Slide No: 1

Active RC Filters

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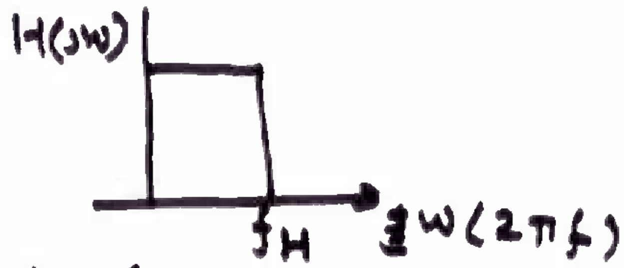
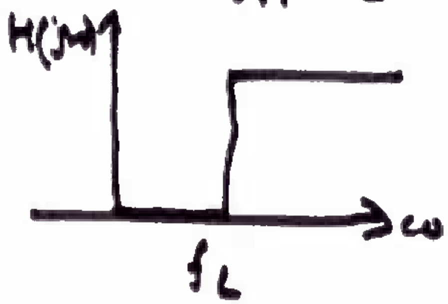
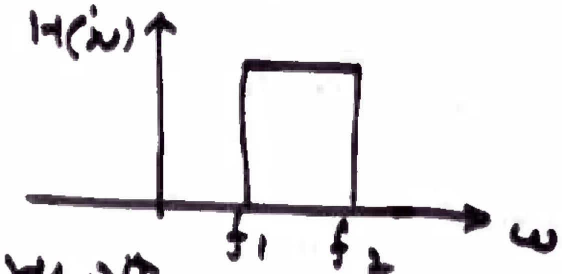
1. Low Pass



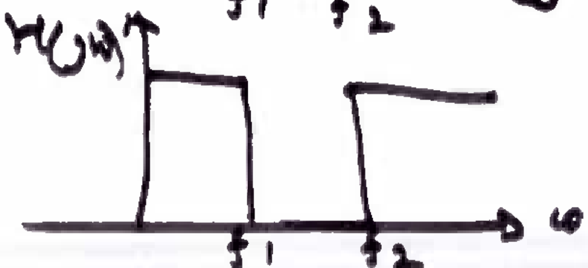
2 High Pass



3 Band Pass

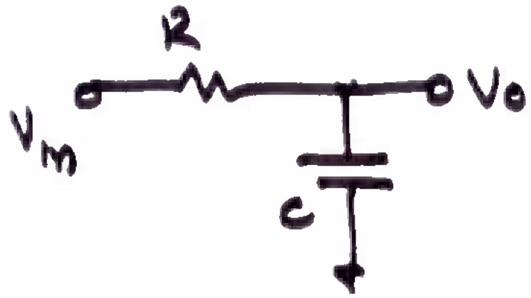


4 Band Reject



Slide No: 2

Active Networks



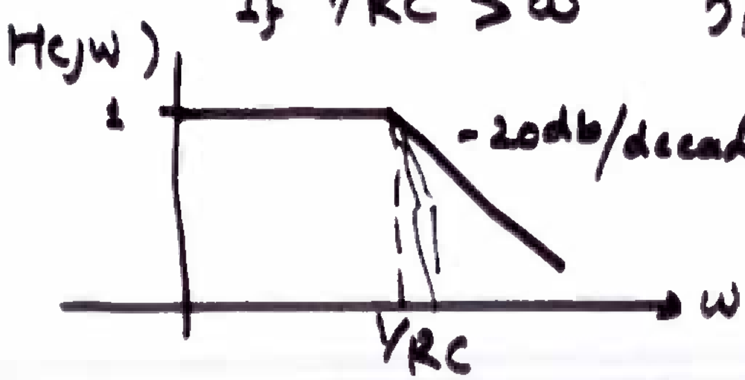
$$\therefore \frac{V_o(s)}{V_{in}(s)} = H(s)$$

$$= \frac{1/cs}{1/cs + R} = \frac{1}{1 + RCs}$$

or $$H(s) = \frac{1}{1 + s/(1/RC)}$$ $\therefore \omega_0 = 1/RC$ is the Pole

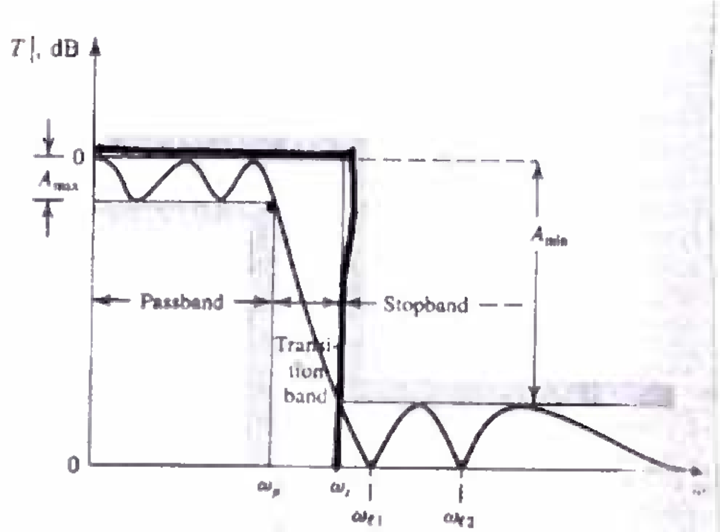
For $\omega < \omega_0$ $$|H(s)| = \left| \frac{RC}{jw + RC} \right| = \frac{RC}{\sqrt{w^2 + R^2C^2}}$$

If $1/RC > \omega$ then $|H(jw)| = 1$



Ideal

Slide No: 3



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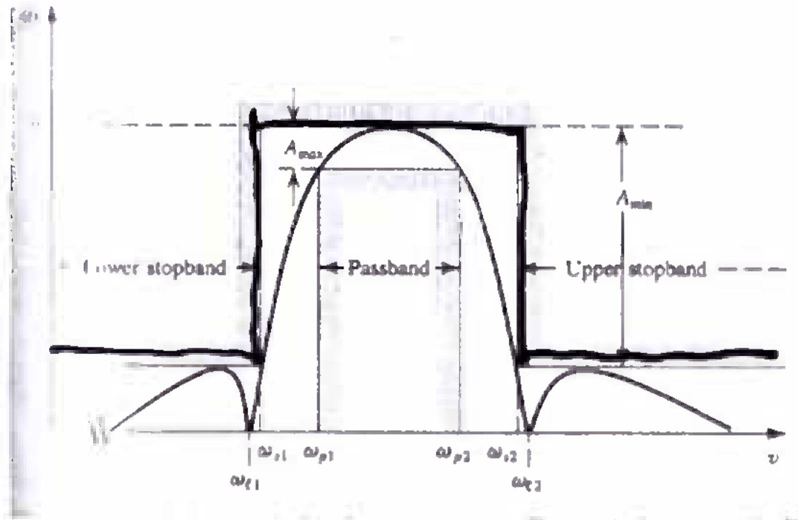

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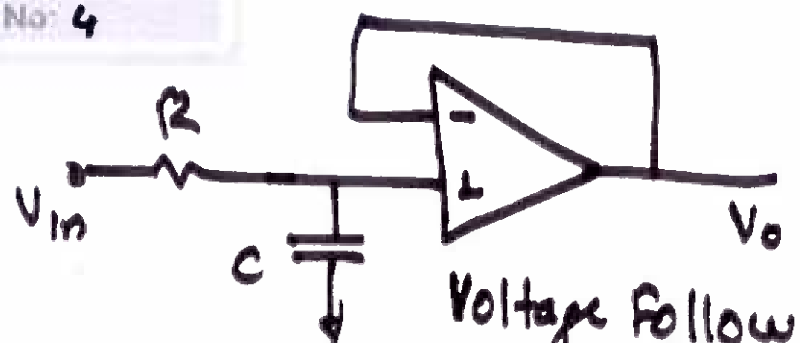
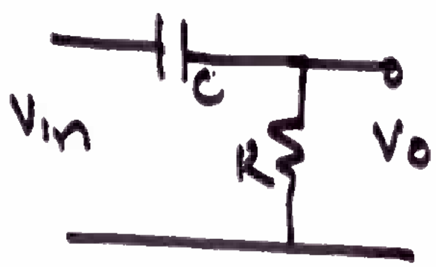


Voltage Follower

Low Pass Filter

(2) High Pass



$$\frac{V_o(s)}{V_{in}(s)} = \frac{R}{R + \frac{1}{Cs}} = \frac{RCs}{RCs+1}$$

$$= \frac{s/\frac{1}{RC}}{\frac{s}{\frac{1}{RC}} + 1} = \frac{(s/\omega_0)}{\left(\frac{s}{\omega_0} + 1\right)}$$

If $\omega > \omega_0$

$H(s) = 1$

where $\omega_0 = 1/RC$ (Pole)

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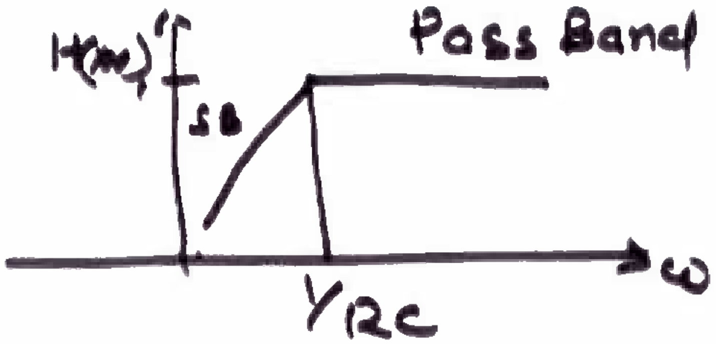

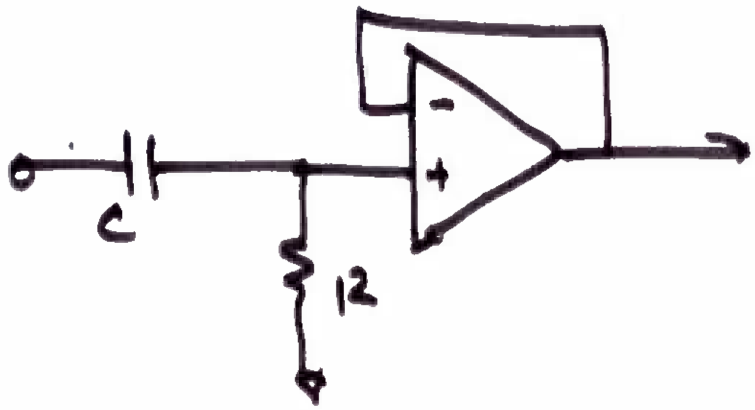

High Pass filter.

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Generalised Two Pole Active Filter

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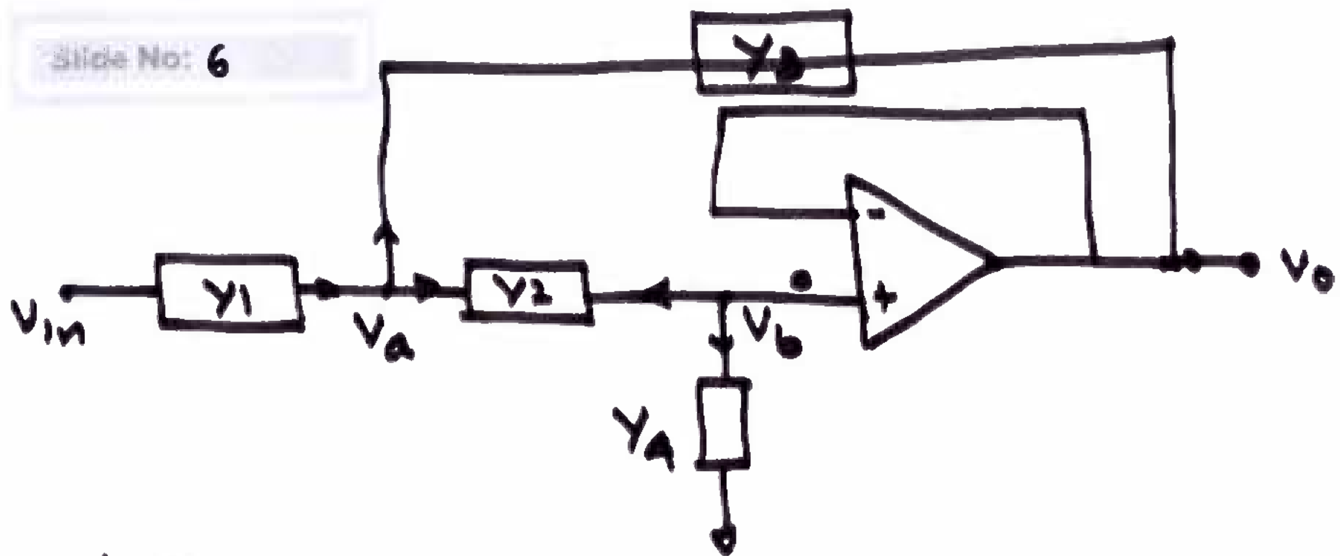

$$Y = G + jB$$
$$Z = R + jX$$

At Node V_a :

$$(V_{in} - V_a) Y_1 = (V_a - V_b) Y_2 + (V_a - V_o) Y_3 \qquad \text{— (i)}$$

At node V_b

$$-(V_b - V_a) Y_2 = V_b \cdot Y_4 \qquad \text{— (ii)}$$

Further $\quad V_o = V_b \qquad$ — (iii)

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From eq. (ii) & (iii)

$$V_a Y_2 = V_b Y_2 + V_b Y_4 = (Y_2 + Y_4) V_b$$

$$\therefore \quad V_a = V_o \left(\frac{Y_2 + Y_4}{Y_2} \right) \quad \text{— (iv)}$$

Substituting V_a in eq (i)

$$V_{in} Y_1 - V_o \left(\frac{Y_2 + Y_4}{Y_2} \right) Y_1 = V_o \left(\frac{Y_2 + Y_4}{Y_2} \right) (Y_2 + Y_3) - V_o Y_1 - V_o Y_3$$

$$\therefore \quad \frac{V_o(s)}{V_{in}(s)} = T(s) = \frac{Y_1 Y_2}{Y_1 Y_2 + (Y_1 + Y_2 + Y_3) Y_4} = H(s)$$

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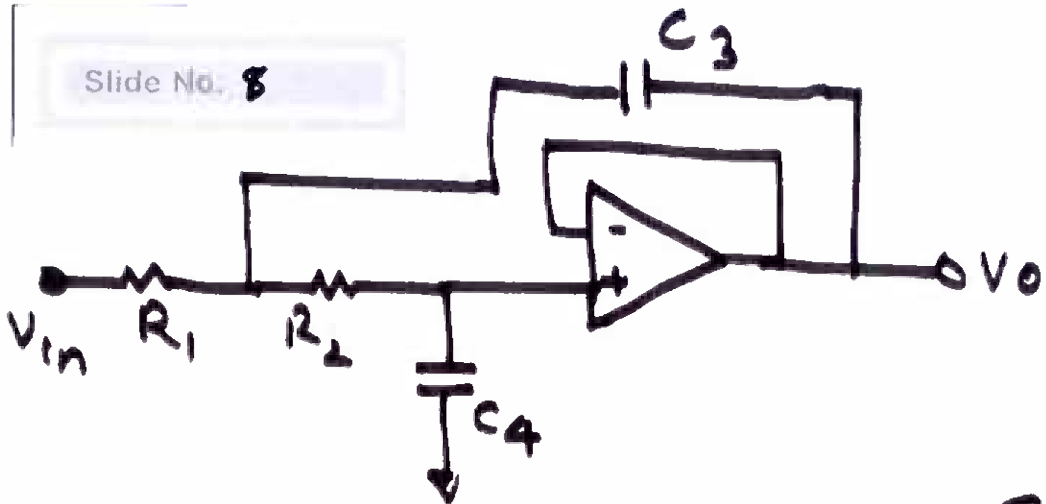

(Example)

$G = \frac{1}{R}$

$Y = (Cs)$

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$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{G_1 G_2}{G_1 G_2 + sC_4(G_1 + G_2 + sC_3)}$$

At $s = j\omega = 0$ $\quad H(s) = \frac{G_1 G_2}{G_1 G_2} = 1$

At $s = j\omega \to \infty$ $\quad H(s) \to 0$

This is a Butterworth Filter

We assume $R_1 = R_2 = R$

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We have then

$$H(s) = \frac{1/R^2}{\frac{1}{R^2} + sC_4\left(\frac{2}{R} + sC_3\right)}$$

$$= \frac{1}{1 + 2RC_4 s + C_4 C_3 R^2 s^2}$$

Define

$\tau_3 = RC_3$

$\tau_4 = RC_4$

$$|H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2\tau_3\tau_4)^2 + (2\omega\tau_4)^2}}$$

For Maximally Flat Filter $\frac{d|H(j\omega)|}{d\omega} \to 0$

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Differentiating $|H(j\omega)|$ and putting it $= 0$, we get

$$0 = 4\omega\tau_4\left[-\tau_3(1-\omega^2\tau_3\tau_4) + 2\tau_4\right]$$

And at $\omega = 0$ $\quad \tau_3 = 2\tau_4$ or $C_3 = 2C_4$

Substituting this in $|H(j\omega)|$ function

$$|H(j\omega)| = \frac{1}{\left[1 + 4(\omega\tau_4)^4\right]^{1/2}}$$

Cut-off or pole occurs when $|H(j\omega)| = 1/\sqrt{2}$ $\quad$ (-3 db)

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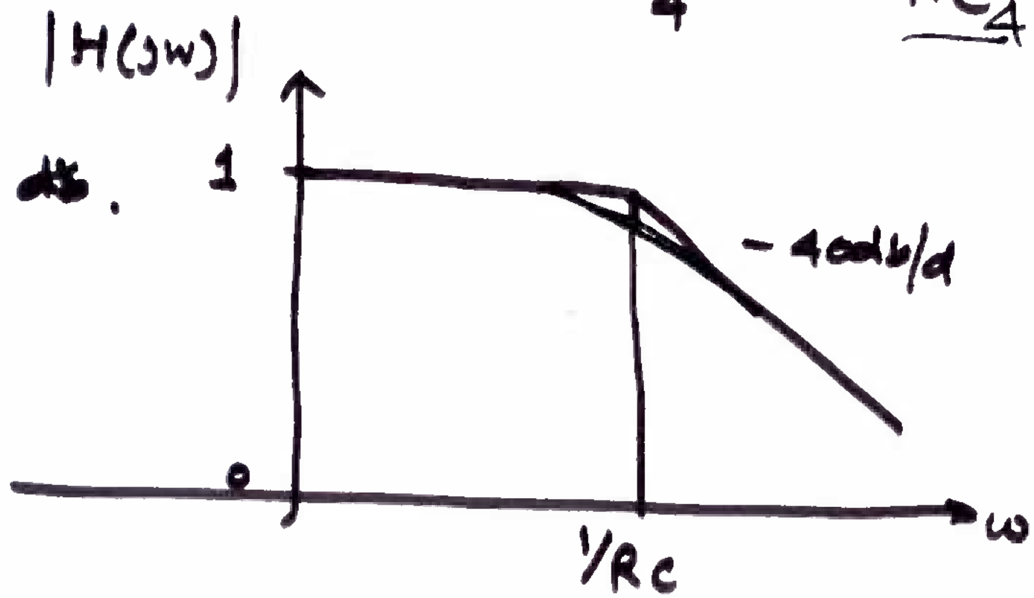
$$\therefore \omega_{pole} = \frac{1}{\sqrt{2}\,\tau_4} = \frac{1}{\sqrt{2}\,RC_4}$$

Using Bode's criterion

$$\omega_{pole} = \frac{1}{RC_4} = \underline{\frac{1}{RC_4}}$$



Biquadratic Function

$$H(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

Low Pass $\quad \dfrac{K}{s^2 + (\omega_0/Q)s + \omega_0^2}$ or $\dfrac{K(s+z)}{s^2 + (\omega_0/Q)s + \omega_0^2}$

High Pass $\quad \dfrac{Ks^2}{s^2 + (\omega_0/Q)s + \omega_0^2}$ or $\dfrac{Ks(s+z)}{s^2 + (\omega_0/Q)s + \omega_0^2}$

Bandpass $\quad \dfrac{K\cdot s}{s^2 + (\omega_0/Q)s + \omega_0^2}$

Band Reject $\quad \dfrac{K(s^2+\omega_0^2)}{s^2 + (\omega_0/Q)s + \omega_0^2}$

$\left\{ \frac{1}{2Q} = k \right.$ is called Damping Factor

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Then using KCL at nodes, we get

$$H(s) = \frac{A_{vo}}{R_1R_2C_1C_2s^2 + s\left[C_2(R_1+R_2) + R_1C_1(1-A_{vo})\right] + 1}$$

From Tr. Fn for LP Filter

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1C_1(1-A_{vo}) + (R_1+R_2)C_2}$$

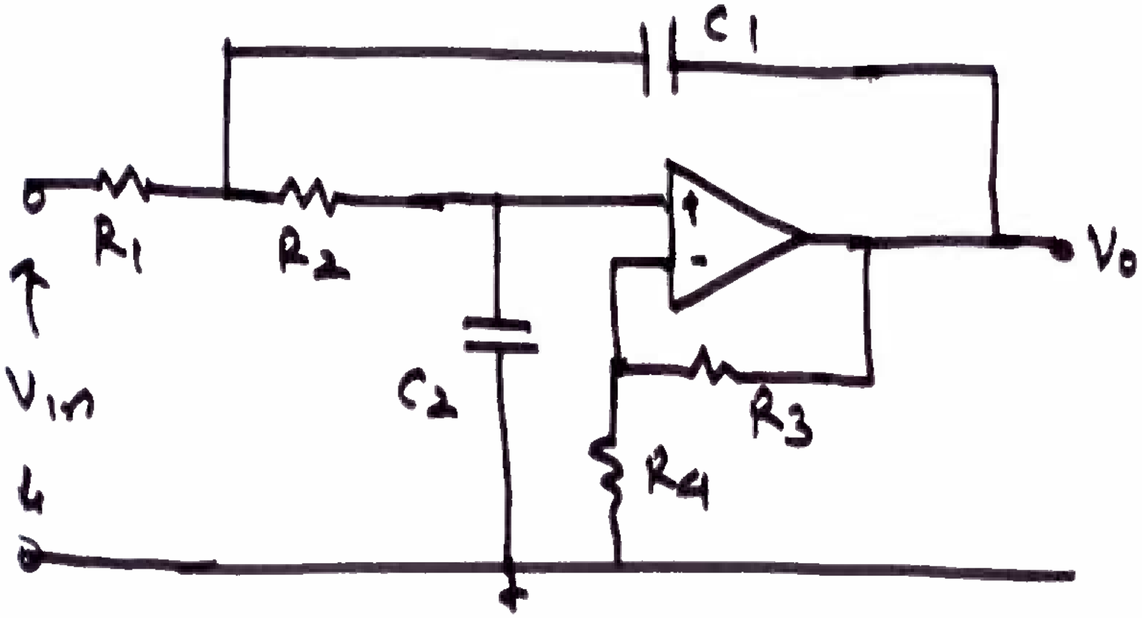
If we choose $R = R_1 = R_2$ & $C_1 = C_2 = C$

Then $$H(s) = \frac{A_{vo}}{R^2C^2s^2 + RC(3-A_{vo})s + 1}$$

& $\omega_0 = \frac{1}{RC}$ & $Q = \frac{1}{3-A_{vo}}$ for Stability $A_{vo} \leq 3$

Low Pass $\quad H(s) = \dfrac{H_0}{\dfrac{s^2}{\omega_0^2} + \dfrac{1}{Q}\cdot\left(\dfrac{s}{\omega_0}\right) + 1}$

where $K/\omega_0^2 = H_0$



Low Pass - Section

$A_v(0) = \left(1 + \dfrac{R_3}{R_4}\right)$

Biquad Filter Implementation using OTA g_m/C or g_m-C Filter.

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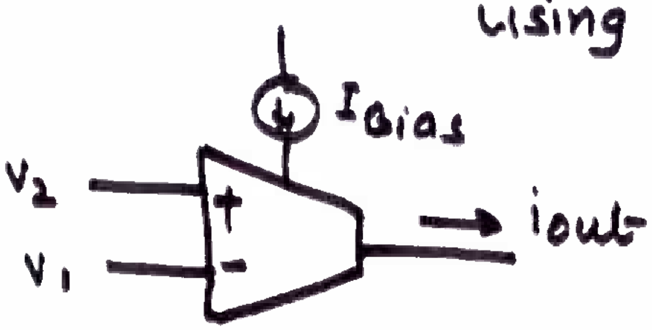
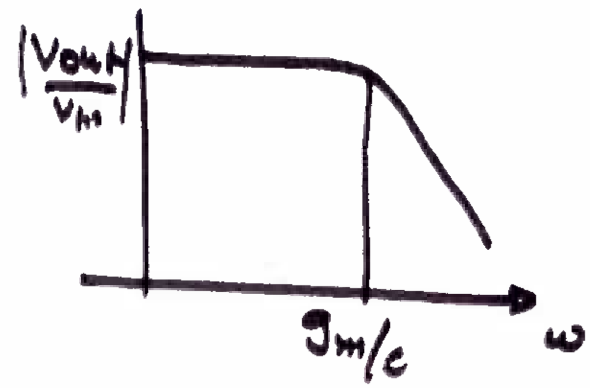
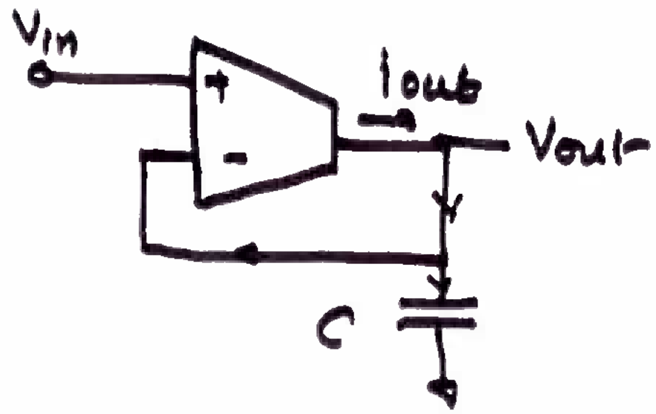


$$G_m = g_m K$$

$$g_m = \sqrt{2\beta'\left(\frac{W}{L}\right) I_{Bias}}$$





$$V_{out} = i_{out} \cdot \frac{1}{Cs}$$

$$\frac{i_{out}}{V_{in}} = G_M = g_m \quad \text{if } K = 1$$

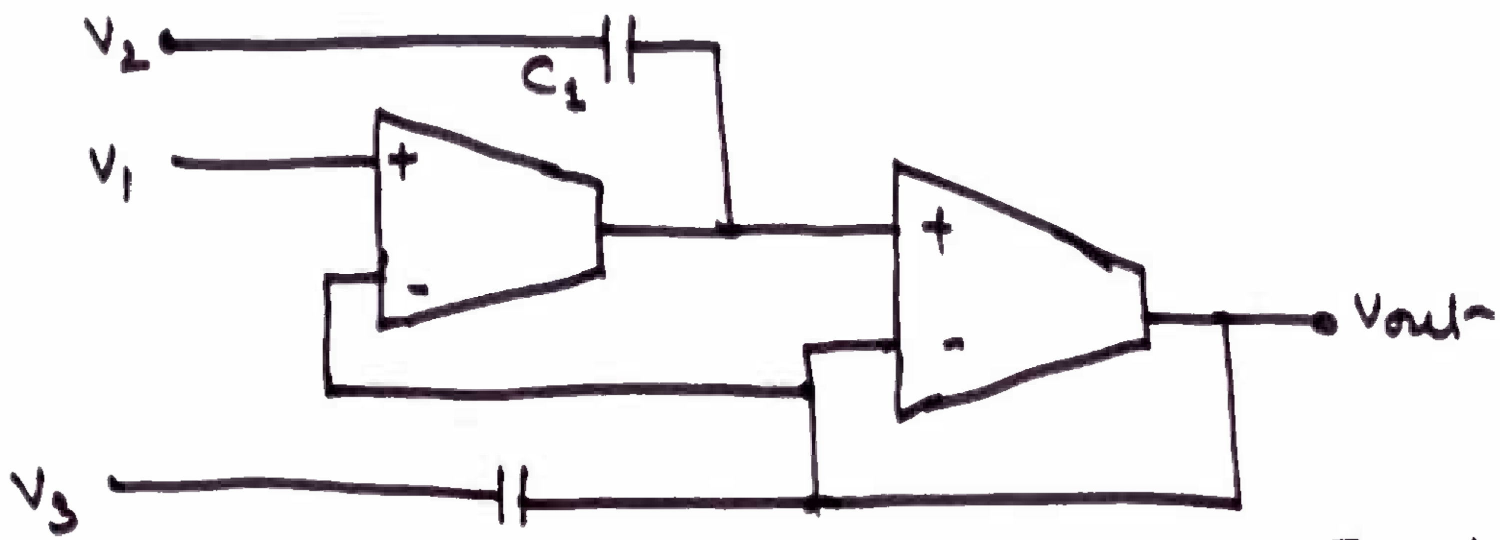
However $i_{out} = g_m (V_{in} - V_{out})$ for OTA

This is low Pass Filter.

$$\therefore \frac{V_{out}}{V_{in}} = \frac{1}{1 + Cs\frac{1}{g_m}}$$

$$\text{Pole} = \omega_0 = \frac{1}{C/g_m} = \frac{g_m}{C}$$

Universal Filter



Filter	Input Conditions	Transfer Fn
LP	$V_{in} = V_1,\ V_2 = 0,\ V_3 = 0$	$g_m^2 / [s^2 C_1 C_2 + s C_1 g_m + g_m^2]$
HP	$V_{in} = V_3,\ V_1 = V_2 = 0$	$s^2 C_1 C_2 / [s^2 C_1 C_2 + s C_1 g_m + g_m^2]$
BP	$V_{in} = V_2,\ V_1 = V_3 = 0$	$s C_1 g_m / [s^2 C_1 C_2 + s C_1 g_m + g_m^2]$
BR	$V_{in} = V_1 = V_3,\ V_2 = 0$	$(s^2 C_1 C_2 + g_m^2) / [s^2 C_1 C_2 + s C_1 g_m + g_m^2]$