Small Signal Low Frequency MOS MODEL

Small Signal means amplitude of signal is very much smaller than (amplitude) or value of DC bias.

i.e.

$$v_{GS} = V_{GS} + v_{gs}$$

$$v_{DS} = V_{DS} + v_{ds}$$

$$i_{DS} = i_{ds} + I_{DS}$$

For a MOSFET with small signal input at Gate (V_{GS}) we have

$$i_{DS} = I_{DS} + i_{ds} = \frac{\beta}{2}\left[(V_{GS} + v_{gs}) - V_{TH}\right]^2 (1 + \lambda V_{DS})$$

Assuming $\lambda \to 0$

$$i_{DS} = \frac{\beta}{2}(V_{GS} - V_T)^2 + \frac{\beta}{2}(v_{gs})^2 + 2 \cdot \frac{\beta}{2}(V_{GS} - V_T)\, v_{gs}$$

For small signal case $v_{gs}^2 \to$ small $\to$ Neglected

$$\therefore \quad i_{DS} = I_{DS} + i_{ds}$$

$$= \frac{\beta}{2}(V_{GS} - V_T)^2 + \beta(V_{GS} - V_T)\, v_{gs}$$

$$\therefore \quad i_{ds} = \beta(V_{GS} - V_T)\, v_{gs}$$

$$\frac{i_{ds}}{v_{gs}} = g_m = \beta(V_{GS} - V_T) = \frac{1}{2}\beta \frac{(V_{GS} - V_T)^2 \times 2}{(V_{GS} - V_T)}$$

$$= \sqrt{2\beta I_{DS}}$$

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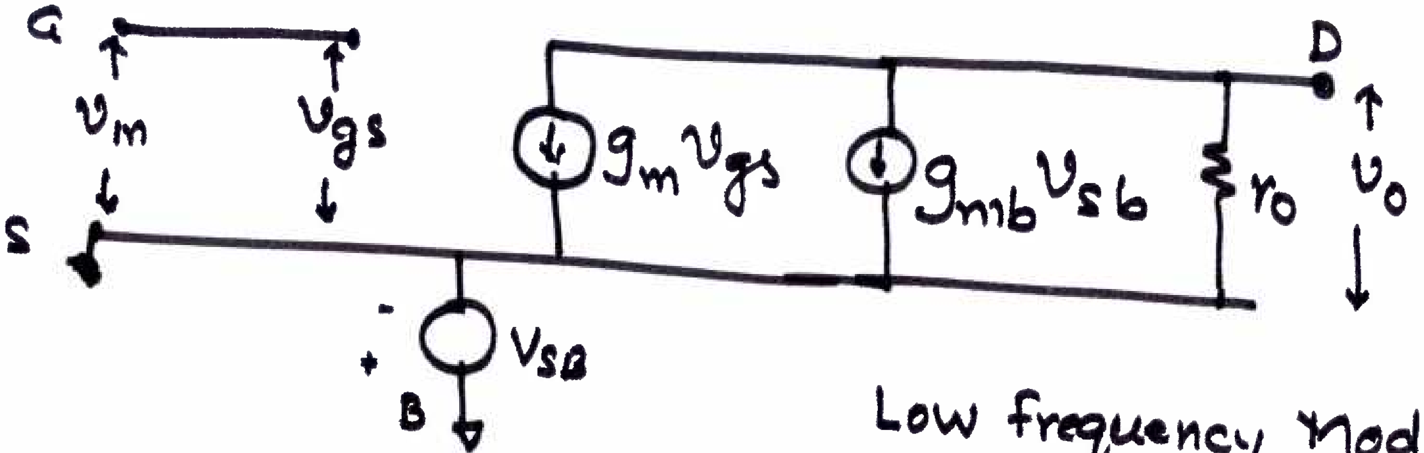
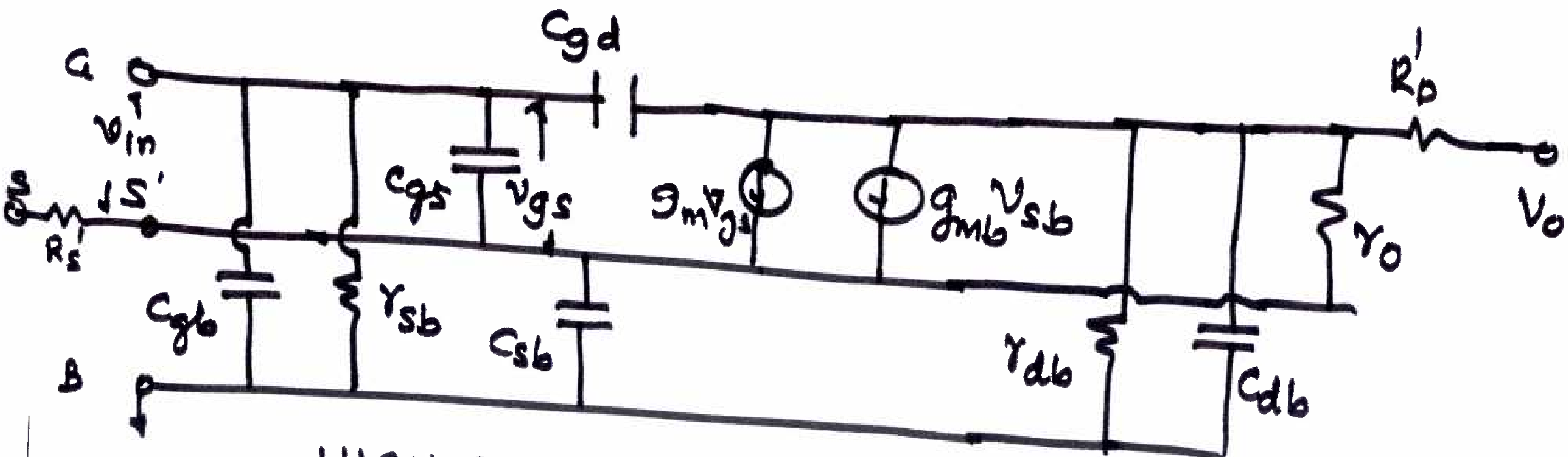
Small Signal MODEL

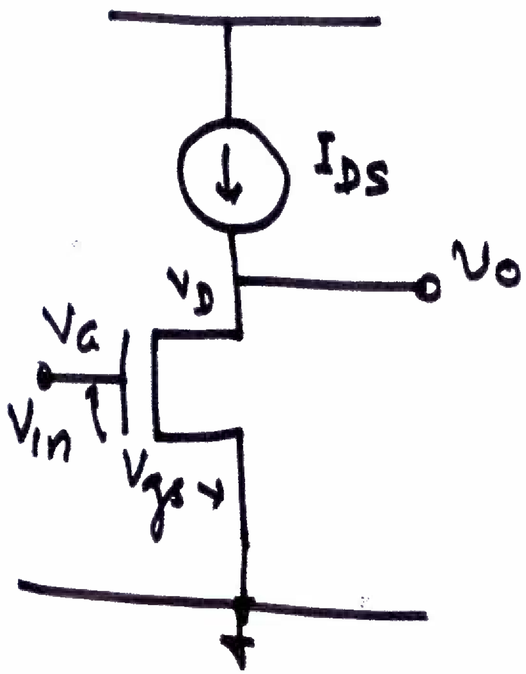


Low frequency Model



HIGH FREQUENCY SMALL SIGNAL MOS MODEL

Common Source Amplifier in an IC



Amplifier with constant current Biasing (I_{DS} const.)

We have $I_{DS} = f(V_G \,\&\, V_D)$

$$\therefore \Delta I_{DS} = \frac{\partial I_{DS}}{\partial V_G}.\Delta V_G + \frac{\partial I_{DS}}{\partial V_D}.\Delta V_D$$

By definition $g_m = \dfrac{\partial I_{DS}}{\partial V_G}$ & $g_o = \dfrac{\partial I_{DS}}{\partial V_D}$

$$\therefore \Delta I_{DS} = g_m \Delta V_G + g_o \Delta V_D$$

or $$dI_{DS} = g_m dV_G + g_o dV_D$$

But $v_{in} = dV_G$ and $v_o = dV_D$

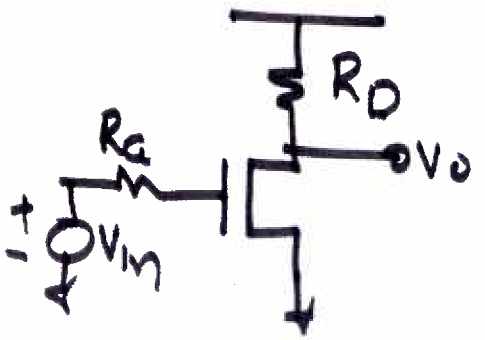
$$\therefore \quad dI_{DS} = g_m v_{in} + g_o v_o$$

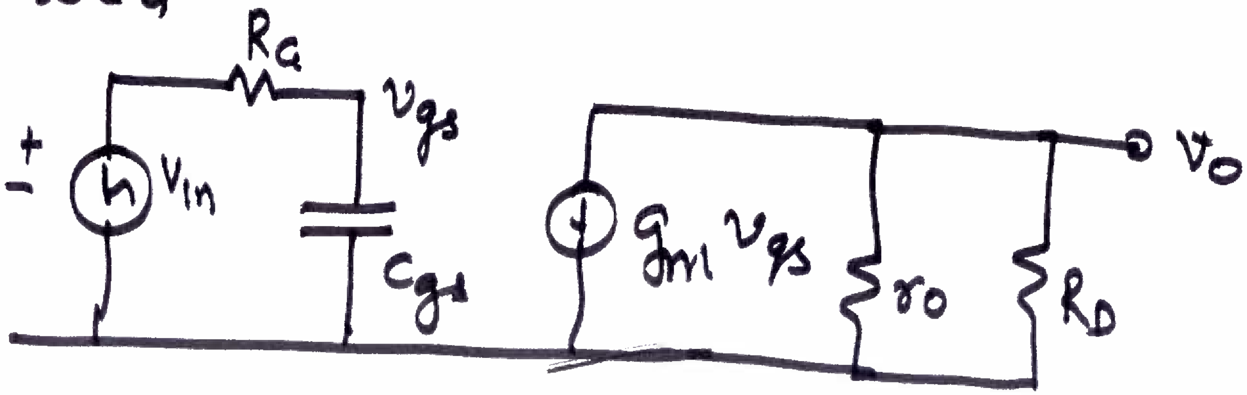
As Amplifier is biased by Constant Current Source I_{DS}, Hence $dI_{DS} = 0$

$$\therefore \quad 0 = g_m v_{in} + g_o v_o$$

$$\text{or} \quad \frac{v_o}{v_{in}} = -\frac{g_m}{g_o} = -g_m r_o$$

If we take First Order Model of MOSFET with Resistive load





If $r_{od} = r_o$

Then $A_{vo} = -g_m r_o$ or $\left|\frac{r_o}{A_{vo}}\right| = \frac{1}{g_m}$

We can see that

$$\frac{g_m}{I_{DS}} = \frac{2}{V_{ov}} \quad \& \quad \frac{g_m}{C_{gs}} = \frac{3}{2}\,\mu \frac{V_{ov}}{L^2}$$

Can be termed as Figures of Merit

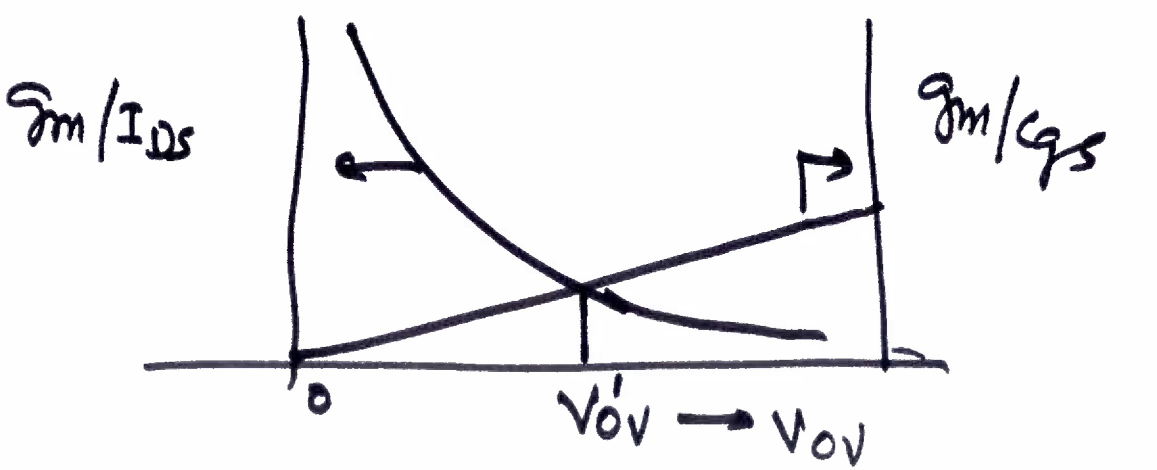
Hence Major decision for any Analog Designer is to choose V_{ov} appropriately, so that
1. Power Dissipation 2 Gain & 3. Bandwidth specs are met

We have observed that Performance of MOSFET Amplifier can be defined by two 'Figure of Merit', namely

$$\frac{g_m}{I_{DS}} \quad \text{and} \quad \frac{g_m}{C_{gs}}$$

where

$$\frac{g_m}{I_{DS}} = \frac{2}{V_{OV}} \quad \text{and} \quad \frac{g_m}{C_{gs}} = \frac{3}{2}\left(\frac{\mu}{L^2}\right) V_{OV}.$$



optimum value of $V_{ov} = V_{ov}'$ from Figure.

Hence Designer should make reasonable correct choice of V_{ov}.

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Another Figure of Merit could be defined as Product

$$FM_3 = \frac{g_m}{I_{DS}} \cdot \frac{g_m}{C_{gs}} = \frac{2}{V_{ov}} \cdot \frac{3}{2}\left(\frac{\mu}{L^2}\right) V_{ov}$$

$$= 3\left(\frac{\mu}{L^2}\right)$$

which is not a function of V_{ov}.

Hence to Improve FM_3, we must reduce Channel Length L.

FM3 essentially represents SPEED & POWER Efficiency together.

Think How about A_v ??

Then

$$H(s) = \frac{v_o(s)}{v_{in}(s)} = -g_m(r_o \| R_D) \cdot \frac{1}{1+sR_G C_{gs}}$$

We can use $g_m = \dfrac{2I_{DS}}{V_{OV}}$

$$C_{gs} = \frac{2}{3} W \cdot L\, C_{ox} \quad (\sim C_{ox})$$

& $r_o = \dfrac{1}{\lambda I_{DS}}$

Then $A_v(s) = H(s) = A_{vo} \dfrac{1}{1+s(R_g C_{gs})}$

where $A_{vo} = -g_m(r_o \| R_D) \simeq -g_m R_D$ if $R_D \ll r_o$

$= g_m r_o$ if $R_D \gg r_o$

↳ Say from Current Source

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From $A_v(s)$, we see we have a pole at $\omega_o = \frac{1}{R_G C_{gs}}$, which is therefore the <u>Bandwidth.</u>

If we write $C_{gs} = \frac{2}{3} W L C_{ox}$

~~Polt~~ $C_{ox} \mu \frac{W}{L} V_{ov} = g_m$

$$\therefore \quad C_{ox} = g_m \left(\frac{L}{W}\right) (V_{ov})^{-1} \frac{1}{\mu}$$

$$C_{ox} = \frac{A_{vo}}{(r_o \| R_D)} \cdot \frac{L}{W} \cdot \frac{1}{V_{ov}} \cdot \frac{1}{\mu}$$

, We define $r_{od} = r_o \| R_D$

$$\therefore \quad C_{gs} = \frac{2}{3} W L \cdot \frac{A_{vo}}{r_{od}} \left(\frac{L}{W}\right) \frac{1}{V_{ov}} \cdot \frac{1}{\mu}$$

$$\therefore \quad \omega_o = \frac{3}{2} \frac{r_{od}}{R_G} \cdot \frac{1}{A_{vo}} \frac{\mu}{L^2} V_{ov}$$

We see the Bandwidth

$$\omega_0 = \underbrace{\frac{3}{2}\cdot\frac{r_{OD}}{R_a}\cdot\frac{1}{A_{vo}}}_{\text{SPECS}}\cdot\underbrace{\frac{\mu}{L^2}}_{\text{Technology}}\cdot\underbrace{V_{ov}}_{\text{Design}} \qquad — (1)$$

We now evaluate Power Dissipation

$$P_D = V_{DD}\cdot I_{DS}$$

$$= \frac{1}{2}\,\frac{V_{DD}}{r_{od}}\cdot A_{vo}\,V_{ov}. \qquad — (2)$$

From (1) & (2) we observe

P_D can be reduced by Reducing V_{ov}

But Reduction in V_{ov} reduces Bandwidth

We have

$$I_{DS} = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)V_{ov}^2$$

Also, we have $g_m = \dfrac{2I_{DS}}{V_{ov}}$

$$\therefore \quad \frac{W}{L} = \frac{g_m}{\mu C_{ox} V_{ov}}$$

$$\text{or} \quad W = (g_m \cdot L)\left(\frac{1}{\mu}\right)\left(\frac{1}{C_{ox}}\right)\left(\frac{1}{V_{ov}}\right)$$

If we fix g_m & L, then for lower V_{ov} we get larger W – width of the Transistor

But $C_{gs} = \frac{2}{3} W \cdot L \cdot C_{ox}$, will increase if W increases

Which will reduce Bandwidth.

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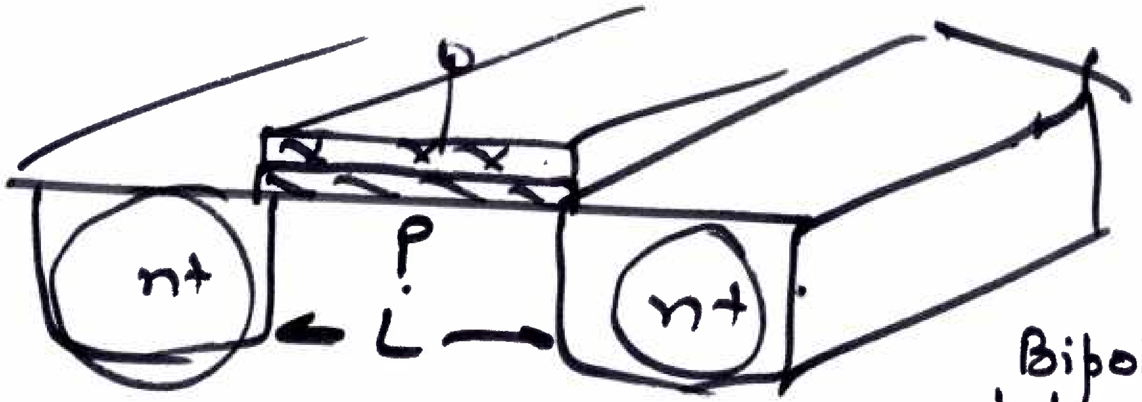
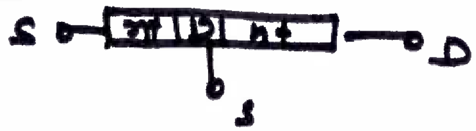
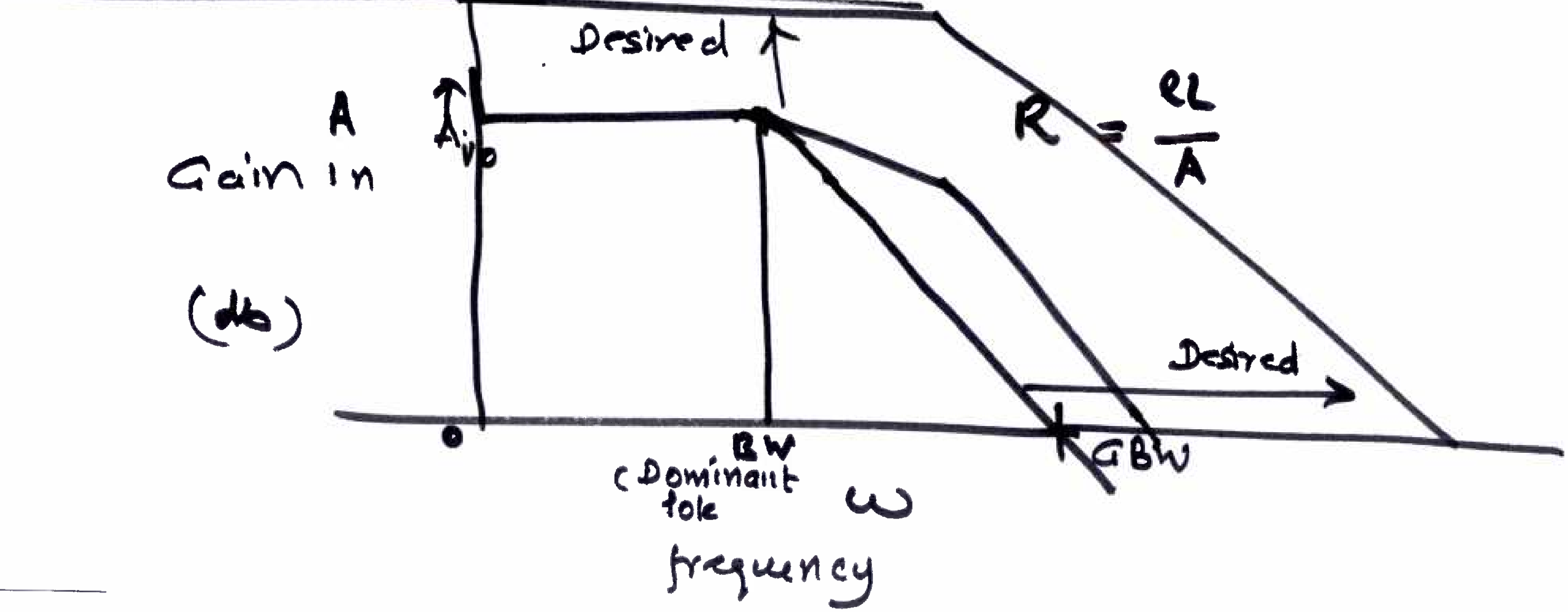


Bipolar lateral Transistor



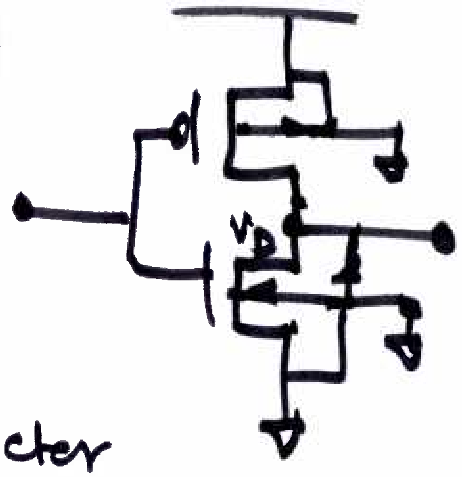


$R = 40K$ (We want!)

Then

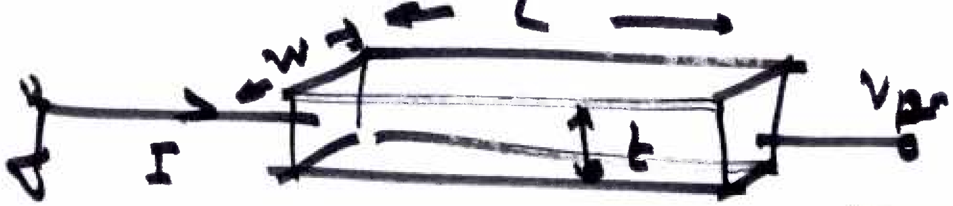
$$\boxed{\frac{L}{W} = 200}$$

If $R_S \rightarrow 200\ \Omega/\square$

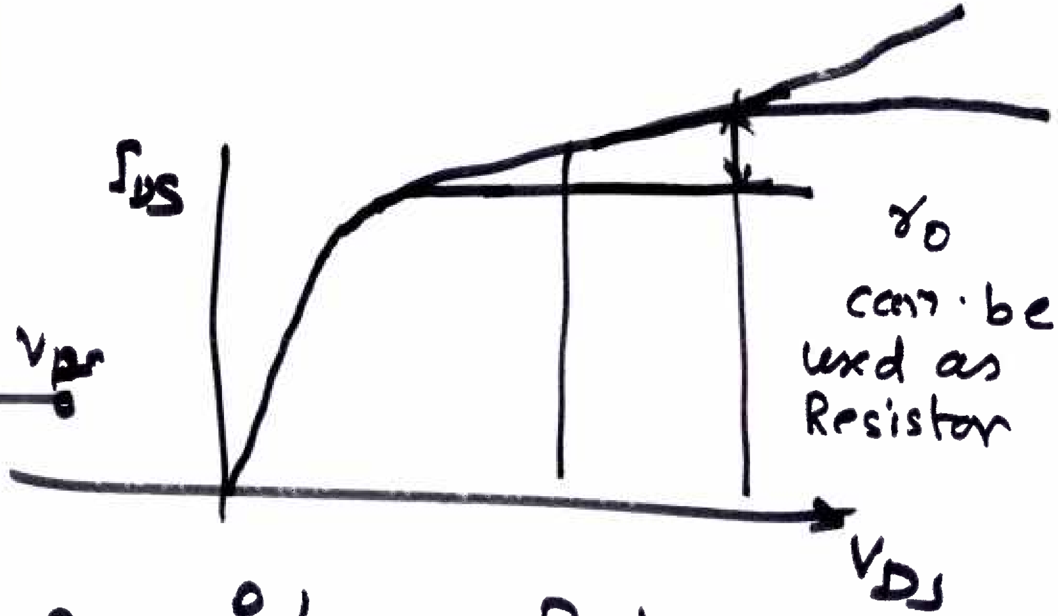


Meandering

Semiconductor Resistor



$$\frac{V}{I} = R$$



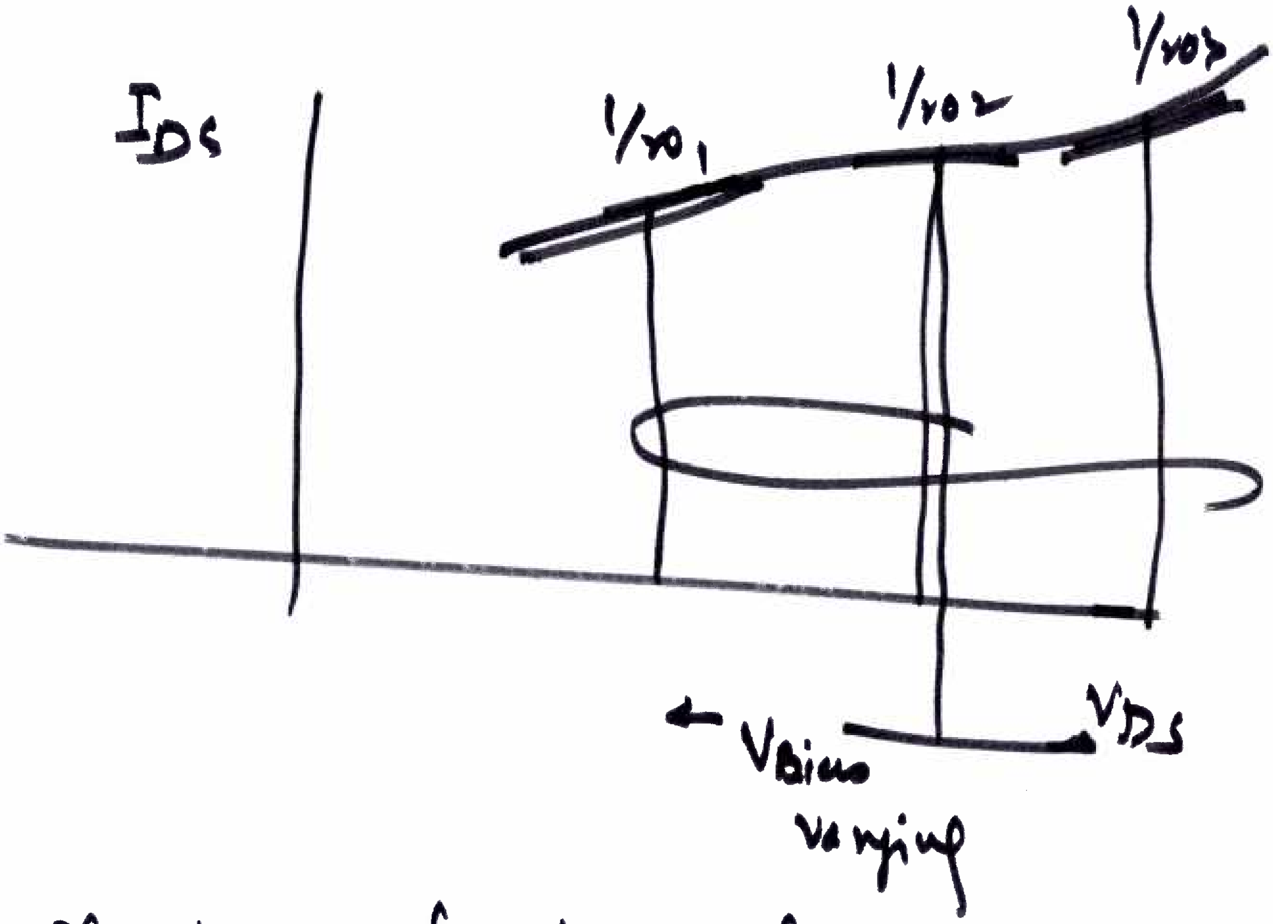
r_o can be used as Resistor

$$R = \frac{\rho L}{A} = \frac{\rho \cdot L}{W \cdot t}$$

$$R = R_s \left(\frac{L}{W}\right)$$

R_s = Sheet Resistance = $\left(\frac{\rho}{t}\right)$

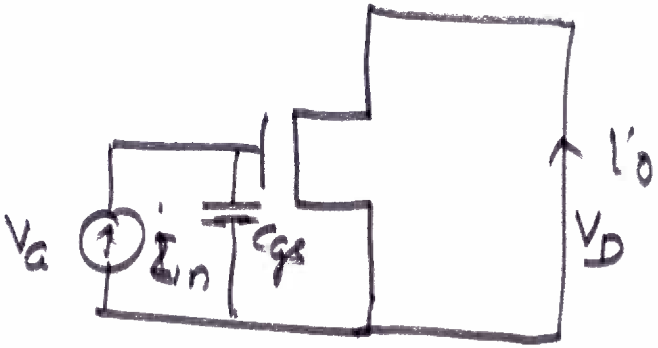
r_o is function of V_{DS} in Large Signal Case



r_o is a function of V_{DS} if V_{DS} swing is high, or V_{Bias} is too low or too high.

Concept of f_T (or ω_T)

f_T is a very standard Figure of Merit which is the frequency at which common source current gain i_o/i_{in} is <u>UNITY</u>.



$$v_{gs} C_{gs} \omega = i_{in}$$

$$g_m v_{gs} = i_o$$

$$\therefore \quad \frac{i_o}{i_{in}} = 1 = \frac{g_m}{\omega C_{gs}} = \frac{3}{2} \frac{\mu V_{ov}}{L^2} \cdot \frac{1}{\omega}$$

or $$\omega_T = 2\pi f_T = \frac{g_m}{C_{gs}} = \frac{3}{2} \frac{\mu V_{ov}}{L^2}$$

$$\therefore \quad f_T = \frac{3}{2\pi} \left(\frac{\mu}{L^2}\right) V_{ov}$$

$$\simeq 10 \text{ GHz}$$

Concept of ω_{max}.

Typically $\omega_{max} = \frac{\omega_T}{10}$ — Lumped Model Validation.

or $\omega_{max} = \frac{1}{2}\sqrt{\left(\frac{1}{R_a C_{gd}}\right)\omega_T}$

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In evaluation of f_T, we have used Long Channel Model for MOSFET.

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Exptal observations

1. f_T measured at large V_{ov}, In Weak Inversion and in V.Strong Inversion show lower values compared to theoretical value.

Possible reason :

In Short channel devices, g_m is lower at larger V_{ov}. It reduces g_m/I_{DS} term, as well as f_T $(= g_m/C_{gs})$. In Short channel case Source-Sub-Drain forms a BJT which is stronger in Weak & Strong Inversion.

In short channel devices, lateral fields and velocity saturation affect the g_m/I_{DS}

It is known that approx. expressions which can be used in short channel devices based Analog Circuits are :—

$$I_{DS} = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)V_{ov}^2\left(\frac{1}{1+\frac{V_{ov}}{E_c \cdot L}}\right)$$

$$\cong \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)V_{ov}\left(\frac{E_c \cdot L \cdot V_{ov}}{E_c L + V_{ov}}\right)$$

Typical $E_c \simeq 6\times10^6$ V/m for 0.25μ.

$= 1.5\times10^6$ V/m for 0.13μ.

Then $$\frac{g_m}{I_{DS}} = \frac{2}{V_{ov}} \cdot \frac{1}{1+\frac{V_{ov}}{E_c \cdot L}}$$

For $E_c L = 2V$ & $V_{ov} = 0.2V$

$$g_m/I_{DS} = \left(\frac{2}{V_{ov}}\right)\underline{\underline{(0.9)}}$$ ← correction factor