Alternative Derivation for BER of Wireless Channel

An approximate BER derivation for BPSK in Rayleigh fading can be obtained as follows. The instantaneous BER as shown in the lecture is given as,

$$Q\left(\sqrt{\frac{a^2 P}{\sigma_n^2}}\right).$$

We can now employ the following property of the Q –function

$$Q(x) \leq \frac{1}{2} e^{-\frac{x^2}{2}}.$$

Employing this property, the instantaneous BER can be simplified as,

$$Q\left(\sqrt{\frac{a^2 P}{\sigma_n^2}}\right) \leq \frac{1}{2}\exp\left(-\frac{1}{2}\frac{a^2 P}{\sigma_n^2}\right).$$

Averaging this over the Rayleigh distribution of a, a bound for the BER can be obtained as,

$$\begin{aligned}
\text{BER} &\leq \int_0^\infty 2a\exp\left(-a^2\right)\times\frac{1}{2}\exp\left(-\frac{1}{2}\frac{a^2 P}{\sigma_n^2}\right)da \\
&= \int_0^\infty a\exp\left(-a^2\left(1+\frac{1}{2}\frac{P}{\sigma_n^2}\right)\right)da \\
&= \frac{1}{2}\frac{1}{1+\frac{1}{2}\frac{P}{\sigma_n^2}}
\end{aligned}$$

At high SNR i.e. as $P \to \infty$, the above BER can be simplified as,

$$
\begin{aligned}
\mathrm{BER} &= \frac{1}{2}\frac{1}{1+\frac{1}{2}\frac{P}{\sigma_n^2}} \\
&\approx \frac{1}{2}\frac{1}{\frac{1}{2}\frac{P}{\sigma_n^2}} \\
&= \frac{1}{\mathrm{SNR}}
\end{aligned}
$$

which is similar to the exact BER derivation at high SNR i.e. the BER decreases as 1/SNR.