maybe nonlinear) under common mode (symmmetric) excitation

- Nodes in each half at identical voltages (symmetry)
- Fold over the circuit and analyze the half circuit

Common mode feedback

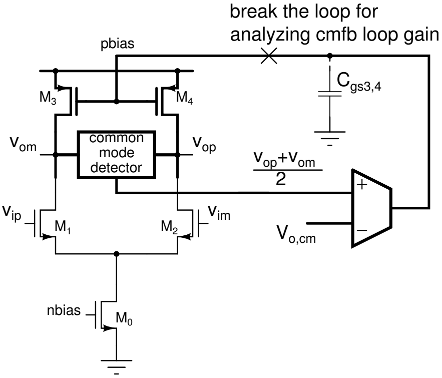


Fully differential opamp

Common mode feedback circuit

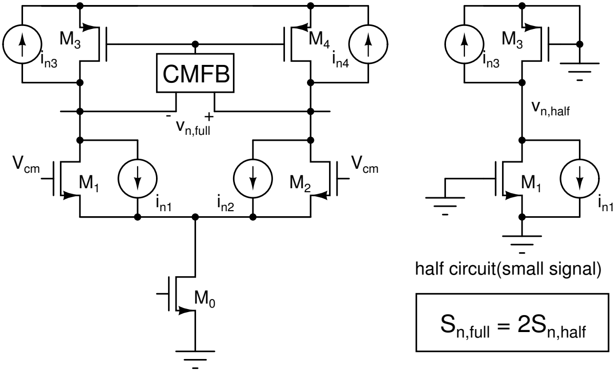
- Common mode feedback circuit for setting the bias
- Detect the output common mode and force it to be $V_{o,cm}$ via feedback

Common mode feedback loop



- Common mode feedback loop has to be stable
- Analyze it by breaking the loop and computing the loop gain with appropriate loading at the broken point
- Apply a common mode step/pulse in closed loop and ensure stability

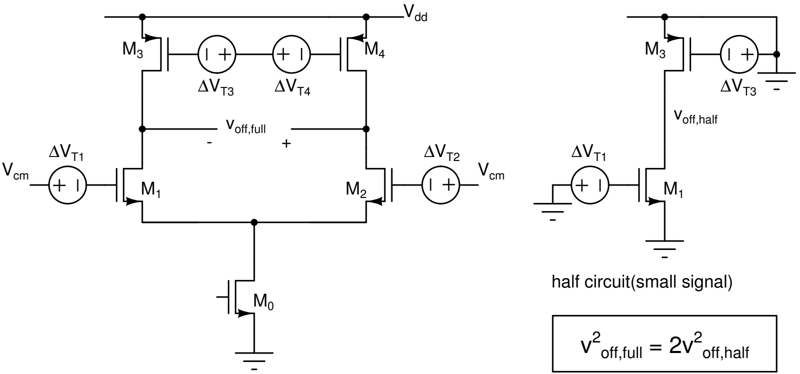
Fully differential circuits: Noise



$$\boxed{S_{n,full} = 2S_{n,half}}$$

- Calculate noise spectral density of the half circuit
- Multiply by 2×

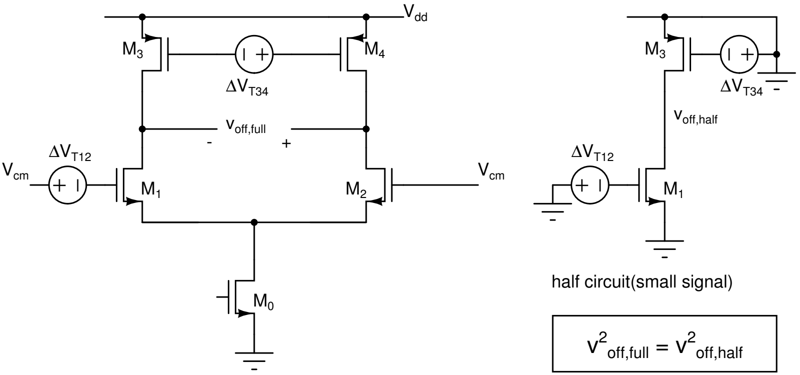
Fully differential circuits: Offset



$$v^2_{off,full} = 2v^2_{off,half}$$

- Calculate mean squared offset of the half circuit
- Multiply by 2× if mismatch (e.g. ΔV_T) wrt ideal device is used

Fully differential circuits: Offset



$$\boxed{v^2_{off,full} = v^2_{off,half}}$$

- Calculate mean squared offset of the half circuit
- Multiply by 1 × if mismatch between two real devices is used