An introduction to Information Theory

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Lecture #6A: The Asymptotic Equipartition Property

Outline of the lecture

- Law of large numbers

Law of large numbers Typical sequences Asymptotic Equipartition Property

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- Law of large numbers
- Typical sequences

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- Law of large numbers
- Typical sequences
- Asymptotic Equipartition Property

Outline

1. Law of large numbers
2. Typical sequences
3. Asymptotic Equipartition Property

Law of large numbers

- *Weak Law of Large Numbers:* Let $X[n]$ be an independent random sequence with mean μ_X and variance σ_X^2 defined for $n \geq 1$. Define

$$\hat{\mu}_X[n] \triangleq (1/n)\sum_{k=1}^{n} X[k] \ \ \forall n \geq 1$$

Law of large numbers Typical sequences Asymptotic Equipartition Property

Law of large numbers

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$$\hat{\mu}_X[n] \triangleq (1/n)\sum_{k=1}^{n} X[k] \ \ \forall n \geq 1$$

- Then $\hat{\mu}_X[n] \to \mu_X$ in probability as $n \to \infty$.

Tychebycheff Inequality

- If X is a real-valued random variable with mean μ_X and variance σ_X^2.

Law of large numbers Typical sequences Asymptotic Equipartition Property

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- If X is a real-valued random variable with mean μ_X and variance σ_X^2.
- Let A denote the event $|X - \mu_X| \geq \epsilon$, and A^c the complementary event ($|X - \mu_X| < \epsilon$), then

$$\begin{aligned} \sigma_X^2 &= E[(X-\mu_X)^2|A]P(A) + E[(X-\mu_X)^2|A^c]P(A^c) \\ &\geq E[(X-\mu_X)^2|A]P(A) \end{aligned}$$

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- Whenever A occurs, $(X-\mu_X)^2 \geq \epsilon^2$, so that

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- Hence

$$\sigma_X^2 \geq \epsilon^2 P(A)$$

- Alternatively,

$$P(|X-\mu_X| \geq \epsilon) \leq \frac{\sigma_X^2}{\epsilon^2}, \quad \forall \epsilon > 0$$

Weak law of Large Numbers

- Sample mean

$$E[\hat{\mu}_X[n]] = (1/n)\sum_{i=1}^{n} E[X[i]] = \mu_X$$

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- Using Tychebycheff Inequality, we get

$$P(|\hat{\mu}_X[n] - \mu_X| \geq \epsilon) \leq \frac{\sigma_X^2}{n\epsilon^2}$$

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Law of large numbers Typical sequences Asymptotic Equipartition Property

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- Thus

$$\lim_{n\to\infty} P(|\hat{\mu}_X[n] - \mu_X| \leq \epsilon) = 1$$

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Weak law of Large Numbers

- Let Y be the indicator random variable for the event A, i.e. $Y = 1$ when A occurs, and 0 otherwise.

$$E[Y] = P(A)$$

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- Since $Y^2 = Y$, this implies that

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- Hence

$$Var[Y] = P(A)[1 - P(A)]$$

Weak law of Large Numbers

- Then

$$\hat{\mu}_Y[n] = n_A/n$$

where n_A is the number of times, the event A has occurred.

Weak law of Large Numbers

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$$\hat{\mu}_Y[n] = n_A/n$$
where n_A is the number of times, the event A has occurred.
- From Tychebycheff Inequality, we get
$$P\left(\left|\frac{n_A}{n} - P(A)\right| \geq \epsilon\right) \leq \frac{P(A)[1 - P(A)]}{n\epsilon^2}$$

Outline

1 Law of large numbers

2 Typical sequences

3 Asymptotic Equipartition Property

Typical sequences

- Consider a sequence of $L = 20$ bits emitted by a discrete memoryless source (DMS) with

$$P_U(0) = \frac{3}{4} \text{ and } P_U(1) = \frac{1}{4}$$

Law of large numbers Typical sequences Asymptotic Equipartition Property

Typical sequences

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$$P_U(0) = \frac{3}{4} \text{ and } P_U(1) = \frac{1}{4}$$

- Which one of the following is the "real" sequence?

$$\begin{array}{ll} (1) & 1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1 \\ (2) & 1,0,1,0,1,0,0,0,0,0,0,0,0,0,1,1,0,0,0,1 \\ (3) & 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0 \end{array}$$

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- Probability of occurrence of the sequences

$$\begin{aligned} (1)\ P_{U_1,\cdots,U_{20}}(u_1,\cdots,u_{20}) &= (1/4)^{20} \\ (2)\ P_{U_1,\cdots,U_{20}}(u_1,\cdots,u_{20}) &= (1/4)^{20}(3)^{14} \\ (3)\ P_{U_1,\cdots,U_{20}}(u_1,\cdots,u_{20}) &= (1/4)^{20}(3)^{20} \end{aligned}$$

Typical sequences

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- Let $\mathbf{u} = [u_1, u_2, \cdots, u_L]$ denote possible values of **U**, i.e. $u_j = \{a_1, a_2, \cdots, a_K\}$ for $1 \leq j \leq L$.

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- Let $n_{a_i}(u)$ denotes the number of occurrence of the letter a_i in the sequence **u**. Then **u** is an $\epsilon-$typical output sequence of length L for this K-ary DMS if

$$(1-\epsilon)P_U(a_i) \leq \frac{n_{a_i}(\mathbf{u})}{L} \leq (1+\epsilon)P_U(a_i), \quad 1 \leq i \leq K$$

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Typical sequences

- Consider a binary DMS with $P_U(0) = 3/4$ and $P_U(1) = 1/4$. Let's choose $\epsilon = 1/3$. Then a sequence $\mathbf{u}$ of length $L = 20$ is ϵ-typical if and only if both
$$\frac{2}{3} \cdot \frac{3}{4} \leq \frac{n_0(u)}{20} \leq \frac{4}{3} \cdot \frac{3}{4}$$
and
$$\frac{2}{3} \cdot \frac{1}{4} \leq \frac{n_1(u)}{20} \leq \frac{4}{3} \cdot \frac{1}{4}$$

Law of large numbers Typical sequences Asymptotic Equipartition Property

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$$\frac{2}{3} \cdot \frac{1}{4} \leq \frac{n_1(u)}{20} \leq \frac{4}{3} \cdot \frac{1}{4}$$
- Equivalently, $\mathbf{u}$ is ϵ-typical if and only if both
$$10 \leq n_0(u) \leq 20$$
and
$$4 \leq n_1(u) \leq 6$$

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Typical sequences

- *Property 1:* If **u** is an ϵ-typical output sequence of length L from a K-ary DMS with entropy H(U) in bits, then

$$2^{-(1+\epsilon)LH(U)} \leq P_{\mathbf{U}}(\mathbf{u}) \leq 2^{-(1-\epsilon)LH(U)}$$

Law of large numbers Typical sequences Asymptotic Equipartition Property

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- *Proof:* From the definition of a DMS, we have

$$P_{\mathbf{U}}(\mathbf{u}) = \prod_{j=1}^{L} P_U(u_j) = \prod_{i=1}^{K} [P_U(a_i)]^{n_{a_i}(\mathbf{u})}$$

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Law of large numbers Typical sequences Asymptotic Equipartition Property

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$$P_{\mathbf{U}}(\mathbf{u}) \geq 2^{-(1+\epsilon)LH(U)}$$

- Similar arguments can be used to prove

$$P_{\mathbf{U}}(\mathbf{u}) \leq 2^{-(1-\epsilon)LH(U)}$$

Typical sequences

- *Property 2:* The probability, 1-P(F), that the length L output sequence **U** from a K-ary DMS is ϵ-typical satisfies

$$1 - P(F) > 1 - \frac{K}{L\epsilon^2 P_{\min}}$$

where $P_{\min}$ is the smallest positive value of $P_U(u)$.

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- Interested to show that for large L, the output sequence **U** of the DMS is certain to be $\epsilon-$typical.

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- Interested to show that for large L, the output sequence **U** of the DMS is certain to be $\epsilon-$typical.
- We will use Tchebycheff inequality

$$P\left(\left|\frac{n_A}{n} - P(A)\right| \geq \epsilon\right) \leq \frac{P(A)[1-P(A)]}{n\epsilon^2}$$

Typical sequences

- Let B_i denote the event that **U** takes on value **u** such that the condition for ϵ-typical sequence is not satisfied. Then,

$$\begin{aligned} P(B_i) &= P\left(\left|\frac{n_{a_i}(\mathbf{u})}{L} - P_U(a_i)\right| > \epsilon P_U(a_i)\right) \\ &\leq \frac{P_U(a_i)[1-P_U(a_i)]}{L[\epsilon P_U(a_i)]^2} \end{aligned}$$

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\end{aligned}
$$

- Simplifying, we have

$$P(B_i) \leq \frac{1 - P_U(a_i)}{L\epsilon^2 P_U(a_i)}$$

Law of large numbers Typical sequences Asymptotic Equipartition Property

Typical sequences

- Let P_{min} is the minimum non-zero value of $P_U(u)$, we get,

$$P(B_i) < \frac{1}{L\epsilon^2 P_{\text{min}}}$$

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Typical sequences

- Let $P_{\min}$ is the minimum non-zero value of $P_U(u)$, we get,

$$P(B_i) < \frac{1}{L\epsilon^2 P_{\min}}$$

- Let F be the failure event that **U** is not ϵ-typical. Since F occurs in atleast one of the events, $B_i, 1 \leq i \leq K$, using union bounds we get

$$\begin{aligned} P(F) &\leq \sum_{i=1}^{K} P(B_i) \\ &< \frac{K}{L\epsilon^2 P_{\min}} \end{aligned}$$

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Typical sequences

- *Property 3:* The number M of ϵ-typical sequence **u** from a K-ary DMS with entropy H(U) in bits satisfies

$$\left(1 - \frac{K}{L\epsilon^2 P_{\min}}\right) \cdot 2^{(1-\epsilon)LH(U)} < M \leq 2^{(1+\epsilon)LH(U)}$$

where $P_{\min}$ is the smallest positive value of $P_U(u)$.

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Typical sequences

- *Property 3:* The number M of ϵ-typical sequence $\mathbf{u}$ from a K-ary DMS with entropy H(U) in bits satisfies

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- *Proof:*

$$1 = \sum_{\forall \mathbf{u}} P_{\mathbf{U}}(\mathbf{u}) \geq M \cdot 2^{-(1+\epsilon)LH(U)}$$

Law of large numbers Typical sequences Asymptotic Equipartition Property

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- *Proof:*

$$1 = \sum_{\forall \mathbf{u}} P_{\mathbf{U}}(\mathbf{u}) \geq M \cdot 2^{-(1+\epsilon)LH(U)}$$

- This gives the upper bound

$$M \leq 2^{(1+\epsilon)LH(U)}$$

Typical sequences

- Total probability of the ϵ-typical sequences is 1 - P(F), so

$$1 - P(F) \leq M \cdot 2^{-(1-\epsilon)LH(U)}$$

Law of large numbers Typical sequences Asymptotic Equipartition Property

Typical sequences

- Total probability of the ϵ-typical sequences is 1 - P(F), so

$$1 - P(F) \leq M \cdot 2^{-(1-\epsilon)LH(U)}$$

- This gives the lower bound

$$M > \left(1 - \frac{K}{L\epsilon^2 P_{\text{min}}}\right) \cdot 2^{(1-\epsilon)LH(U)}$$

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Asymptotic Equipartition Property

- Property 3 says that when L is large and ϵ is small, there are roughly $2^{LH(U)}$ $\epsilon-$typical sequences $\mathbf{u}$.

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Asymptotic Equipartition Property

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- Property 1 says each of these $\epsilon-$typical sequences has probability equal to $2^{-LH(U)}$.
- Property 2 says that the total probability of these $\epsilon-$ typical sequences is very nearly 1.
- These three properties are known as *asymptotic equipartition property (AEP)* of the output sequence of a DMS.