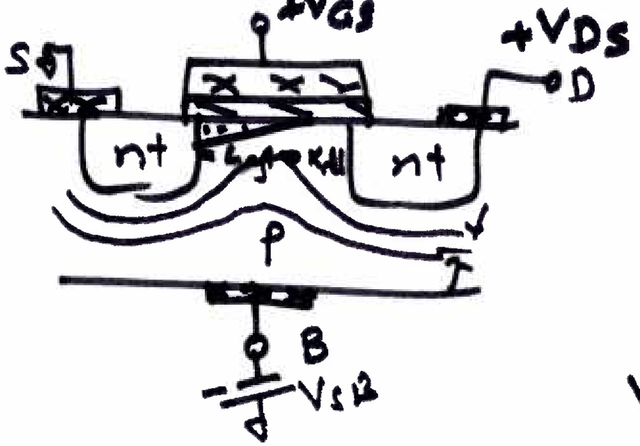
# Analog MOS Model

① $V_T$ Variation with Substrate Bias $V_{SB}$




As $V_{SB}$ increases(-), Depletion Layer increases, which in turn increases $Q_B$. Hence $V_T$ increases.

$V_T$ with $V_{SB}$ is given by

$$V_T = V_{T0} + \gamma \left[ (V_{SB} + 2\phi_F)^{1/2} - (2\phi_F)^{1/2} \right]$$

where $V_T$ is Threshold Voltage with $V_{SB} \rightarrow -ve$
$V_{T0}$ is Threshold Voltage with $V_{SB} = 0$

& $$\gamma = \frac{1}{C_{ox}} \sqrt{2 K_s \epsilon_0 q N_{a,d}}$$

Let us find variation of $V_T$ with $V_{SB}$

$$\frac{\partial V_T}{\partial V_{SB}} = 0 + \frac{\gamma}{2}\left(V_{SB} + 2\phi_F\right)^{-1/2}$$

(2) The Saturation parameter $\lambda$, has been described as $\frac{\lambda'}{L}$ and treated constant ($\neq f(V_{DS})$)

However $\lambda$ has two components which shows that each is function of $V_{DS}$

$$\lambda = \lambda_c + \lambda_m$$

$\lambda_c \rightarrow$ Related to Channel length modulation

and $\lambda_m \rightarrow$ Related to Field Dependent Mobility

We observe that $x_c$ is function of depletion layer widths, $x_{dl}$.

As $$x_{dl} = \sqrt{\frac{2K_s\epsilon_0}{qN_a}(\psi_s + V_{SB}')}$$

where $$V_{SB}' = V_{SB} + V_{DS}$$

or $$x_{dl} = \sqrt{\frac{2K_s\epsilon_0}{qN_a}}\left(\psi_s + V_{SB} + V_{DS}\right)^{1/2}$$

$$x_{dl} = K_1'\left(\psi_s + V_{SB} + V_{DS}\right)^{+1/2}$$

$$\therefore \frac{dx_{dl}}{dV_{DS}} = K_1' \frac{d}{dV_{DS}}\left(\psi_s + V_{SB} + V_{DS}\right)^{1/2}$$

or $$\frac{dx_{dl}}{dV_{DS}} = K_1' \cdot \frac{1}{2}(\psi_s + V_{SB} + V_{DS})^{-1/2}$$

$$= \sqrt{\frac{K_s\epsilon_0}{2qN_a}\frac{1}{(\psi_s + V_{SB} + V_{DS})}}$$

$$= K_1''(\psi_s + V_{SB} + V_{DS})^{-1/2}$$

$$= \frac{K_1''}{(\psi_s + V_{SB})^{1/2}}\left[1 + \frac{V_{DS}}{\psi_s + V_{SB}}\right]^{-1/2}$$

We know after Pinch off, increase of $V_{DS}$, increases $x_{dl}$

$$\therefore \quad L_{eff} = L_{Mask} - x_{dl}$$

& $$\lambda_C = \left[\frac{1}{L_{eff}} \cdot \frac{dx_{dl}}{dV_{DS}}\right]$$

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We have $I_{DS}$ in Saturation

$$I_{DS} = \frac{1}{2}\beta'\left(\frac{W}{L_{eff}}\right)(V_{GS}-V_T)^2$$

$$= \frac{1}{2}\beta'\left(\frac{W}{L}\right)\frac{L}{L_{eff}}(V_{ov})^2$$

$$L_{eff} = L - x_{dl} \quad \text{or} \quad \frac{L}{L_{eff}} = 1 + \frac{x_{dl}}{L_{eff}}$$

$$I_{DS} = \frac{1}{2}\beta\left(1 + \frac{x_{dl}}{L_{eff}}\right)V_{ov}^2$$

We define $\lambda_c = \frac{1}{L_{eff}} \cdot \frac{dx_{dl}}{dV_{DB}}$

or $\lambda_c = \frac{1}{L - x_{dl}} \cdot \frac{dx_{dl}}{dV_{DS}}$

$$\text{or} \quad \lambda_c = \frac{1}{L\left(1 - \frac{x_{dl}}{L}\right)} \cdot \frac{K_1''}{(\psi_s + V_{SB})^{1/2}} \left(1 + \frac{V_{DS}}{(\psi_s + V_{DS})}\right)^{-1/2}$$

$$= \frac{1}{L\left[1 - \frac{1}{L}\sqrt{\frac{2k_s\epsilon_0}{qN_a}(\psi_s + V_{SB})}\right]} \times$$

$$\times \frac{K_1''}{(\psi_s + V_{SB})^{1/2}} \left(1 + \frac{V_{DS}}{\psi_s + V_{DS}}\right)^{-1/2}$$

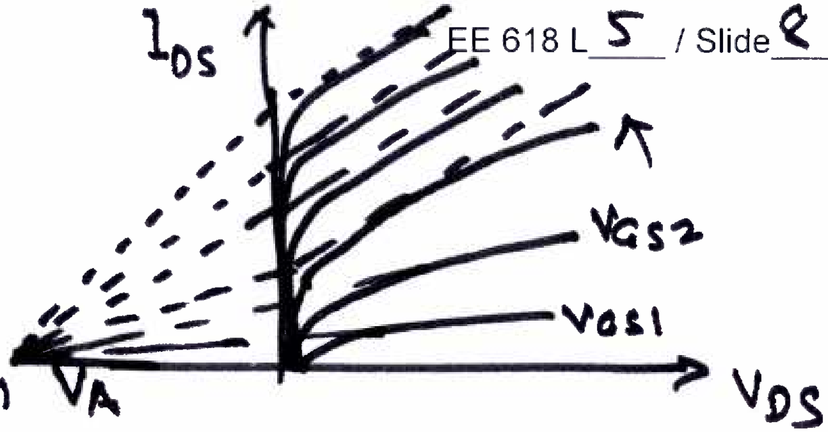
$$\lambda_c = \frac{\lambda'}{L} = \frac{1}{L}\left[\frac{1}{1 - \frac{1}{L}\sqrt{\frac{2k_s\epsilon_0}{qN_a}(\psi_s + V_{SB})}} \cdot \frac{K_1''}{(\psi_s + V_{SB})^{1/2}} \cdot \left(1 + \frac{V_{DS}}{\psi_s + V_{SB}}\right)^{-1/2}\right]$$

Mobility Dependent $\lambda$ is given by

$$\lambda_m = \frac{\mu(0) - \mu(V_{DS})}{\mu(0) \cdot V_{DD}}$$

$\mu(0)$ = Low Field Mobility

$\mu(V_{DS})$ = Mobility at $V(y) = V_{DS}$




However overall $\lambda = \lambda_c + \lambda_m$

$$I_{DS} = \frac{1}{2}\beta'\left(\frac{W}{L}\right)(V_{GS} - V_T)^2(1 + \lambda V_{DS})$$

$$= I_{DS}'(1 + \lambda V_{DS}) = I_{DS}'\left(1 + \frac{V_{DS}}{V_A}\right)$$

where $V_A$ is called Early Voltage $= \frac{1}{\lambda}$

Revisiting $V_T$ relationship with $V_{SB}$

We have

$$V_T = V_{T0} + \gamma\left[(V_{SB}+2\phi_F)^{1/2} - (2\phi_F)^{1/2}\right]$$

$$\frac{\partial V_T}{\partial V_{SB}} = 0 + \gamma\left[\frac{1}{2}(V_{SB}+2\phi_F)^{-1/2}\right]$$

$$= \frac{1}{2}\gamma\left[(V_{SB}+2\phi_F)\right]^{-1/2}$$

Lets see variation of $I_{DS}$ with $V_{BB}$

We have $\dfrac{\partial I_{DS}}{\partial V_{SB}} = -\dfrac{\partial I_{DS}}{\partial V_{BS}}$

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$$I_{DS} = \frac{1}{2}\beta\,(V_{GS} - V_T)^2\,(1 + \lambda V_{DS})$$

$$\cong \frac{1}{2}\beta\,(V_{GS} - V_T)^2 \qquad \text{Taking } \lambda \text{ Small}$$

$$\text{or} \quad \frac{\partial I_{DS}}{\partial V_{SB}} = 2\beta\,(V_{GS} - V_T)\left(-\frac{\partial V_T}{\partial V_{SB}}\right)$$

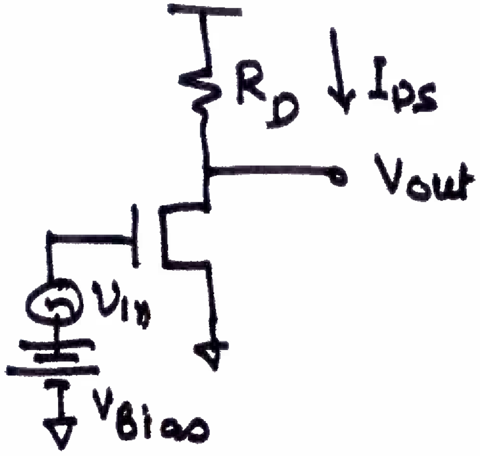
$$= 2\beta\,(V_{GS} - V_T)\cdot\frac{1}{2}\gamma\,(V_{SB} + 2\phi_F)^{-1/2}$$

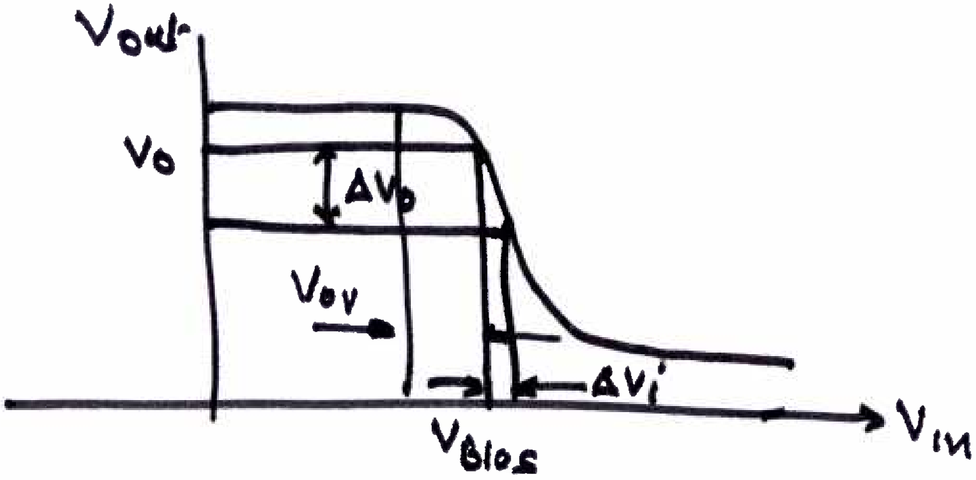
$$= 2\beta\,(V_{GS} - V_T)\;\frac{\gamma}{2(V_{SB} + 2\phi_F)^{1/2}}$$

$$= \underline{2\beta\,(V_{GS} - V_T)}\cdot\eta \qquad g_{mb} = g_m \cdot \eta$$

$$\text{where } \eta = \frac{\gamma}{2(V_{SB} + 2\phi_F)^{1/2}} \simeq 0.6$$

# Take a case of Simple MOS Amplifier




$V_{in} = V_{Bias} + V_{signal}$

or $\Delta V_{in} = V_{in} - V_{Bias}$

$i_{DS} = I_{DS} + \Delta I_{DS}$

$\Delta I_{DS}$ is ac current

Clearly $\Delta v_o$ = ac output

$= -\Delta I_{DS} \cdot R_D$

But $\dfrac{\Delta I_{DS}}{\Delta V_{in}} = g_m$

$\therefore \Delta v_o = -g_m R_D \Delta V_{in}$

or Gain $A_v = \dfrac{\Delta v_o}{\Delta V_{in}} = -g_m R_D$

Representation of our Amplifier is :


Since MOS Transisistor in Saturation acts like <u>VCCS</u>

We write

$$\Delta V_{out} = \left[\frac{1}{2}\beta V_{ov}^2 - \frac{1}{2}\beta\left(V_{ov} + \Delta V_{in}\right)^2\right] R$$

or

$$\Delta V_{out} = -\frac{1}{2}\beta R\left[V_{ov}^2 + \Delta V_{in}^2 + 2V_{ov}\cdot\Delta V_{in} - V_{ov}^2\right]$$

$$= -\frac{1}{2}\beta R\left[\Delta V_{in}^2 + 2V_{ov}\Delta V_{in}\right]$$

$$= -\frac{1}{2}\beta R\, 2V_{ov}\cdot\Delta V_{in}\left[1 + \frac{\Delta V_{in}}{2V_{ov}}\right]$$

or $\Delta V_{out} = -\frac{1}{2}\beta V_{ov}^2 R \; \frac{2\Delta V_{in}}{V_{ov}} \left(1 + \frac{\Delta V_{in}}{2 V_{ov}}\right)$

$$= - I_{DS} R \frac{2\Delta V_{in}}{V_{ov}} \left(1 + \frac{\Delta V_{in}}{2 V_{ov}}\right)$$

$$= - \frac{2 I_{DS}}{V_{ov}} . R \; \Delta V_{in} \left(1 + \frac{\Delta V_{in}}{2 V_{ov}}\right)$$

or Gain $= A_V = \frac{\Delta V_o}{\Delta V_{in}} = - \frac{2 I_{DS}}{V_{ov}} . R \left(1 + \frac{\Delta V_{in}}{2 V_{ov}}\right)$

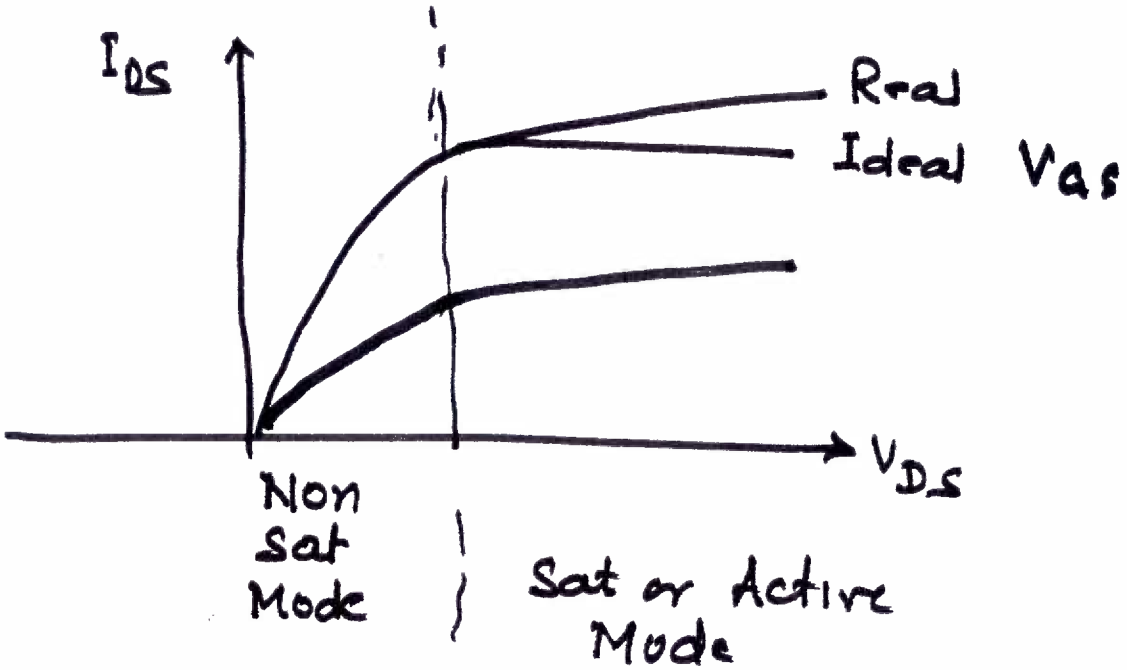
If ac Signal $\Delta V_{in} << 2 V_{ov}$

Then $A_V = - \frac{2 I_{DS}}{V_{ov}} . R = - g_m R$

$$\therefore \boxed{g_m = + \frac{2 I_{DS}}{V_{ov}}}$$

# Small Signal Output Resistance $r_o$






$$\frac{1}{r_o} = \left.\frac{\partial I_{DS}}{\partial V_{DS}}\right|_{V_{GS}}$$

$r_o$ is Finite in Real case

as $I_{DS} = f(V_{DS})$ in Sat Mode

$$I_{DS} = \frac{\beta}{2}\left[(V_{GS}-V_T)^2\right]\left[1+\lambda V_{DS}\right]$$

$\lambda$ is Saturation Parameter

① Channel Length Modulation ② In reality

$\lambda$ is a 'Fudge Factor' to fit the $I_{DS}-V_{DS}$ charact.

$$r_o^{-1} = g_o = \frac{\partial I_{DS}}{\partial V_{DS}} =$$

$$\frac{2I_{DS}}{\beta V_{OV}^2} = 1+\lambda V_{DS} \quad ; \quad \lambda = \left(\frac{2I_{DS}}{\beta V_{OV}^2} - 1\right)\frac{1}{V_{DS}}$$

$$\frac{\partial I_{DS}}{\partial V_{DS}} = \frac{1}{2}\beta V_{OV}^2 \cdot 0 + \frac{1}{2}\beta V_{OV}^2 \lambda$$

or $$\frac{1}{r_o} = \frac{1}{2}\beta V_{OV}^2 \lambda$$

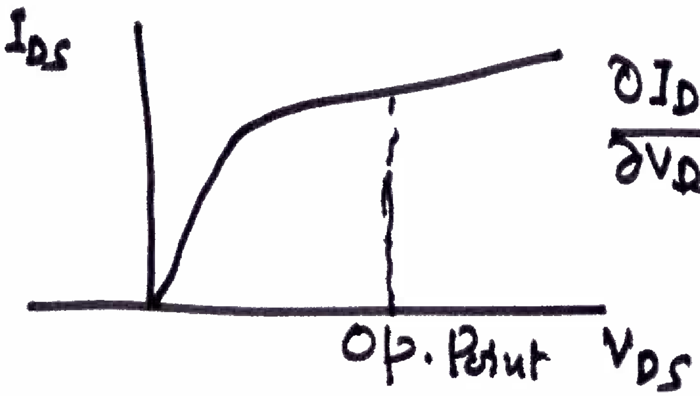
$$= \frac{1}{2}\beta V_{OV}^2 \frac{(1+\lambda V_{DS})}{(1+\lambda V_{DS})} \cdot \lambda$$

$$= I_{DS} \frac{\lambda}{1+\lambda V_{DS}} = I_{DS}\lambda(1-\lambda V_{DS}) \quad \lambda \text{ small}$$

$$= I_{DS}\lambda - I_{DS}\lambda^2 V_{DS} \cong I_{DS}\lambda$$

$$\therefore r_o = \frac{1}{I_{DS}\lambda} \quad \text{where } \lambda = \frac{\lambda'}{L} \; ; \left(\lambda' = \sqrt{\frac{2}{q N_{sub}}}\right)$$

# Behavior of $r_o$




$$\frac{\partial I_{DS}}{\partial V_{DS}} = \text{const}$$



Aim : (a) Make $\frac{\partial I_{DS}}{\partial V_{DS}}$ constant in region of Interest

(b) Be remain in Small Signal Domain

However Bias Operating Point and larger $V_{DS}$ swing may see $r_o = f(V_{DS})$

Designer Should Note :

1. $r_o$ evaluation is Tentative.

2. It depends upon Technology Parameters

3. It depends upon Operating Point

4 It depends upon Output Swing

$r_o$ is therefore showing Non Linear Behaviour.

⇒ Use Proper Bounds in SPICE Simulations.

# Analog Design Issues in CS Amplifier

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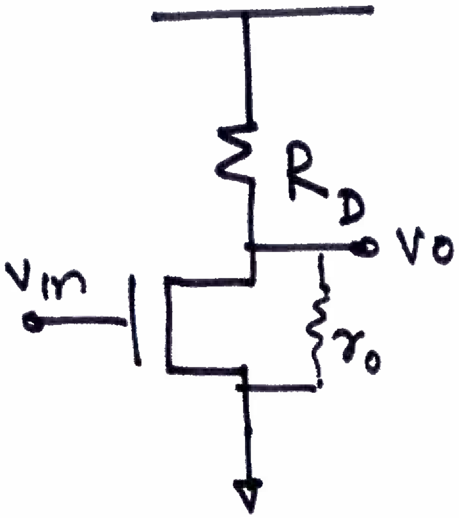


We have seen

$$g_m = \frac{2I_{DS}}{V_{OV}} = \partial I_{DS} / \partial V_{OV}$$

& Gain $= -g_m(R_D \| r_o)$

An MOSFET in saturation, has $I_{DS}-V_{DS}$ as

$$I_{DS} = \frac{1}{2}\beta (V_{OV})^2 (1+\lambda V_{DS}) = \frac{1}{2}\beta(1+\lambda V_{DS}) \cdot V_{OV}^2$$

or $$V_{OV}^2 = \frac{2I_{DS}}{\beta(1+\lambda V_{DS})}$$ or

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \frac{\partial I_{DS}}{\partial V_{OV}} = \beta(1+\lambda V_{DS})V_{OV} = \frac{2I_{DS}}{V_{OV}}$$

$$\therefore \quad g_m = \frac{2 I_{DS}}{V_{ov}} \quad \text{or} \quad g_m \propto \frac{1}{V_{ov}} \longrightarrow (1)$$



We look at $I_{DS}-V_{DS}$ characteristics again

$$I_{DS} = \frac{1}{2} \beta \left[ V_{ov}^2 \right] (1+\lambda V_{DS})$$

We assume $\lambda$ very small & introduce a technology parameter $\alpha$

Then $I_{DS} = \frac{\beta}{2\alpha} \cdot V_{ov}^2$ Typically $\alpha \rightarrow 1$

or more precisely

$$I_{DS} = \frac{\beta}{2\alpha} (V_G - V_T - \alpha V_s)^2 \simeq \frac{\beta}{2\alpha} \cdot V_{ov}^2$$

Now $\frac{\partial I_{DS}}{\partial V_{GS}} = \frac{\partial I_{DS}}{\partial V_{ov}} = g_m = \frac{\beta}{\alpha} \cdot V_{ov}$

Now $g_m \propto V_{ov} \longrightarrow (2)$

We rewrite $I_{DS}-V_{DS}$ equation

$$I_{DS} = \frac{\beta}{2\alpha} V_{ov}^2$$

or $$V_{ov} = \sqrt{\frac{2\alpha I_{DS}}{\beta}}$$

But we just derive $$g_m = \frac{\beta}{\alpha} V_{ov}$$

or $$g_m = \frac{\beta}{\alpha}\sqrt{\frac{2\alpha I_{DS}}{\beta}} = \sqrt{\frac{2\beta}{\alpha} I_{DS}}$$

or to say $$g_m \neq f(V_{ov})$$

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# Design of Amplifier with MOSFETs



DESIGN PARAMETERS ARE

| Small Signal Parameters ↓ | $(W, L, V_{ov})$ | $(W, L, I_{DS})$ | $(V_{ov}, L, I_{DS})$ |
|---|---|---|---|
| $g_m$ | $\frac{\mu C_{ox} W V_{ov}}{\alpha L}$ | $\sqrt{2\beta I_{DS}/\alpha}$ | $\frac{2 I_{DS}}{V_{ov}}$ |
| $r_o$ | $\frac{2L^2\alpha}{\lambda' \mu C_{ox} W V_{ov}^2}$ | $\frac{L}{\lambda' I_{DS}}$ | $\frac{L}{\lambda I_{DS}}$ |
| $A_0$ | $\frac{2L}{\lambda' V_{ov}}$ | $\frac{1}{\lambda'}\sqrt{\frac{2\beta L^2}{\alpha I_{DS}}}$ | $\frac{2L}{\lambda' V_{ov}}$ |
| GBW | $\frac{\beta V_{ov}}{\alpha C_{Total}}$ | $\frac{\sqrt{2\beta I_{DS}/\alpha}}{C_{Total}}$ | $\frac{2 I_{DS}}{V_{ov} C_{Total}}$ |
| What you do → | Fix $V_{ov}$, Vary $I_{DS}$ | Fix $I_{DS}$, vary $V_{GS}$ | Fix $V_{ov}$ and $I_{DS}$, Vary W |