Slide No: 1

Butterworth Filter

If $H(s)$ is Transfer F^n $V_o(s)/V_{in}(s)$

given by $H(s) = \dfrac{A(s)}{B(s)}$

And if this has only Poles but 'no' zeros $A(s)$ const $= H_0$

Then $H(s) = \dfrac{H_0}{B(s)}$

Then the F^n $B^2(\omega) = 1 + \epsilon^2\left(\dfrac{\omega}{\omega_0}\right)^{2n}$ is called

Butterworth Polynomial

$\epsilon \to 1$

The Filters using Butterworth F^n are called

'Maximally Flat' Kind, i.e. Ripple is v. low.

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$$|H(j\omega)| = \frac{H_0}{\sqrt{1+\epsilon^2\left(\frac{\omega}{\omega_0}\right)^{2N}}}$$

At $\omega = \omega_0$, $\quad H(j\omega_0) = \frac{H_0}{\sqrt{1+\epsilon^2}}$

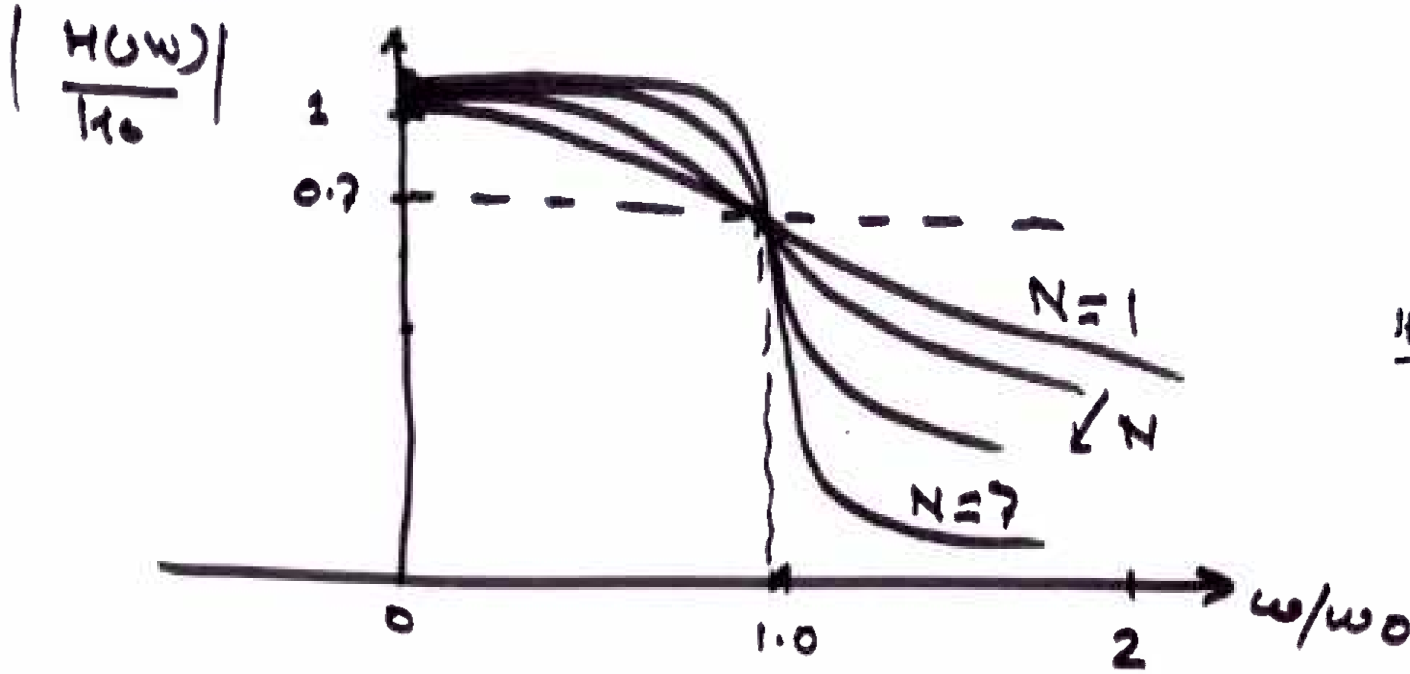
ϵ gives measure of Maximum Transmission A_{max}

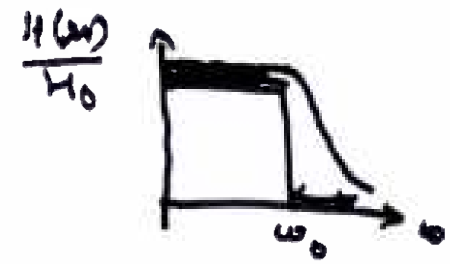
$$A_{max} = 20 \log (1+\epsilon^2)^{1/2}$$

$$\text{or} \quad \epsilon = \sqrt{10^{A/10} - 1} \quad \text{gives Max. Transmission}$$

If $\epsilon = 1$ $\quad \left|\frac{H(j\omega)}{H_0}\right|^2 = \frac{1}{2}$ $\quad$ & $\underline{A_{min} = 0}$ $\quad$ Passband terminates at $\omega = \omega_0$

Slide No. 3





At $\omega > \omega_0$

$$\left|\frac{H(jw)}{H_0}\right| = -20\log\left[\frac{1}{\sqrt{1+\epsilon^2(\omega/\omega_0)^{2N}}}\right]$$

$$= 10\log\left[1+\epsilon^2\left(\frac{\omega}{\omega_0}\right)^{2N}\right]$$

∴ Larger N means larger Attenuation

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Example: For a LP Butterworth Filter, we need Attenuation of 40 db ~~and~~ at $\frac{\omega}{\omega_0} = 2$. We use $\epsilon = 1$.

Then
$$\left| \frac{H(j\omega)}{H_0} \right|^2 = \frac{1}{1 + (\omega/\omega_0)^{2N}}$$

Given
$$\frac{H(j\omega)}{H_0} = \frac{1}{100} = 0.01$$

$$\therefore \quad 10^{-4} = \frac{1}{1 + 2^{2N}} \quad \text{or} \quad 2^{2N} = 10^4 - 1 \simeq 10^4$$

$$\text{or} \quad 2N \log 2 = 4 \qquad \text{or} \quad N = \frac{2}{\log 2} = \frac{2}{0.3010} \simeq 6.64 \simeq \underline{7}$$

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Chebyshev Polynomial is

$$C_N(\omega) = \cos\left(N \cos^{-1} \frac{\omega}{\omega_0}\right) \quad \omega \le \omega_0$$

$$= \cosh\left(N \cosh^{-1}\left(\frac{\omega}{\omega_0}\right)\right) \quad \underline{\omega \ge \omega_0}$$

Given $\left|\frac{H(j\omega)}{H_0}\right|^2 = (-40\,db)^2 = 10^{-4}$

$$\therefore \quad 10^{-4} = \frac{1}{1 + (0.5089)^2 \underline{C_N^2(2)}}$$

or $$C_N^2(2) = \frac{10^4 - 1}{(0.5089)^2} = \underline{3.86 \times 10^4}$$

$$C_N(2) = \sqrt{3.86} \times 10^2$$

or $$196.5 = \cosh\left(N \cosh^{-1}(2)\right)$$

Solving. $N = 4.53 \simeq 5$

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The Ripple frequency ω_e is related to '-3db' cut-off frequency ω_H as

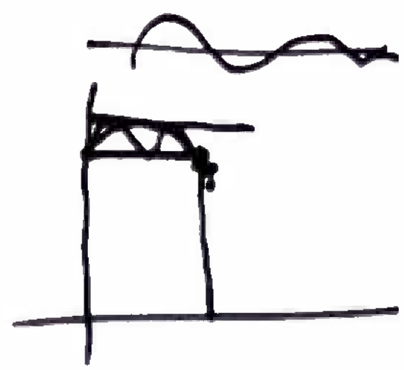
$$\omega_H = \omega_c \cosh\left(\frac{1}{N}\cosh^{-1}\frac{1}{\epsilon}\right)$$

For 1db ripple with say $N=5$ $\quad \underline{\omega_H} = 1.03\,\underline{\omega_c}$

Example: $r = 1$db, α at $\frac{\omega}{\omega_0} = 2$ $\therefore$ Attenuation is 40 db.

Since $r = 1$db, $\epsilon = 0.5089$

$$\therefore \left|\frac{H(j\omega)}{H_0}\right|^2 = \frac{1}{1 + (0.5089)^2 C_n^2(2)}$$



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Chebyshev Filters are 'All Pole' Filters and has larger 'Ripple' but sharper fall for Lower Number of sections (N) compared to Butterworth.

N represents number of Poles.

Parameter ϵ is related to Passband Ripple γ in dbs by

$$\epsilon^2 = 10^{(\gamma/10)} - 1$$

For 0.5 db Ripple ($\gamma = 0.5$), $\epsilon = 0.3493$

& For 1.0 db Ripple ($\gamma = 1.0$), $\epsilon = 0.5089$

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The Chebyshev Filters

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If the Transfer F^n has the form of Magnitude as:

$$|H(jw)| = \frac{1}{\sqrt{1+\epsilon^2 \cos^2\underbrace{\left[N\cos^{-1}\left(\frac{\omega}{\omega_0}\right)\right]}_{C_N(\ )}}} \quad \text{for } \omega \leq \omega_0$$

$$= \frac{1}{\sqrt{1+e^2 \cosh^2\left[N\cosh^{-1}\left(\frac{\omega}{\omega_0}\right)\right]}} \quad \text{for } \omega \geq \omega_0$$

Then Transfer F^n represents Chebyshev Function

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and $\frac{\omega_H}{\omega_C} = \cosh\left(\frac{1}{5}\right)\cosh^{-1}\frac{1}{0.5089}$

$= 1.03$

clearly we have a Sharper Fall at ω_H to w_c with $N=5$ and $\gamma = 1.0$ db

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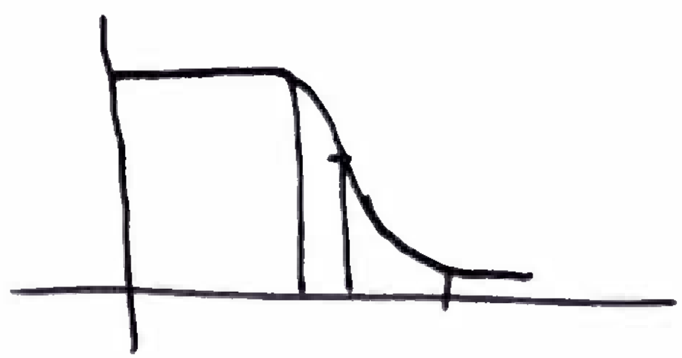
Butterworth Polynomial

n	*Factors of polynomial $B_n(s)$*
1	$(s + 1)$
2	$(s^2 + 1.414s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$(s^2 + 0.518s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s + 1)$
7	$(s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.663s + 1)(s^2 + 1.962s + 1)$



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Chebyshev Polynomial

	0.5-dB ripple ($\epsilon = 0.3493$)
1	$s + 2.863$
2	$s^2 + 1.425\, s + 1.516$
3	$(s + 0.626)\,(s^2 + 0.626s + 1.142)$
4	$(s^2 + 0.351s + 1.064)\,(s^2 + 0.845s + 0.356)$
5	$(s + 0.362)\,(s^2 + 0.224s + 1.036)\,(s^2 + 0.586s + 0.477)$
6	$(s^2 + 0.1554s + 1.024)\,(s^2 + 0.4142s + 0.5475)\,(s^2 + 0.5796s + 0.157)$
7	$(s + 0.2562)\,(s^2 + 0.1014s + 1.015)\,(s^2 + 0.3194s + 0.6657)\,(s^2 + 0.4616s + 0.2539)$
8	$(s^2 + 0.0872s + 1.012)(s^2 + 0.2484s + 0.7413)(s^2 + 0.3718s + 0.3872)(s^2 + 0.4386s + 0.08805)$
	1.0-dB ripple ($\epsilon = 0.5089$)
1	$s + 1.965$
2	$(s^2 + 1.098s + 1.103$
3	$(s + 0.494)\,(s^2 + 0.494s + 0.994)$
4	$(s^2 + 0.279s + 0.987)\,(s^2 + 0.674s + 0.279)$
5	$(s + 0.289)\,(s^2 + 0.179s + 0.988)(s^2 + 0.468s + 0.429)$
6	$(s^2 + 0.1244s + 0.9907)\,(s^2 + 0.3398s + 0.5577)\,(s^2 + 0.4642s + 0.1247)$
7	$(s + 0.2054)\,(s^2 + 0.0914s + 0.9927)\,(s^2 + 0.2562s + 0.6535)\,(s^2 + 0.3702s + 0.2304)$
8	$(s^2 + 0.07s + 0.9942)\,(s^2 + 0.1994s + 0.7236)\,(s^2 + 0.2994s + 0.3408)\,(s^2 + 0.3518s + 0.0702)$

Course Name

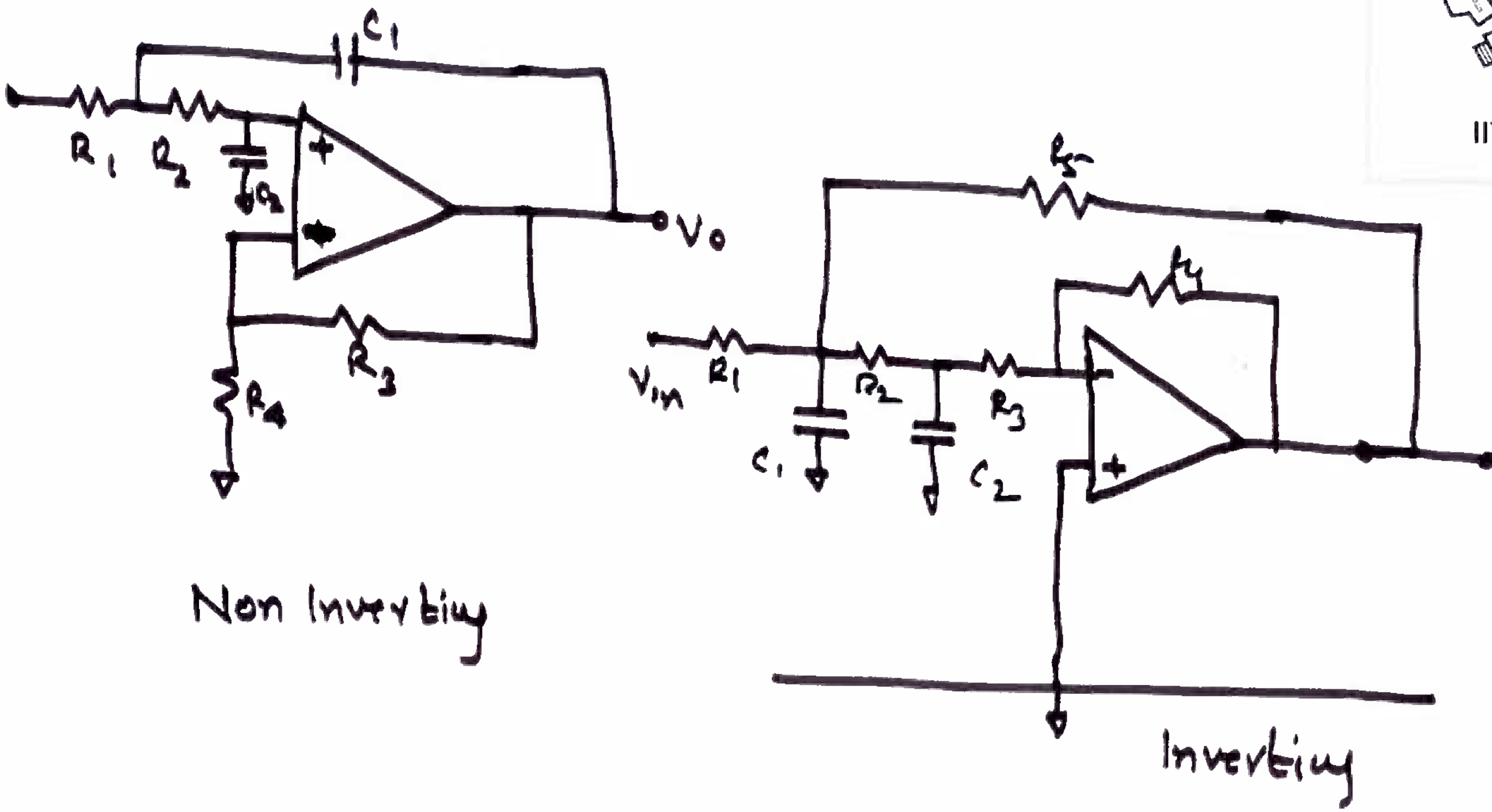
Analog Circuits

Lecture No. 22

Instructor's Name

Prof. A. N. Chandorkar

Sallan - Key Low Pass - Section



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Creation of a Real Single Pole

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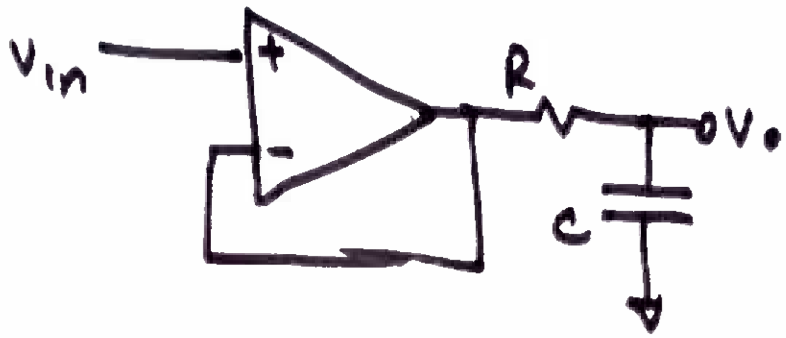
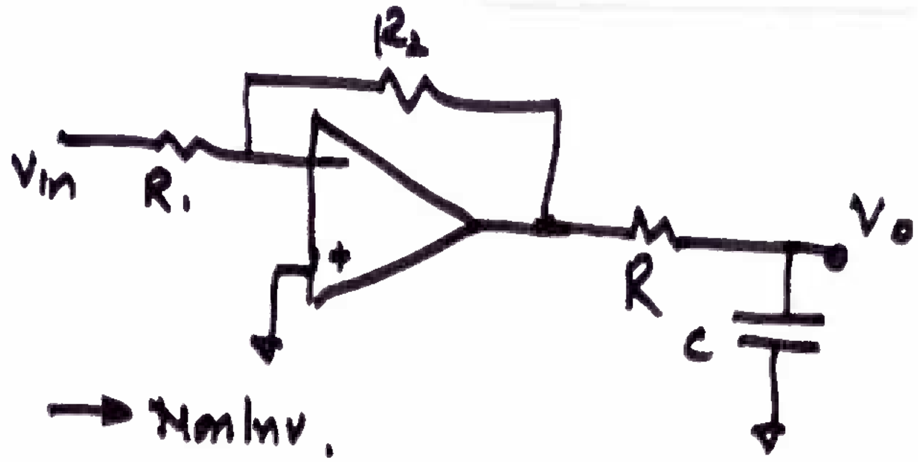




$$\frac{V_0}{V_{in}} = H(s) = \frac{1}{1+RCs} = \frac{1}{1+s/(1/RC)} \rightarrow \text{NonInv.}$$

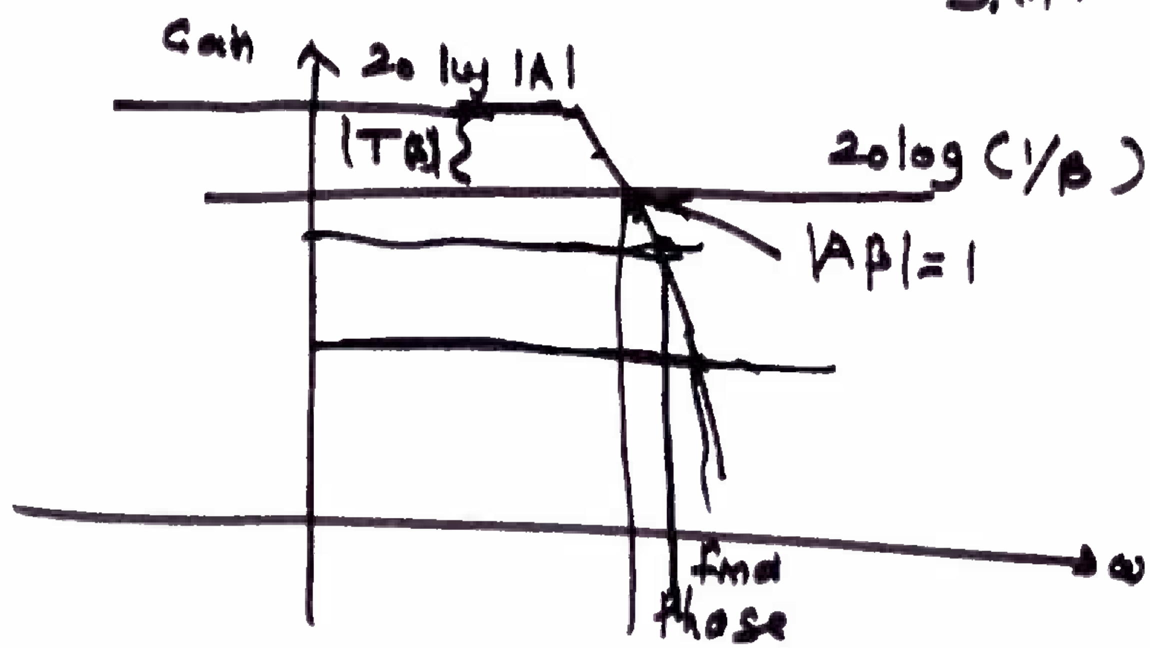
$$\frac{V_0}{V_{in}} = -\left(\frac{R_2}{R_1}\right)\cdot\frac{1}{1+s/(1/RC)}$$

Inverting

Stability : Revisit

We have $A\beta$ = Loop Gain $= \dfrac{A}{1/\beta}$

$$\therefore \quad 20 \log |A\beta| = 20 \log A - 20 \log (1/\beta)$$



If $\phi < -180^\circ$ Stable