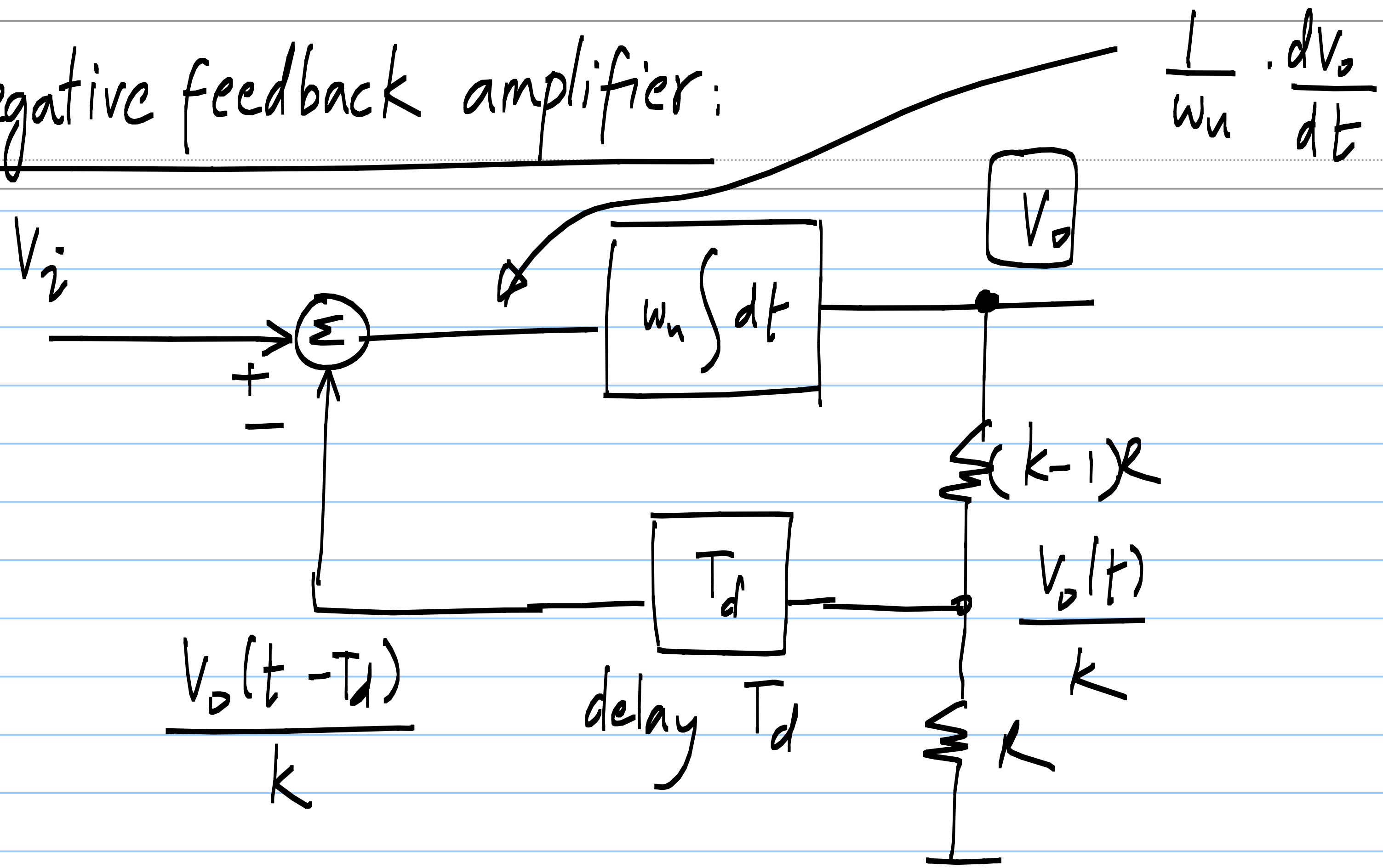
Note Title 12/24/2010

Negative feedback system with delay

- Feedback is delayed
- Comparison is with the actual output some time ago
- Don't know if actual output has reached the target → "overshoot" (go past the target)

→ go below the target → "ringing"

Negative feedback amplifier:



$$\frac{1}{\omega_u} \cdot \frac{dV_o(t)}{dt} = V_i(t) - \frac{V_o(t-T_d)}{k}$$

Note Title 1/18/2011

$$\frac{1}{w_u} \cdot \frac{dV_o}{dt} = V_i(t) - \frac{V_o(t-T_d)}{k}$$

Assume initially $(t<0)$

$V_i = 1V, \quad V_o = k \cdot V \Rightarrow$ steady state

@ $t=0 \quad V_i \longrightarrow 0$

$$\boxed{\frac{1}{w_u} \cdot \frac{dV_o}{dt} = -\frac{V_o(t-T_d)}{k}}$$

Note Title 1/18/2011

$$\frac{1}{\omega_u} \cdot \underbrace{\frac{dV_o}{dt}} = -\frac{V_o(t-T_d)}{k}$$

Assume an exponential form : $V_o(t) = V_p \exp(\sigma t)$

$$\frac{1}{\omega_u} \cdot \underbrace{\cancel{V_p} \cdot \sigma \cancel{\exp(\sigma t)}} = -\frac{V_p \exp(\sigma(t-T_d))}{k}$$

$$= -\frac{\cancel{V_p} \cancel{\exp(\sigma t)} \exp(-\sigma T_d)}{k}$$

$$\boxed{\frac{\sigma}{\omega_u} = -\frac{\exp(-\sigma T_d)}{k}}$$

Note Title 1/18/2011

$$\frac{\sigma}{\omega_u} = -\frac{\exp(-\sigma T_d)}{k}$$

$$\frac{\sigma}{\omega_u/k} + \exp(-\sigma T_d) = 0$$

$$\left(\frac{\sigma}{\omega_u/k}\right) \cdot \left(T_d \cdot \frac{\omega_u}{k}\right)$$

$$\sigma' = \frac{\sigma}{\omega_u/k}$$

$$T = \frac{T_d}{k/\omega_u}$$

$$\boxed{\sigma' + \exp(-\sigma' T) = 0}$$

Note Title 1/18/2011

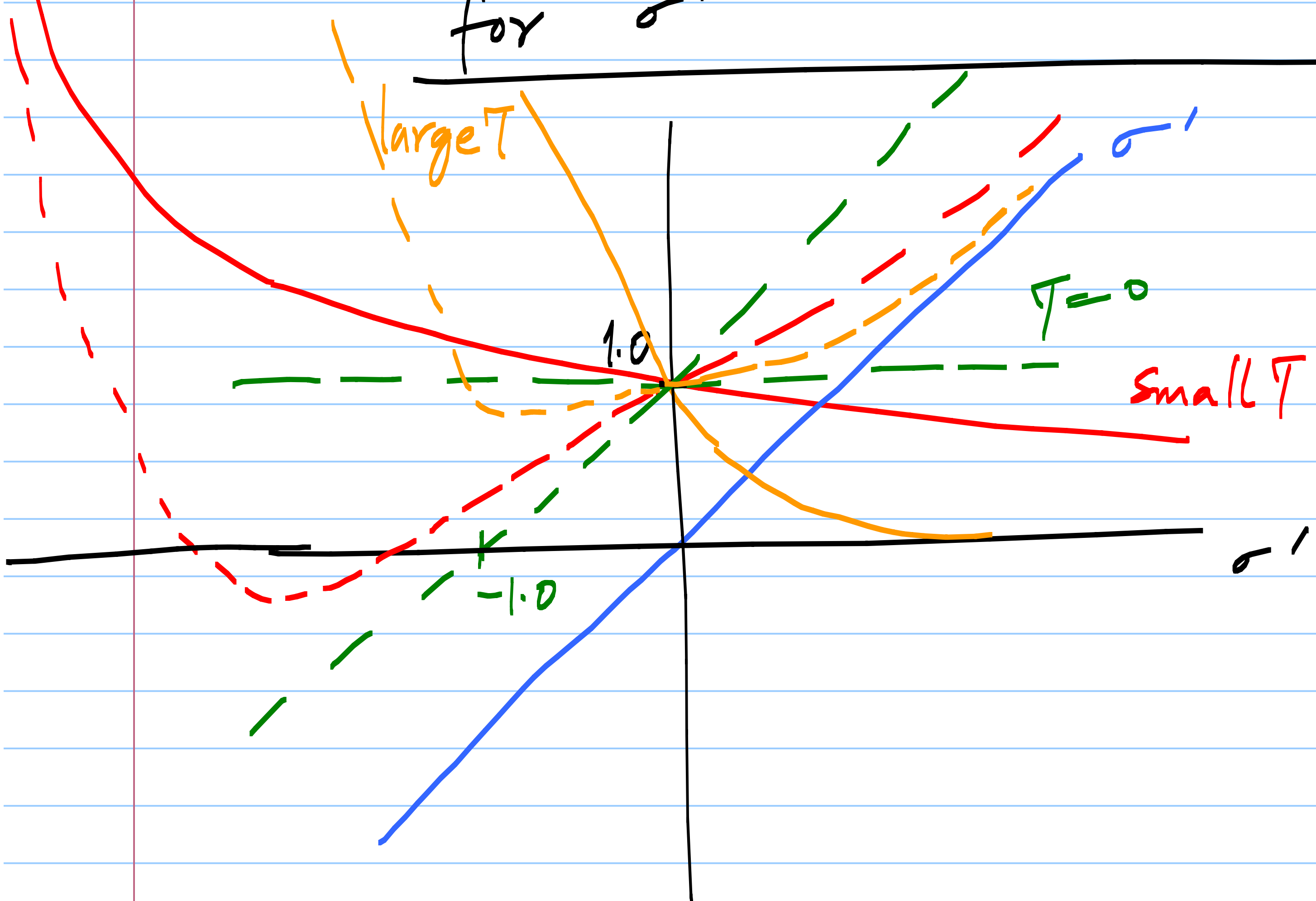
Solve $f(\sigma') = \sigma' + \exp(\sigma' T) = 0$

for σ'

$\sigma' = -1$

$\sigma = -\frac{\omega_n}{k}$

for $T_d = 0$



Note Title 1/18/2011

Negative feedback with delay: $V_p \exp(\sigma t)$

$$\sigma' + \exp(\sigma' T) = 0$$

$$\sigma' = \frac{\sigma}{w_u/k}$$

$$T = \frac{T_d}{k/w_u}$$

* Has two solutions for small T

  $\sigma_1', \sigma_2' \Rightarrow \sigma_1 \;\&\; \sigma_2$

* Has no solutions for large T

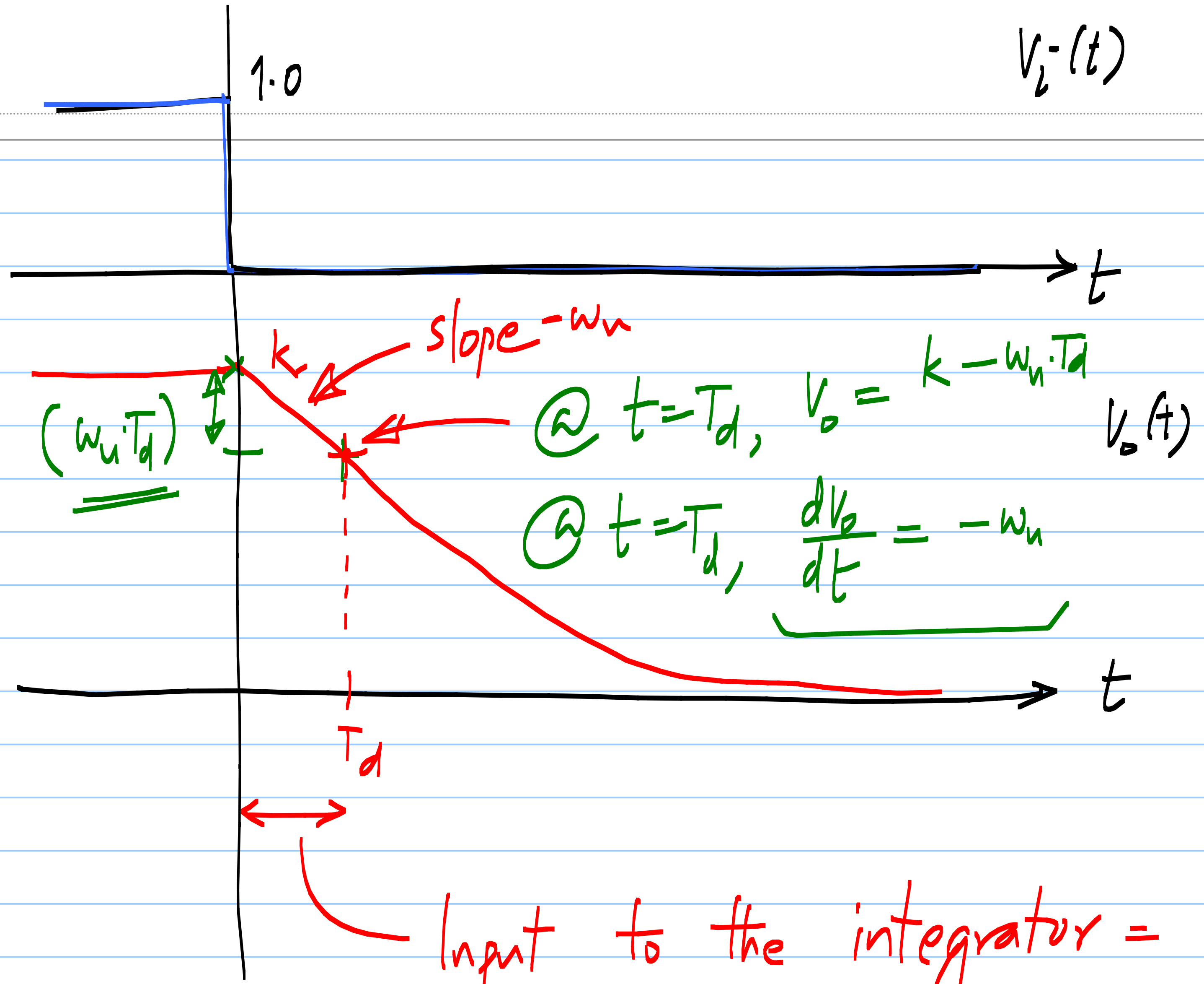
Two solutions σ_1, σ_2

Note Title 1/18/2011

$\Rightarrow$ $\exp(\sigma_1 t)$, $\exp(\sigma_2 t)$ are solutions

to $\underbrace{\frac{1}{w_n}\frac{dV_o}{dt} = -\frac{V_o(t-T_d)}{k}}$

<u>Complete solution:</u> $\underbrace{V_o(t) = A_1 \cdot \exp(\sigma_1 t) + A_2 \cdot \exp(\sigma_2 t)}$



@ $t = T_d$, $V_o = k - \omega_u \cdot T_d$

@ $t = T_d$, $\underbrace{\frac{dV_o}{dt} = -\omega_u}$

Input to the integrator = −1V

Note Title

Solution $V_o(t) = A_1 \exp(\sigma_1 t) + A_2 \exp(\sigma_2 t)$

1/18/2011

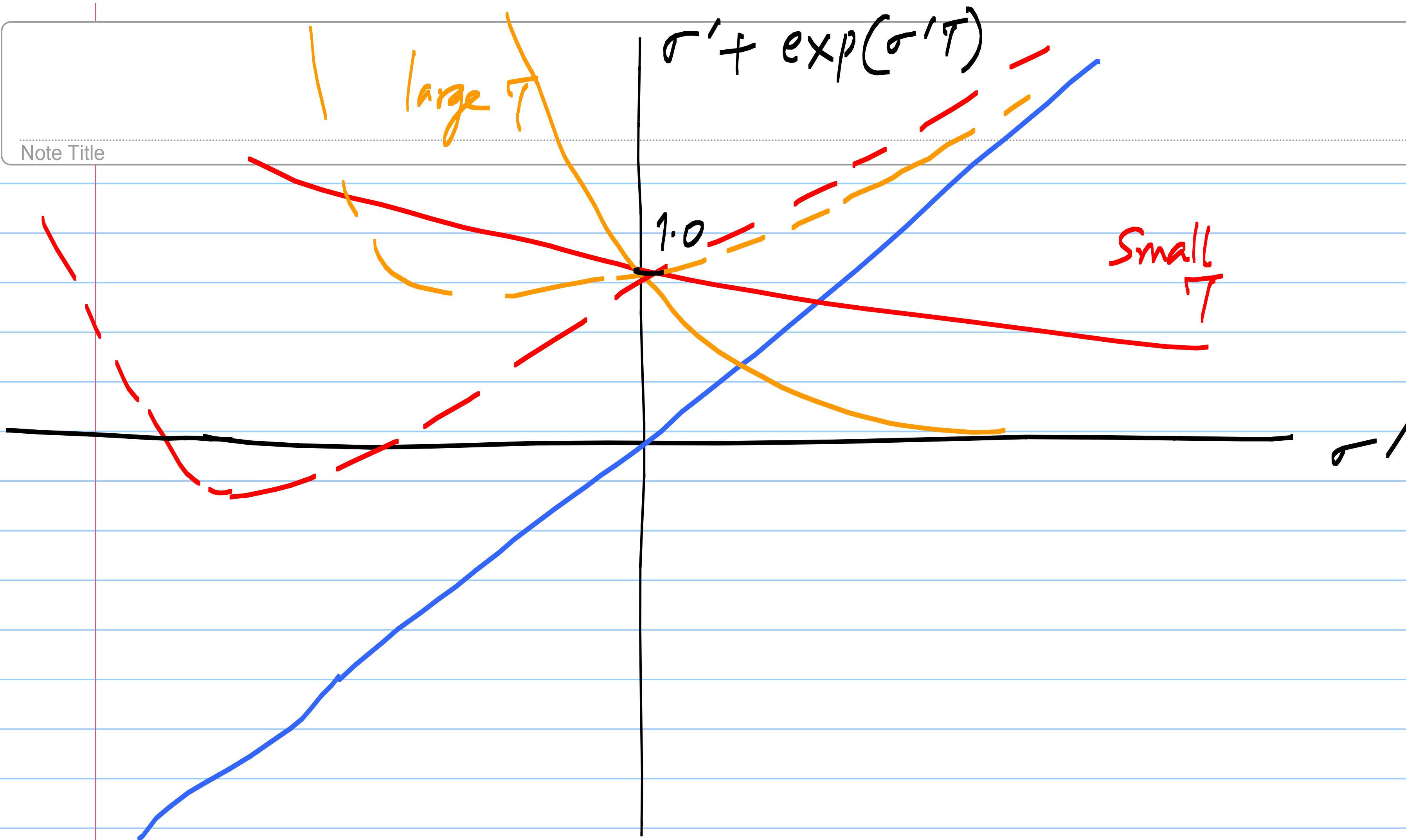
$$V_o(T_d) = k - \omega_u \cdot T_d$$

$$= k\left(1 - \frac{T_d}{\omega_u/k}\right)$$

$$= \underline{k\,(1 - T)}$$

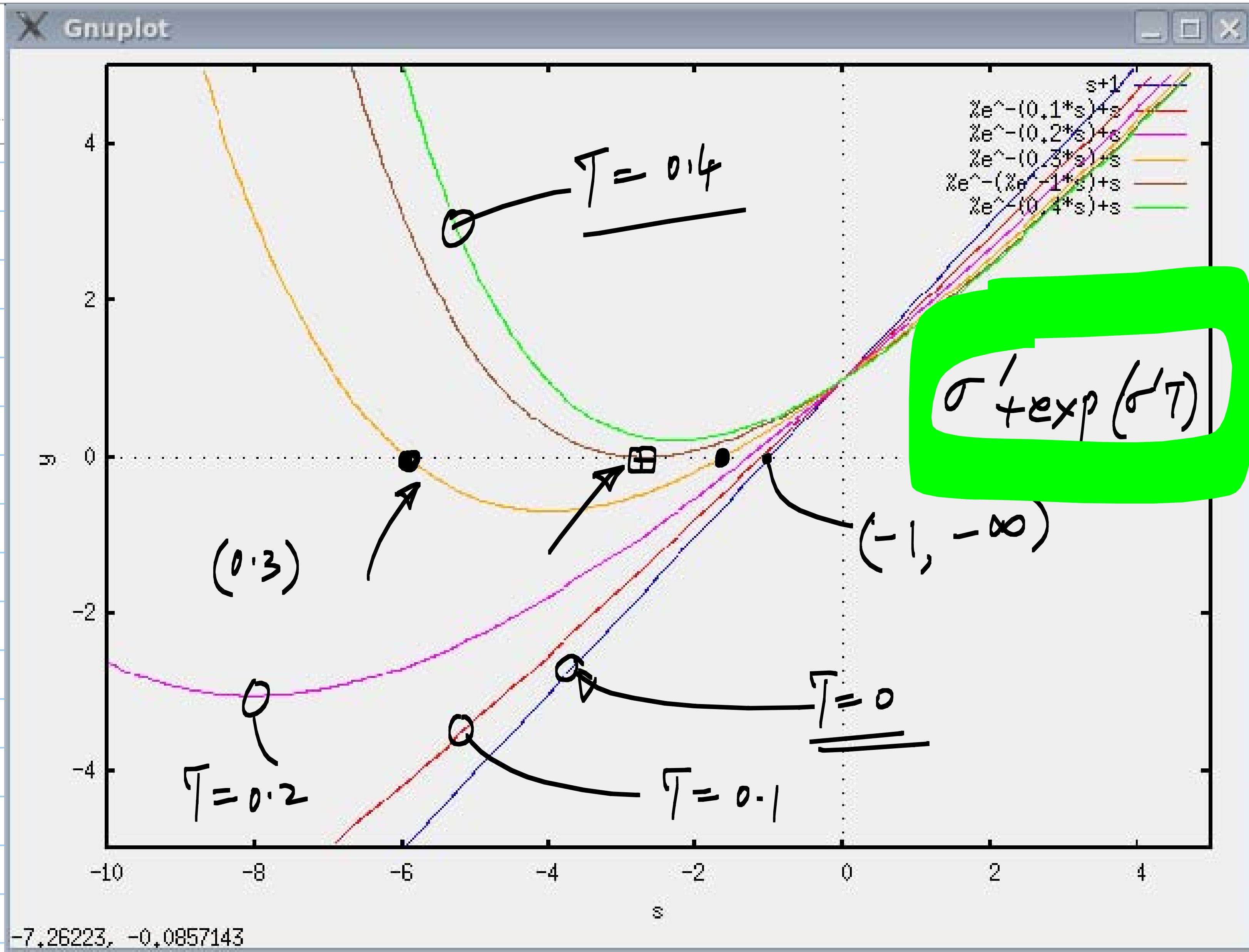
$$\underbrace{\left.\frac{dV_o}{dt}\right|_{t=T_d} = -\omega_u}$$

Note Title 1/18/2011



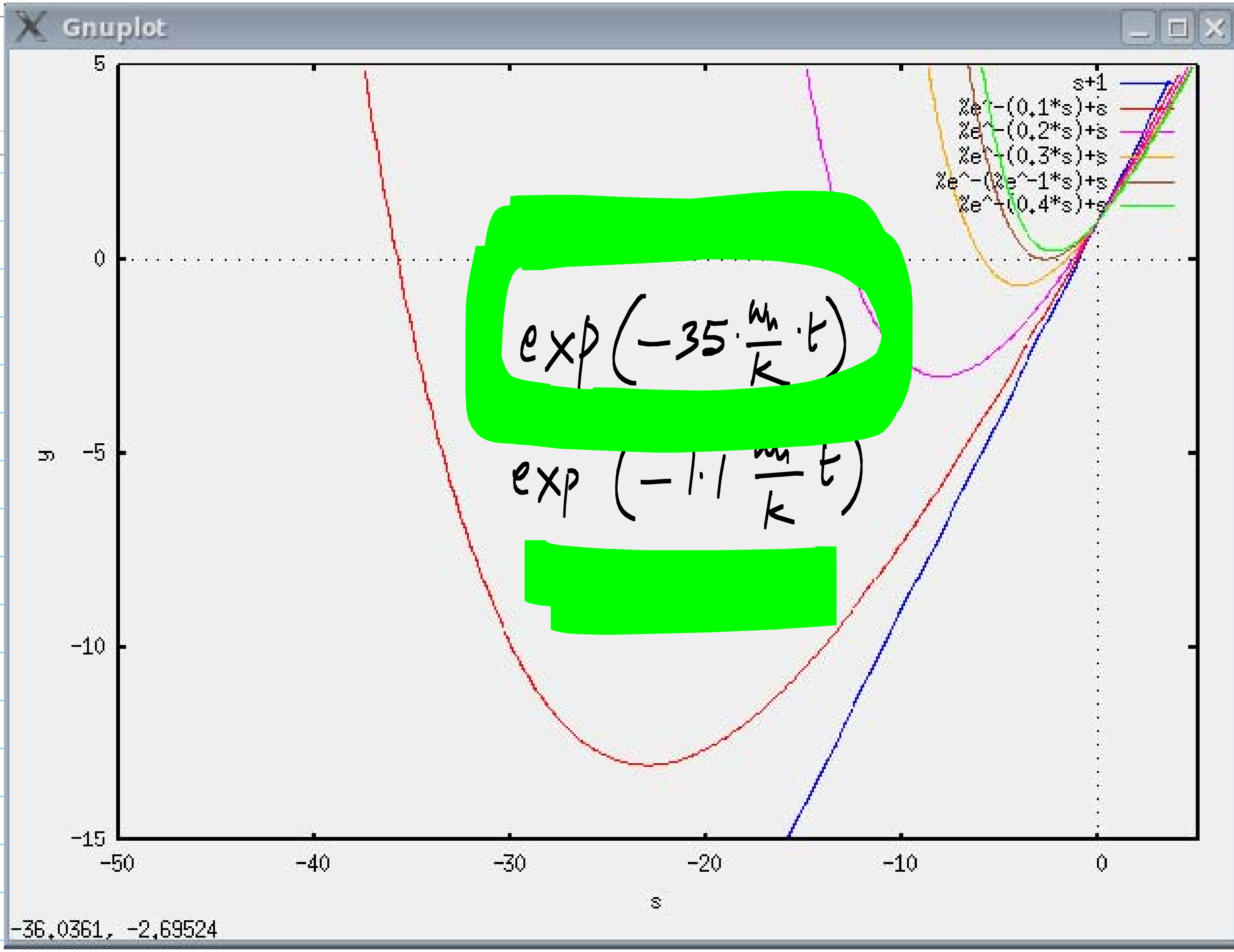
Note Title

1/18/2011



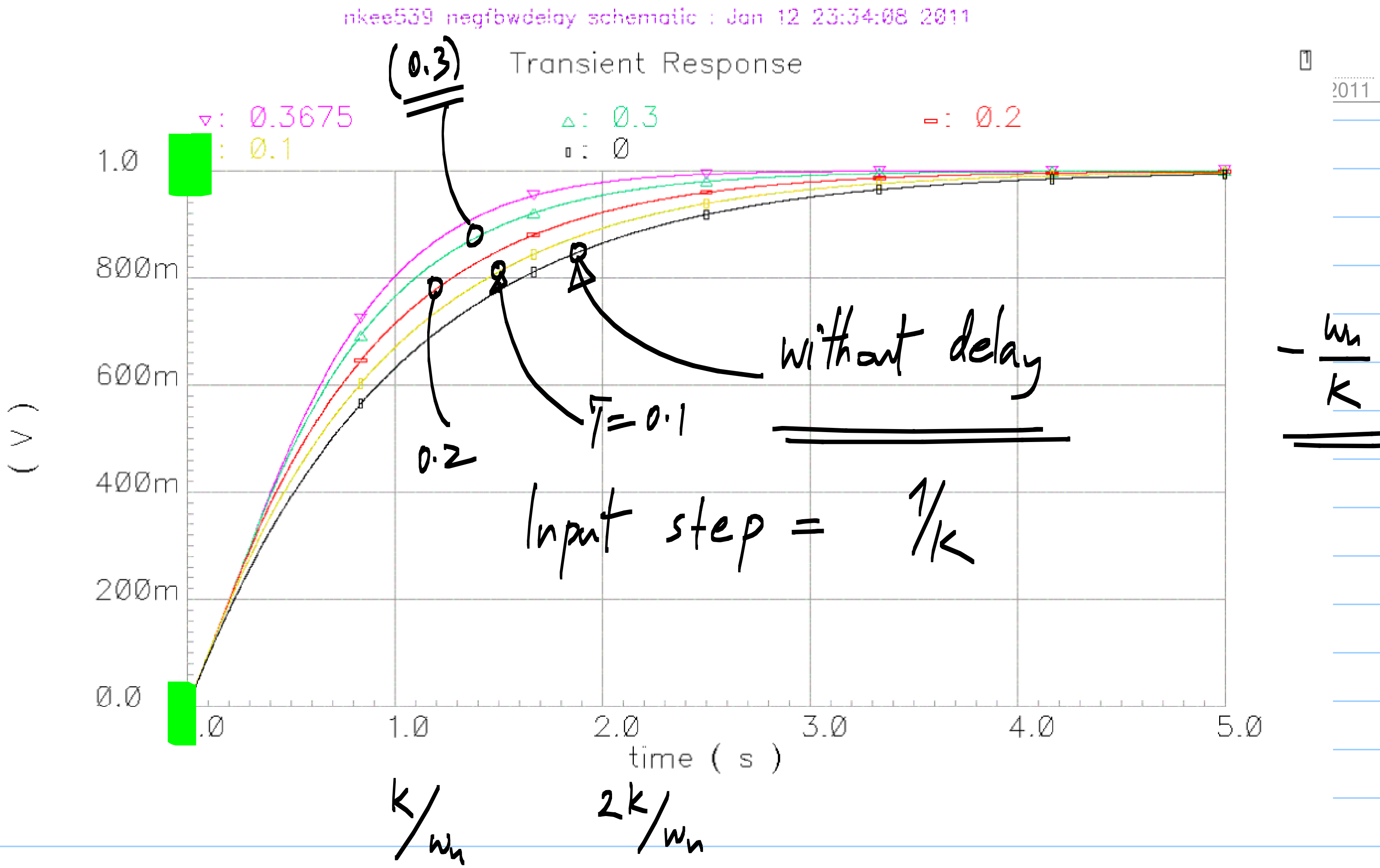
Note Title

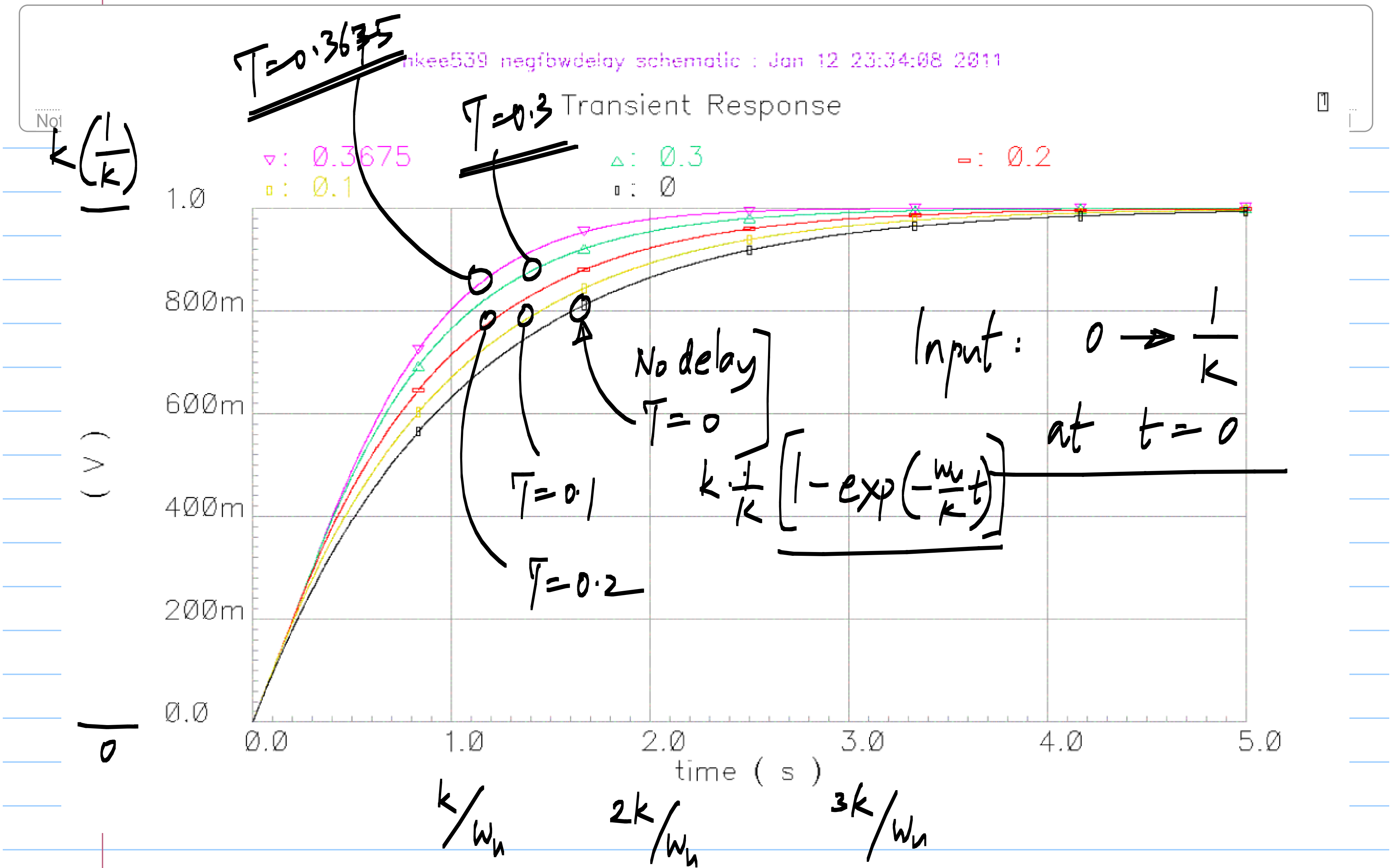
1/18/2011



$$\exp\left(-35\cdot\frac{\omega_n}{k}\cdot t\right)$$

$$\exp\left(-1.1\,\frac{\omega_n}{k}\,t\right)$$


nkee539 negfbwdelay schematic : Jan 12 23:34:08 2011
Transient Response
▽: 0.3675
△: 0.3
▭: 0.2
: 0.1
▯: 0
1.0
800m
600m
400m
200m
0.0
(V)
.0
1.0
2.0
3.0
4.0
5.0
time (s)
(0.3)
0.2
$T = 0.1$
without delay
Input step = $1/K$
$-\frac{\omega_n}{K}$
K/ω_n
$2K/\omega_n$



nkee539 negfbwdelay schematic : Jan 12 23:34:08 2011
Transient Response
▽: 0.3675
△: 0.3
▭: 0.2
▯: 0.1
▯: 0
1.0
800m
600m
400m
200m
0.0
(V)
0.0
1.0
2.0
3.0
4.0
5.0
time (s)
T=0.3675
T=0.3
T=0.1
T=0.2
No delay T=0
k(1/k)
0
Input: 0 → 1/k at t = 0
k·1/k [1 − exp(−ω_u/k t)]
k/ω_u
2k/ω_u
3k/ω_u

Note Title 1/18/2011

$$\sigma' + \exp(-\sigma' T)$$

Substitute σ'

$$\frac{d}{d\sigma'}$$

$$1 - T\exp(-\sigma' T) = 0$$

$$\exp(-\sigma' T) = \frac{1}{T}$$

$$\sigma' T = \ln(T)$$

$$\underbrace{\sigma' = \frac{\ln(T)}{T}}$$

Note Title 1/18/2011

$$\frac{\ln(\tau)}{\tau} + \exp\left(-\frac{\ln(\tau)}{\cancel{\tau}}\cdot\cancel{\tau}\right) = 0$$

$$\frac{\ln(\tau)}{\tau} + \frac{1}{\tau} = 0$$

$$\ln(\tau) = -1$$

$$\tau = \frac{1}{e} = 0.3675$$

$$\boxed{\tau > \frac{1}{e}}$$