An introduction to Information Theory

Adrish Banerjee

Department of Electrical Engineering
Indian Institute of Technology Kanpur
Kanpur, Uttar Pradesh
India

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Lecture #7B: Universal Source Coding-II: Lempel-Ziv Welch Algorithm (LZW)

Outline of the lecture

- LZW encoding

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- Initially the dictionary is initialized to all length-one phrases (q of them).
- Codeword is a pair representing $(C(w) \cdot w_k)$ where $C(w)$ is the code (index in the dictionary) of the prefix of the new phrase and w_k is the innovation symbol.

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- $\lceil \log |D_n| \rceil$ bits can be used to represent codewords where $|D_n|$ is the size of the dictionary at time n.
- Alternatively, Elias coding of positive numbers can be used to represent codewords where smaller indices can be used for recent phrases (reverse order of dictionary).

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- Solution: Encoding procedure is shown in a table (next frame).

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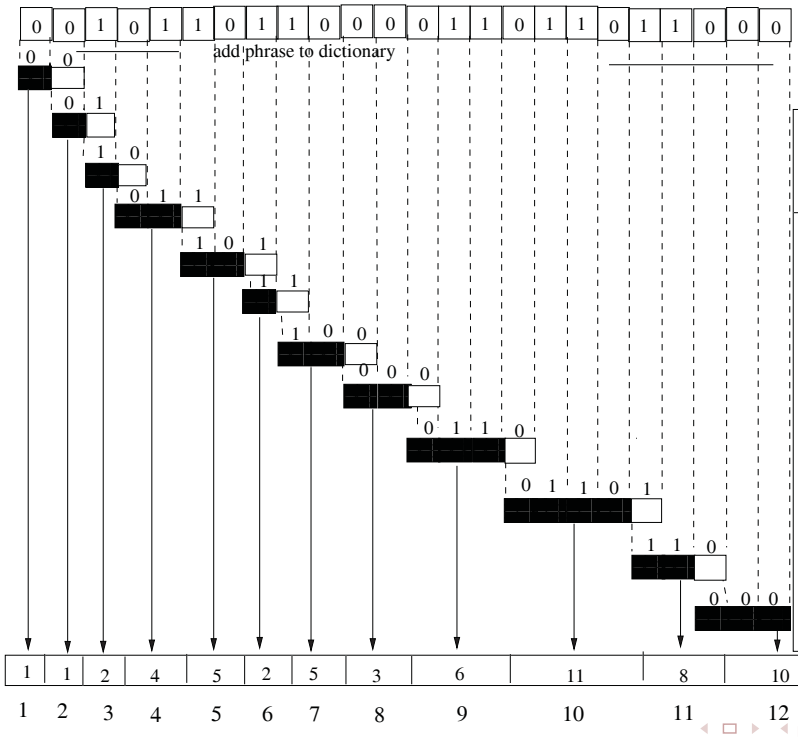

Dictionary

Phrase	C	alt
0	1	1
1	2	2
0 0	3	1.0
0 1	4	1.1
1 0	5	2.0
0 1 1	6	4.1
1 0 1	7	5.1
1 1	8	2.1
1 0 0	9	5.0
0 0 0	10	3.0
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0 1 1 0 1	12	11.1
1 1 0	13	8.0

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- At the beginning of decoding, dictionary is initialized with codewords corresponding to the source alphabet.
- A phrase is added to the decoder dictionary after each new codeword is received except the first codeword.
- Every received codeword represents a prefix of a new entry into the dictionary . The innovation symbol for this entry is determined from the first symbol of the next decoded codeword.

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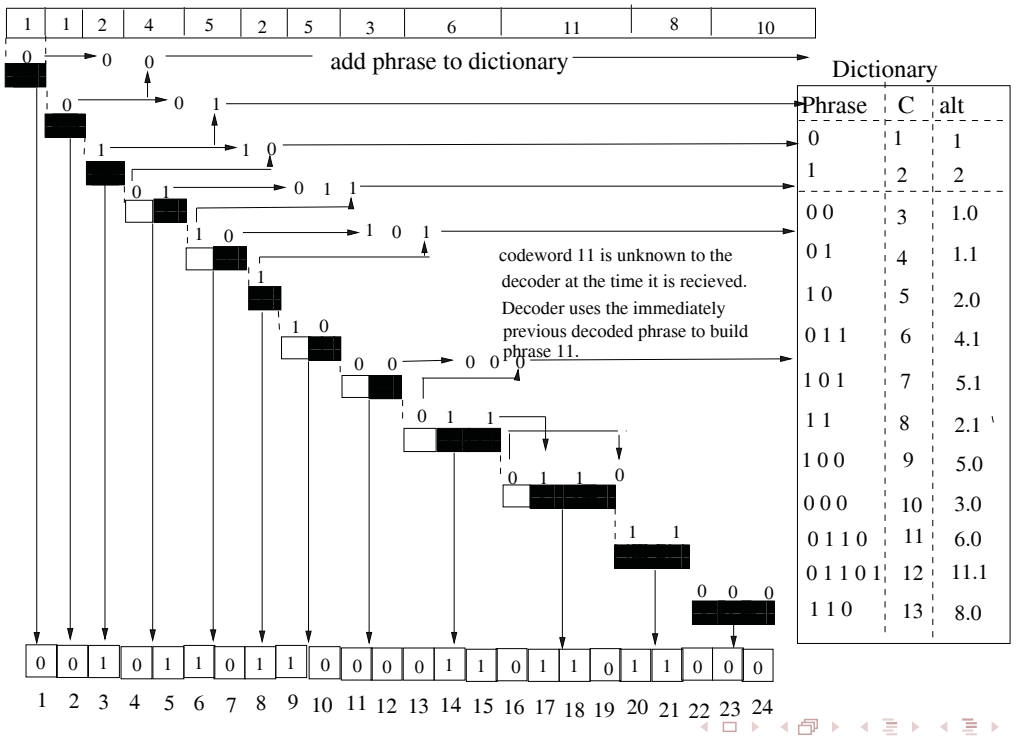

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