Frequency Response of Amplifier

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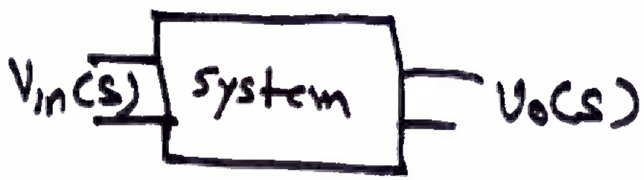
S-Domain Analysis

We know

$$\boxed{j\omega = s}$$

Any Transfer Function of a Network is given as



v_{in} $\quad$ $\dot{i}_o$

i_{in} $\quad$ i_o

i_n $\quad$ v_o

$$H(s) = H(jw) = \frac{v_o(s)}{v_{in}(s)}$$

$$= K \frac{(s-z_1)(s-z_2) - - (s-z_m)}{(s-p_1)(s-p_2) - - (s-p_n)}$$

where z is called 'zero' of the T. function

& p is called 'Pole' of the T. function

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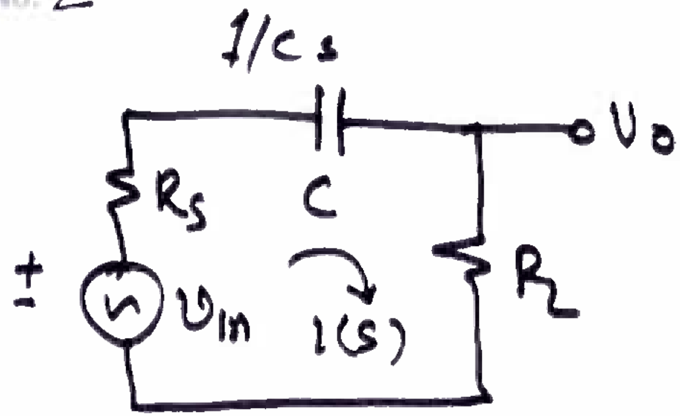

$$v_o(s) = i(s) \cdot R_L$$

$$i(s) = \frac{v_{in}}{R_s + \frac{1}{Cs} + R_L} = \frac{v_{in}\, Cs}{(R_s+R_L)Cs+1}$$

or $$\frac{v_o(s)}{v_{in}(s)} = \frac{R_L C \cdot s}{(R_L+R_s)C \cdot s + 1} = H(s)$$

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(1) If $s = 0$ then $H(s) \rightarrow 0$

$\therefore$ we have a 'zero' at $\omega = 0$

(2) If $(R_L + R_s)C \cdot s = -1$

Then ~~at~~ $H(s) \rightarrow \infty$

$\therefore$ at $S = \dfrac{-1}{(R_L + R_s)C}$ we have a Pole.

We ~~see~~ that RC has unit of Time

$\therefore \tau_s = (R_L + R_s)C$

We can Rewrite $H(s)$ as

$$H(s) = \frac{v_o(s)}{v_{in}(s)} = \underbrace{\frac{R_L}{R_L + R_s}}_{K} \cdot \frac{(R_L + R_s)C\,s}{1 + (R_L + R_s)C \cdot s}$$

$$= K \frac{\tau_s s}{1 + s\tau_s}$$

or

$$A_v(s) = \underbrace{\frac{R_L}{R_L + R_s}}_{①} \cdot \frac{j\omega\tau_s}{1 + j\omega\tau_s} \quad \begin{matrix} \rightarrow ② \\ \rightarrow ③ \end{matrix}$$

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$$\therefore \left| A_v(j\omega) \right| = \frac{R_L}{R_L+R_s} \cdot \left[\frac{\omega \tau_s}{\sqrt{1+\omega^2 \tau_s^2}} \right]$$

Where $\boxed{\omega = 2\pi f}$

$$\therefore \left| A_v(jf) \right| = \frac{R_L}{R_L+R_s} \left[\frac{2\pi f \tau_s}{(1+4\pi^2 f^2 \tau_s^2)^{1/2}} \right]$$

In dbs.

$$\left| A_v(jf) \right|_{db} = 20 \log_{10} \left[\left| A_v(j\omega) \right| \right]$$

$$P_{db} = 10 \log \frac{P_2}{P_1}$$

$$A_{v_{db}} = 20 \log V_2/V_1$$

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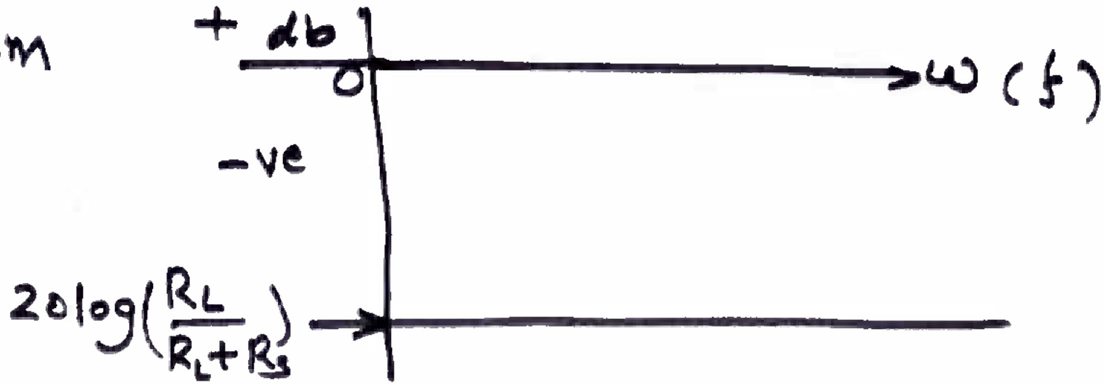
$$\therefore \left|A_v(\omega f)\right|_{db} = 20\log\left(\frac{R_L}{(R_L+R_s)}\right) + 20\log\left(2\pi f \tau_s\right) - 20\log\left[\sqrt{1+(2\pi f \tau_s)^2}\right]$$

Ist Term



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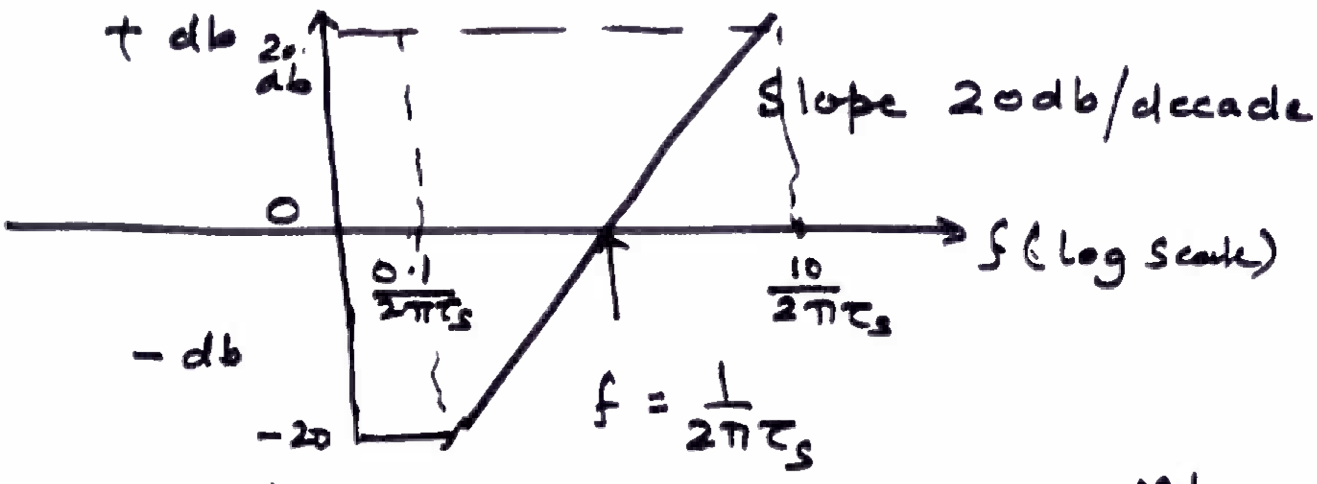
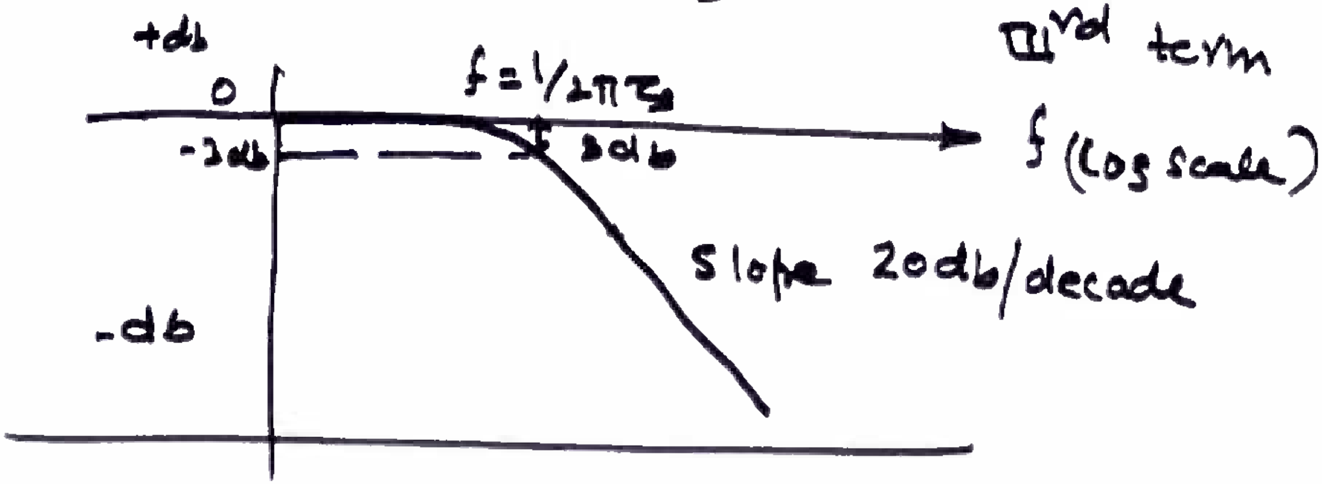
IInd Term

$$f = \frac{1}{2\pi\tau_s}$$



IIIrd term



$$-20\log(\sqrt{2})$$

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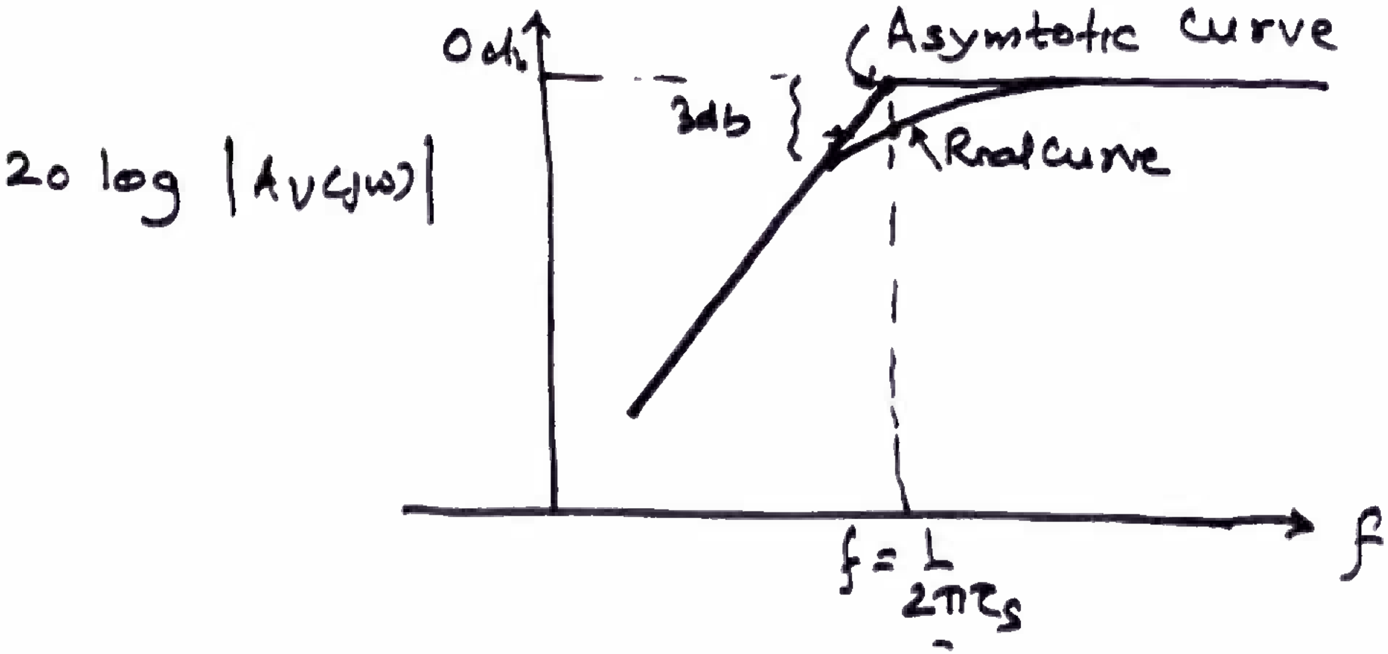
Complete Function '$A_v(j\omega)$'s frequency response can then be plotted as



Corner Frequency

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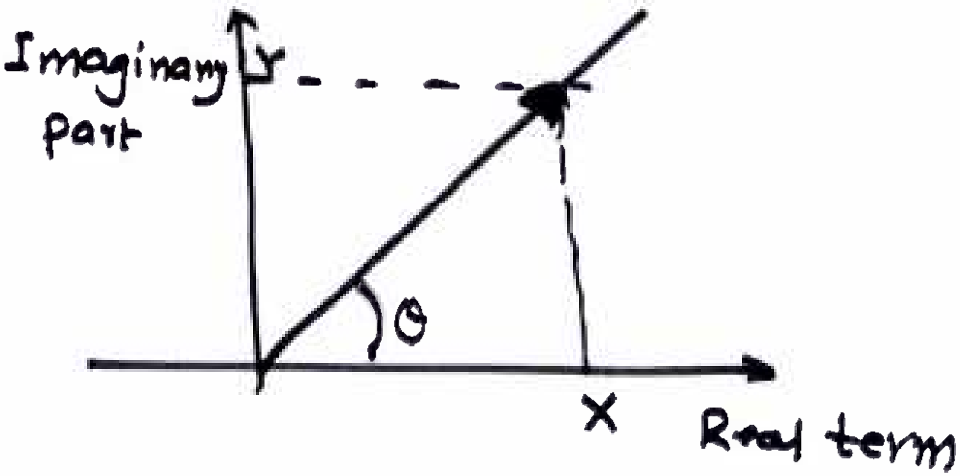
A complex term $= x + jY = f$

Then $|f| = \sqrt{x^2 + y^2}$ — Magnitude

& $\theta = \angle f(j\omega) = \tan^{-1}\frac{Y}{X}$ — Phase.



Phase part

$f = x + jy$

$$\tan^{-1}\frac{y}{x} = \theta$$

If $y = x$ $\quad \tan^{-1} 1$ $\quad$ then $\theta = 45^\circ$

If $y = 0$ $\quad \tan^{-1} 0$ $\quad$ then $\theta = 0^\circ$

If $x = 0$ or $y \to \infty$ $\quad \tan^{-1} \infty$ $\quad$ then $\theta = 90^\circ$

Take a case of frequency response

$$\theta = \tan^{-1}\frac{\omega}{\omega_0}$$

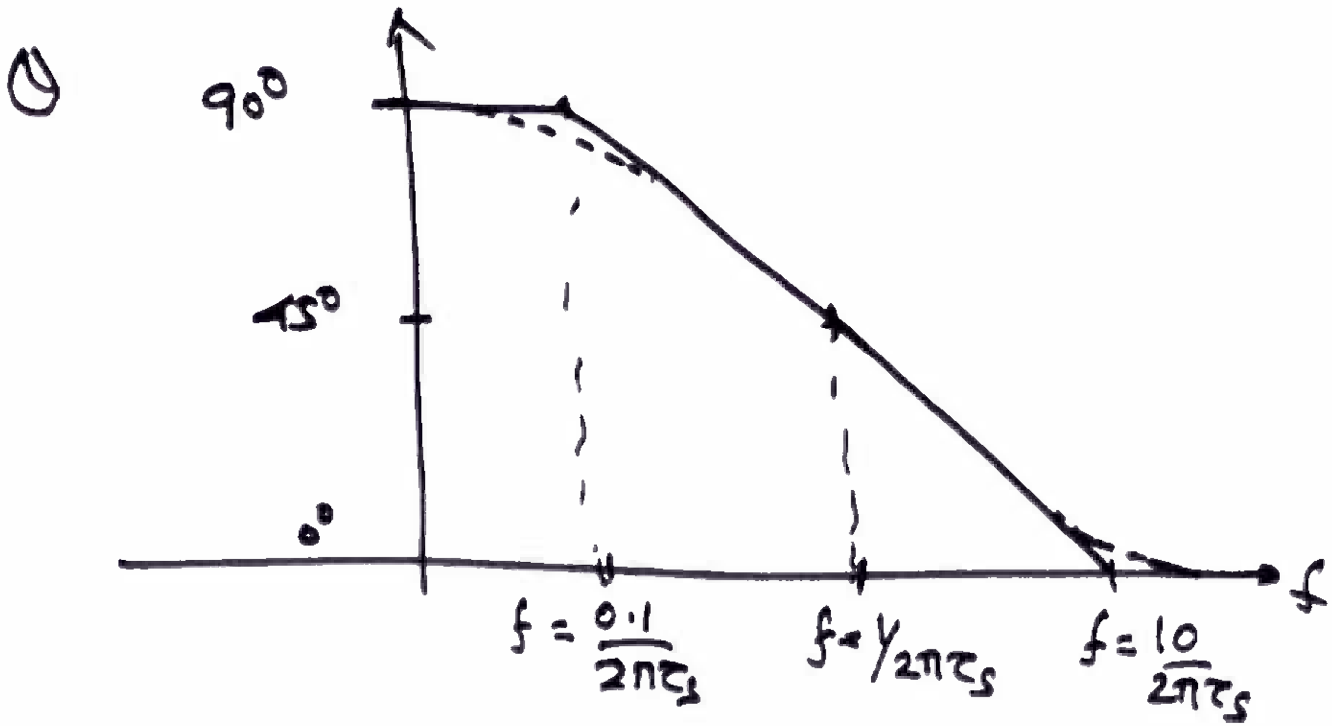
At $\omega = \omega_0$ $\quad \theta = 45^\circ$

At $\omega = 0.1\omega_0$ $\quad \theta = \tan^{-1}(0.1) \to 0^\circ$

At $\omega = 10\omega_0$ $\quad \tan^{-1} 10 = \theta \to 90^\circ$

i.e. θ changes 45°/decade

Phase Response of $A_v(j\omega)$



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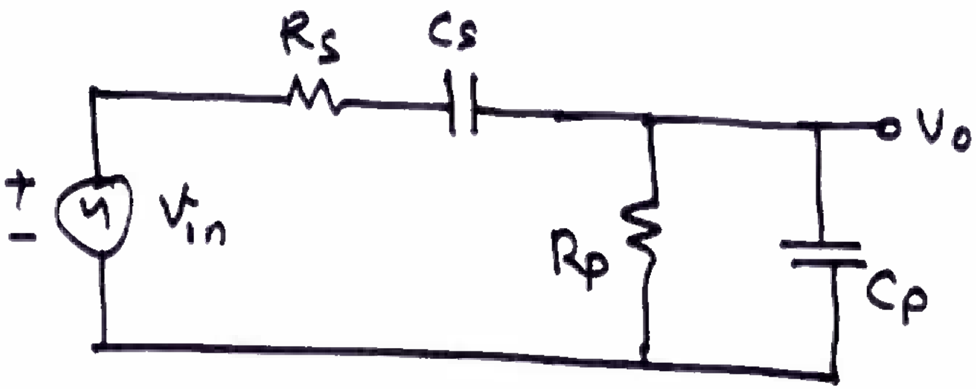
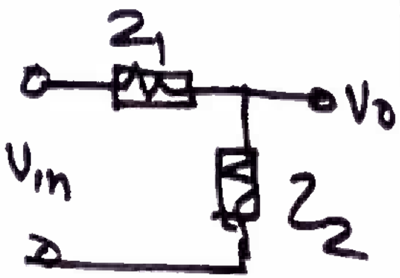
Two Pole Circuit





$$V_o = \frac{Z_2}{Z_1+Z_2} \cdot V_{in}$$

Then $$\frac{V_o(s)}{V_{in}(s)} = A_v(s) = \frac{R_p}{R_p+R_s} \cdot \frac{1}{\left[1 + \frac{R_p}{R_p+R_s}\left(\frac{C_p}{C_s}\right) + \frac{1}{s\tau_s} + s\tau_p\right]}$$

where $$\tau_s = (R_s+R_p)\, C_s$$

$$\tau_p = (R_s \| R_p)\, C_p$$

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τ_s is called <u>open Circuit Time Constant</u>

i.e. when $\frac{1}{C_p \omega} \rightarrow \infty$

$\therefore$ τ_s represents time constant Associated with C_s

Similarly

τ_p is called <u>Short Circuit Time Constant</u>

i.e. when $\frac{1}{C_s S} \rightarrow 0$

$\therefore$ τ_p is time constant associated with C_p

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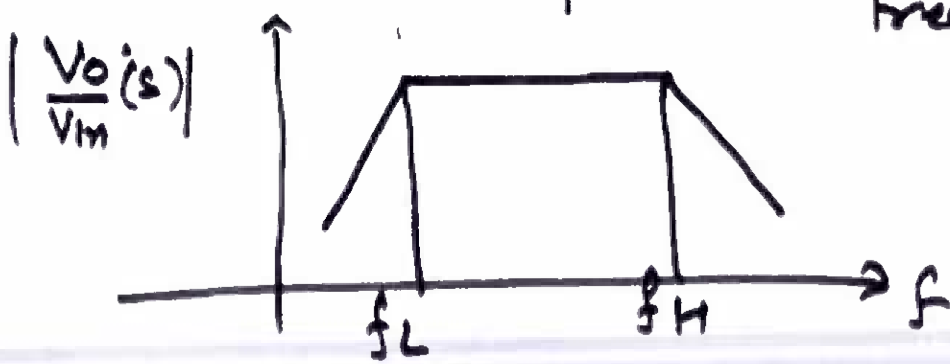
Two Poles are associated with these Two Time constants. We shall see later that

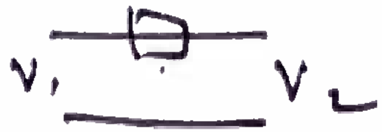
$$f_L = \frac{1}{2\pi\tau_s}$$ & is called Lower Corner (3db) Frequency

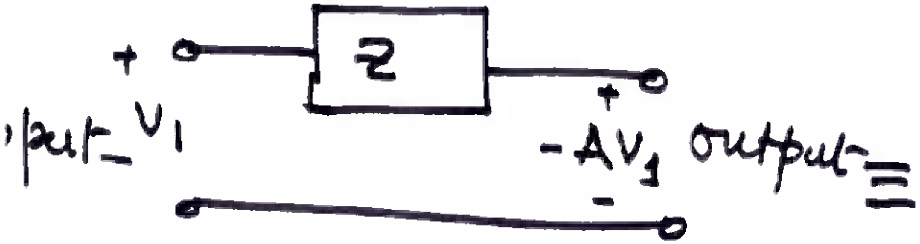
and $$f_H = \frac{1}{2\pi\tau_p}$$ and is called Upper Corner (3db) Frequency.

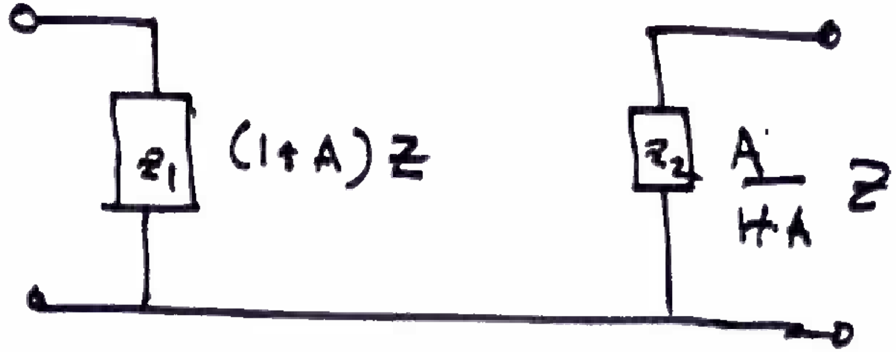


$f_H - f_L$ is called Midband and is essentially Bandwidth.

Miller's Theorem

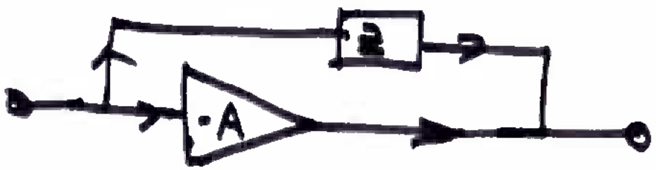






Two Port Network

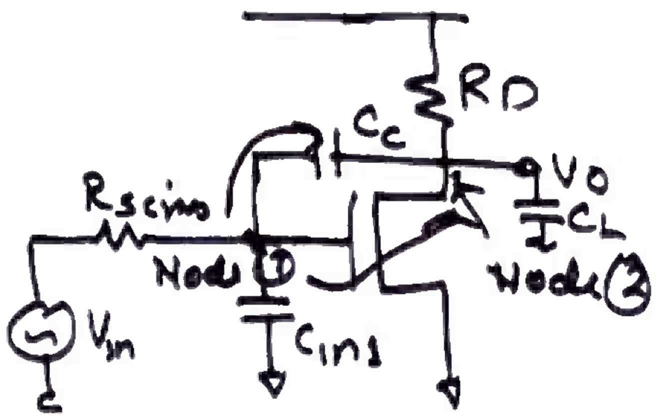
Condition for Validity of Miller's Theorem:



We must have Two Paths from Input to Output

Concept of Pole-Zero (Revisited)

A typical Amplifier is shown here



If C_{in} is input capacitance at Node ①, then C_{in} must contain contribution from C_{in1} and C_c.

C_c is ofcourse capacitance at node ① and node ② too

We can say pole's association is with Time Constant

$$\therefore \quad P_1 = \frac{1}{R_s C_{in}}.$$

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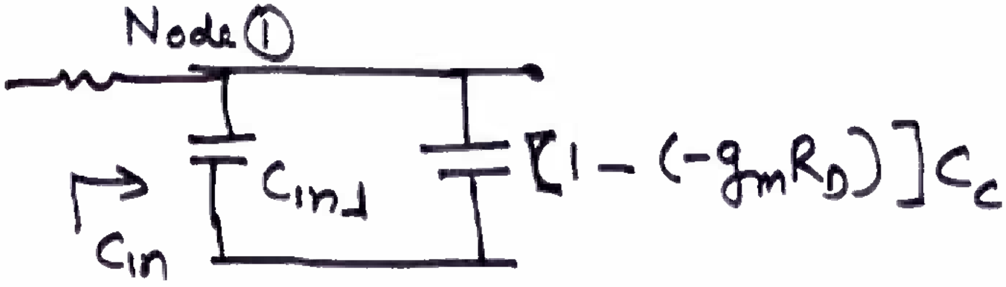
For our Amplifier, the Gain

$$A_v = -g_m R_D$$

$\therefore$ If C_c acts like component (Impedance) between Input & Output, we can convert it's contribution on Input & Output side independently using Miller's theorem

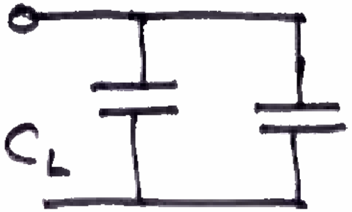
On Input Side



or $C_{in} = C_{in1} + (1 + g_m R_D) C_c$

$\simeq C_{in1} + (g_m R_D) C_c = C_{in1} + |A_v| C_c$

On the Output side then



$C_c \left(\frac{A}{1+A}\right) \simeq C_c$ $\quad \therefore C_{out} = C_L + C_c$

$P_1 = \frac{1}{R_s \cdot C_{in}}$ 1KHz

$\therefore Pole_2 = p_2 = \frac{1}{R_D C_{out}} = \frac{1}{R_D (C_L + C_c)}$

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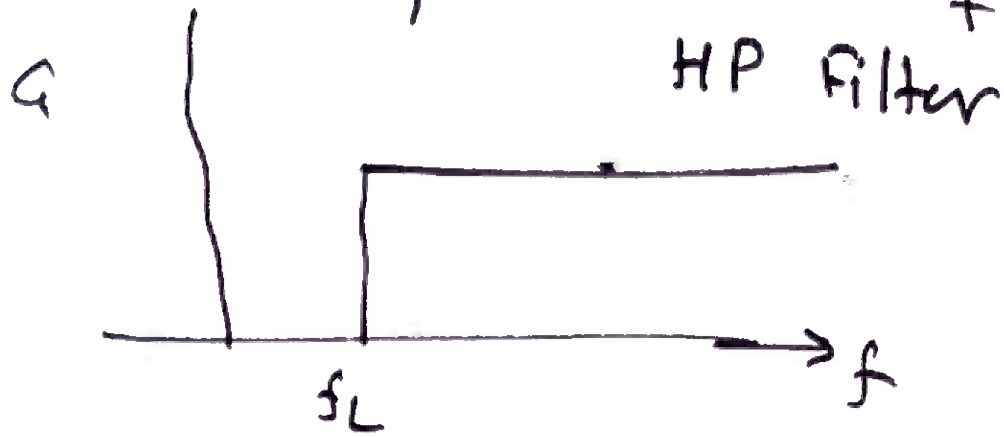

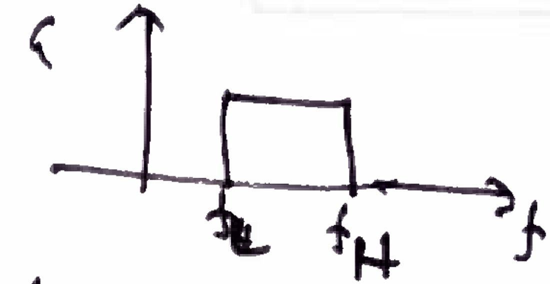

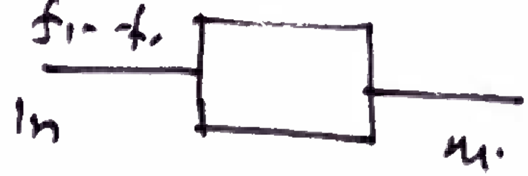

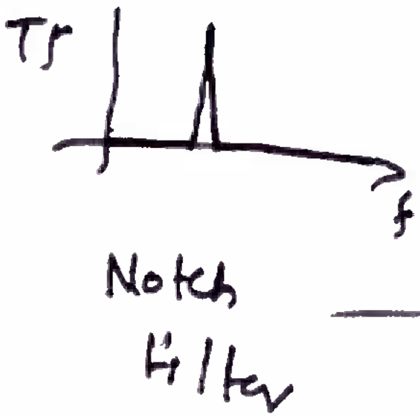

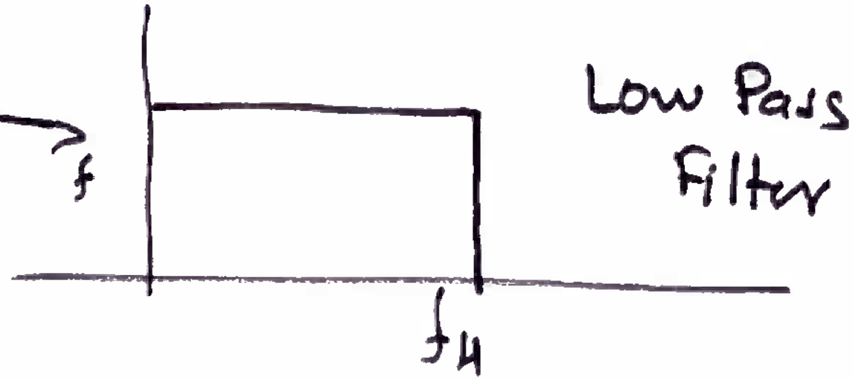

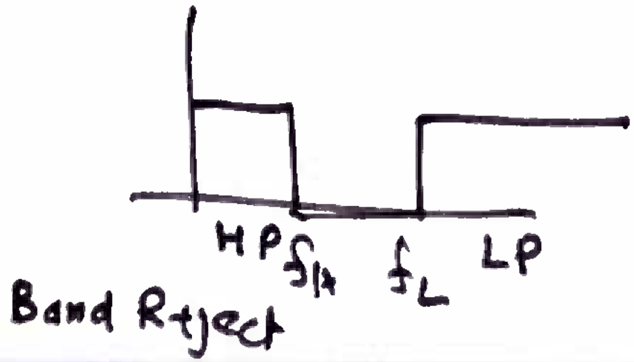

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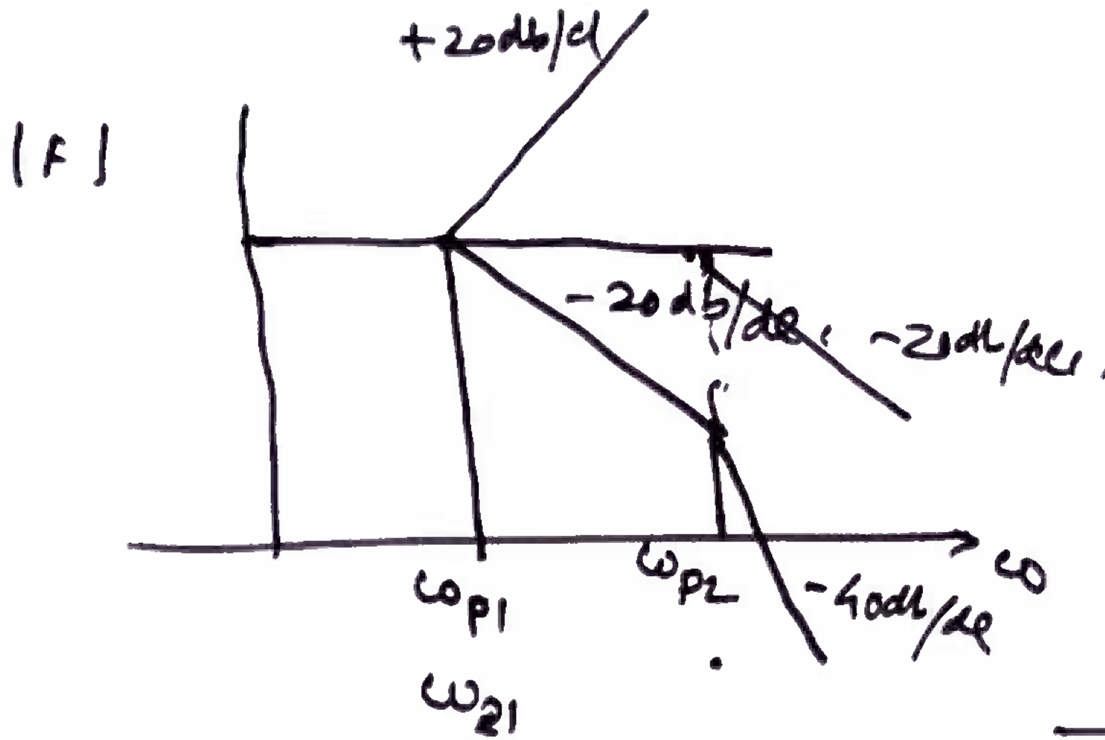
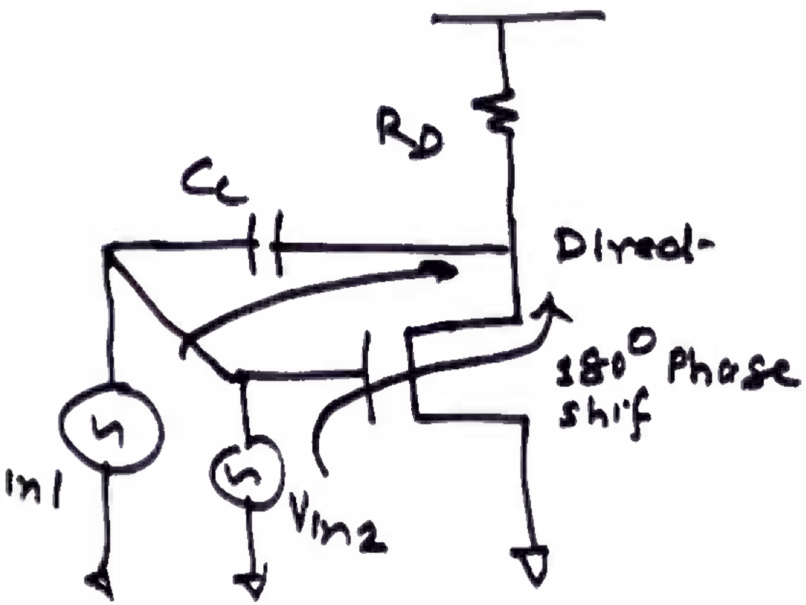


Stability

$$\frac{1}{s^2+as+c}$$

$$= \frac{1}{(s+s_1)(s+s_2)} = \frac{1}{s_1 s_2} \frac{1}{\left(\frac{s}{s_1}+1\right)\left(\frac{s}{s_2}+1\right)}$$

Concept of Zero



If Magnitude of two signals Direct & 180° phase out one are equal, then net $V_o = 0$ due to 180° phase shift

∴ $\boxed{\text{This is called Zero}}$

It can be shown

$$z_1 = \frac{g_m}{C_c}$$