An introduction to Information Theory

Adrish Banerjee

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Indian Institute of Technology Kanpur
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July 18, 2016

Lecture #1A: Introduction

Books

Textbook:

- James L. Massey, Lecture notes on "Applied Digital Information Theory I". (http://www.isiweb.ee.ethz.ch/archive/massey_scr/)

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- Emmanuel Desurvire, "Classical and Quantum Information Theory", 1st Edition, Cambridge University Press, 2009.

Topics to be covered

Information Theory:

- Introduction: Entropy, Relative Entropy, Mutual Information

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- Block to block length coding: Typical sequences

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- Asymptotic Equipartition Property

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Topics to be covered

Information Theory:

- Coding for sources with memory

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- Gaussian Channel
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What to expect from the course

- What are the fundamendal limits of communication?

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What to expect from the course

- What are the fundamendal limits of communication?
- What are the fundamental limits of data compression?

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- What are the fundamendal limits of communication?
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- Practical source compression algorithm.

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What to expect from the course

- What are the fundamendal limits of communication?
- What are the fundamental limits of data compression?
- Practical source compression algorithm.
- Some mathematical techniques.

Outline of the lecture

- Introduction

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- Basic blocks of communication system

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Outline

Introduction

All communications involves three basic steps

- Encoding a message at its source.

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- Transmitting that message through a communication medium.

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All communications involves three basic steps

- Encoding a message at its source.
- Transmitting that message through a communication medium.
- Decoding the message at its destination.

Information Theory

Information Theory answers two fundamental questions in communications

- What is the ultimate data compression?

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Information Theory

Information Theory answers two fundamental questions in communications

- What is the ultimate data compression?
- What is the ultimate transmission rate?

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What is Information?

- A flip of coin with two heads? Does it convey any information?

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What is Information?

- A flip of coin with two heads? Does it convey any information?
- A source produces successive bits of $\pi : 3, 1, 4, 1, 5, 9, 2, 6$. Does it convey any information?

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- No, no uncertainty in the source.

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What is Information?

- A flip of coin with two heads? Does it convey any information?
- A source produces successive bits of $\pi : 3, 1, 4, 1, 5, 9, 2, 6$. Does it convey any information?
- No, no uncertainty in the source.
- Shannon's Information Theory regards only those symbols as information that are not predictable.

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Encoding Information

- Use the statistical structure of a source to represent its output efficiently.

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- Example: A bag contains 50% black balls, 25% red balls, 12.5% blue balls, 12.5% green balls. You are randoming picking a ball from the bag and want to convey the information about the color of the ball.

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- Smart way? black=0, red=10, blue=110, green=111. An average of 1.75 bits/color

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- Can you figure out the color of the balls from the sequence 0110100111?

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- Main principle of data compression: "Only infrmatn esentil to understnd mst b tranmitd."

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- The channel capacity is a measure of the amount of information that can be conveyed between the input X and the output Y of a channel.

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- Example: If the channel capacity of a particular communication link is (say) 2 Gbps. We can communicate over this channel at any desired rate less than 2 Gbps, and achieve arbitrary low error rates.

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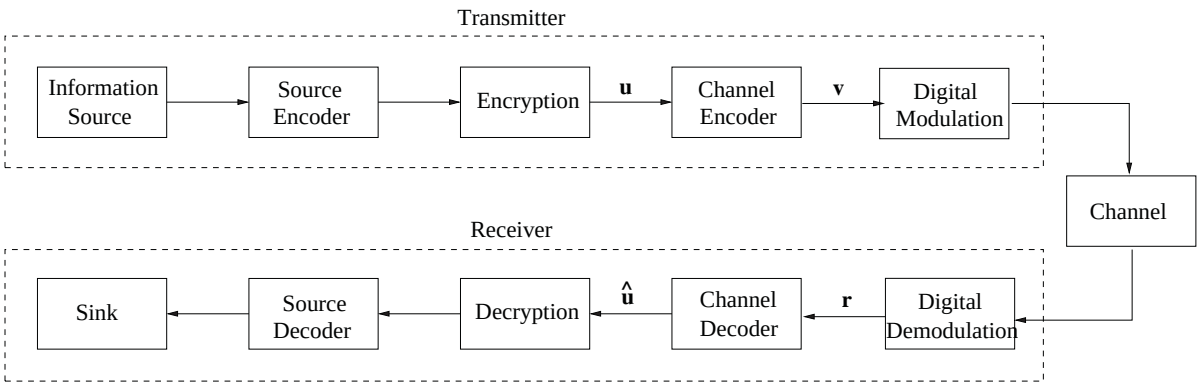
Introduction



Outline

Source Coding

- *Function*: To minimize the number of bits per unit time required to represent the source output.

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- This process is known as *source coding* or *data compression*
- *Examples*: Huffman coding, Lempel-Ziv algorithm.
- The output of the source encoder is referred to as the *information sequence*.

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Encryption

- *Function*: To make source bits transmission secure.

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Encryption

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- This process of converting source bits (message text) into a source stream that looks like meaningless random bits of data (cipher text) is known as *encryption*.

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- *Examples*: Data Encryption Standard (DES), RSA system.

Channel Coding

- *Function*: To correct transmission errors introduced by the channel.

Channel Coding

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- The process of introducing some redundant bits to a sequence of information bits in a controlled manner to correct transmission errors is known as *channel coding* or *error control coding*.

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- *Example*: Repetition code, Reed-Solomon codes, CRC codes.
- The encoded sequence that is the output of the channel encoder is referred to as *codeword*.

Modulation

- *Function*: To map the codewords into waveforms which are then transmitted over the physical medium known as the channel.

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- *Examples*: Phase shift keying (PSK), quadrature amplitude modulation (QAM).

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- *Examples*: Binary erasure channel (BEC), Additive white Gaussian noise (AWGN) channel.

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Demodulation

- *Function*: To convert received noisy waveform to a sequence of bits, which is an estimate of the transmitted data bits. This is known as *hard demodulation*.

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- Soft demodulation has significant improvement over hard demodulation.

Channel Decoding

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- The BER is defined as the expected number of information bit decoding errors per decoded information bit.
- The coded sequences can be broken up into blocks of data *frames*. A frame error occurs if any information bit in that data frame is in error. The decoded FER is the percentage of frames in error.

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Decryption

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Decryption

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- It is in the key that the security of a modern cipher lies, not in the details of the cipher.

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Source Decoding

- *Function*: To reconstruct the original source bits from the decoded information sequence.

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Source Decoding

- *Function*: To reconstruct the original source bits from the decoded information sequence.
- Due to channel errors, the final reconstructed signal may be distorted.

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