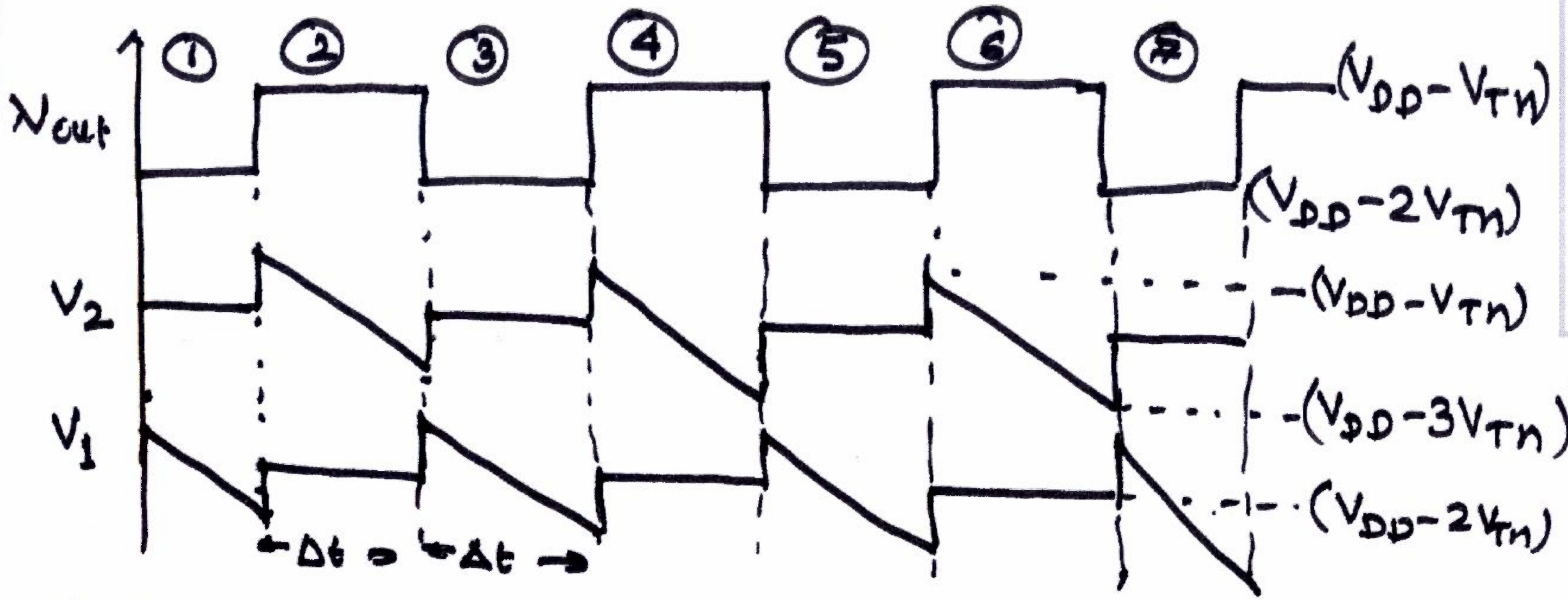
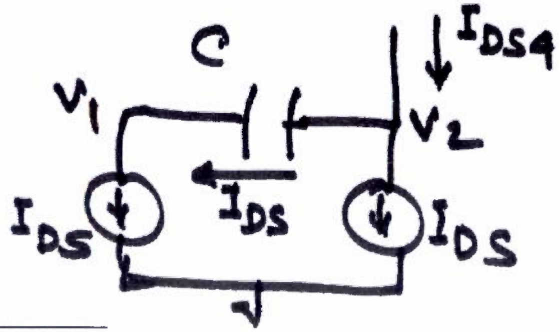


① If M3 is OFF & M4 ~~are~~ is ON

Then $V_{out} = V_{DD} - V_{Tn_{M6}} - V_{Tn_4} = V_{DD} - 2V_{Tn}$

Then no current goes through M3. But M4 must provide $2I_{DS}$ current, as it must cater to I_{DS_2} & I_{DS1} currents. I_{DS1} comes through charging of Capacitor C.



CDEEP
IIT Bombay

Case 2 Now if M3 is ON and M4 is OFF

Then $\overline{V_{out}}$ will be similar to that of V_{out} as in case 1.

Hence V_1 and V_2 will alternate in time frame.

Since I_{DS} charges capacitor in time Δt, hence delivered charge in capacitor C $= I_{DS} \cdot \Delta t$

However capacitor sees change of $V_{DD} - V_{TN} - V_{DD} - 3V_{TN}$

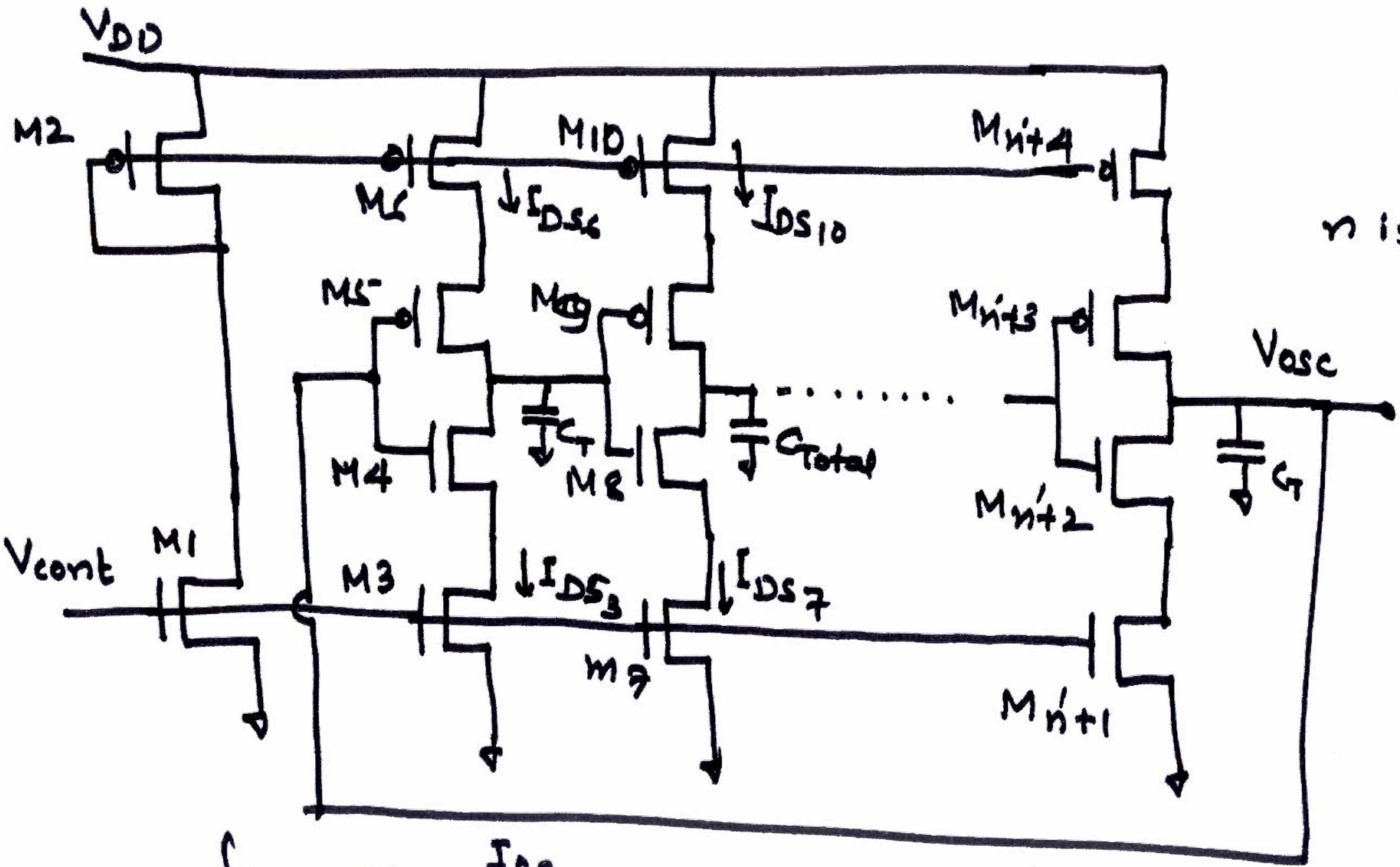
$= 2V_{TN}$

$\therefore$ Charge in Capacitor $= C \cdot 2V_{TN}$

$\therefore \quad \Delta t = \dfrac{2CV_{TN}}{I_{DS}}$. But Frequency $f = \dfrac{1}{2\Delta t} = \dfrac{I_{DS}}{4CV_{TN}}$

Current Starved VCO

This is essentially a Ring Oscillator

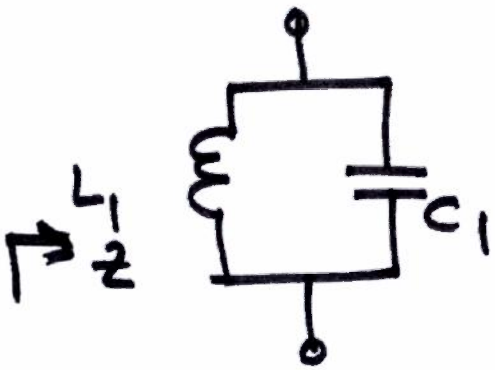


n is no. of Column.

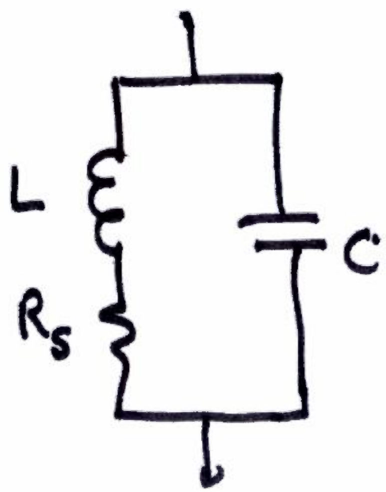
$$f_{osc} = \frac{I_{DS}}{N \cdot C_{Total} \cdot V_{DD}}$$

LC Oscillators

All LC Oscillators use Resonant Tank Circuit to sustain oscillation.



Ideal Inductor $f = \frac{1}{2\pi\sqrt{LC}}$



Series resistance of Inductor coil.

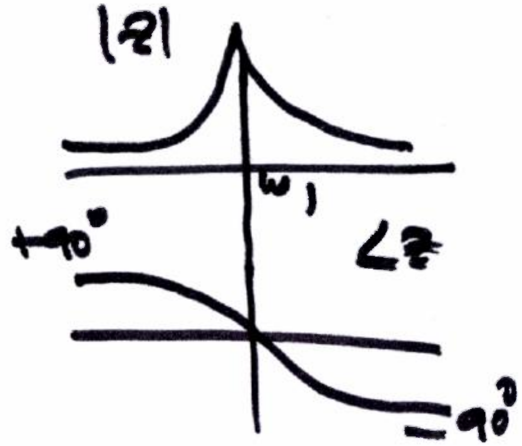
In IC chips, Spiral Inductor Coils are printed to have desired Inductance value

For the Non-Ideal Tank circuit

$$Z_{TC}(s) = \frac{R_s + L_1 s}{1 + L_1 C_1 s^2 + R_s C_1 s} = \left((R_s + j\omega L_1) \parallel \frac{1}{j\omega C_1} \right)$$

$$\therefore \; |Z_{Tc}(j\omega)|^2 = \left| \frac{R_s + j\omega L_1}{1 - L_1 C_1 \omega^2 + j\omega R_s C_1} \right|^2$$

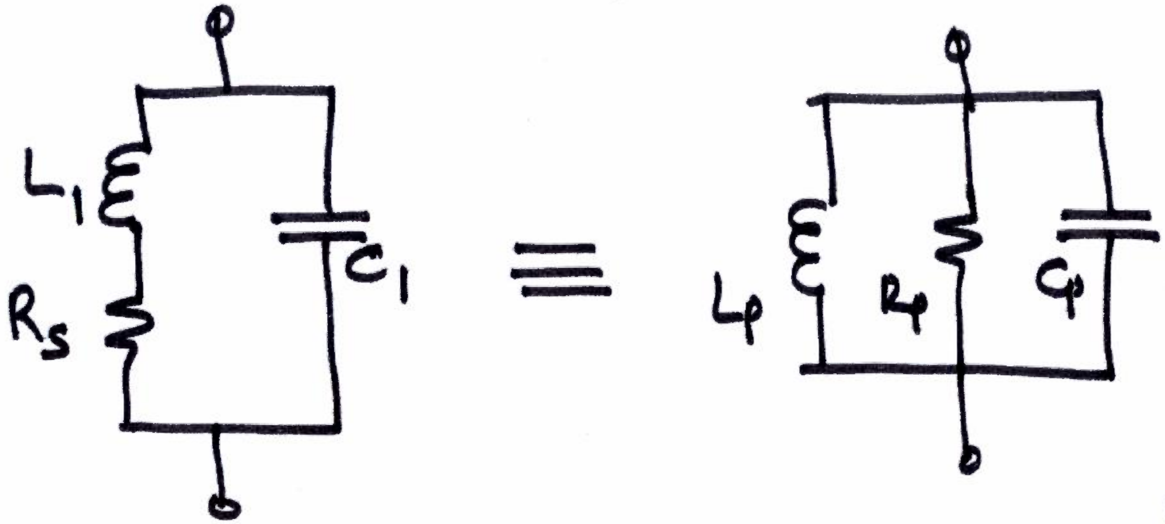
$$= \frac{R_s + L_1^2 \omega^2}{(1 - \omega^2 L_1 C_1)^2 + R_s^2 C_1^2 \omega^2}$$



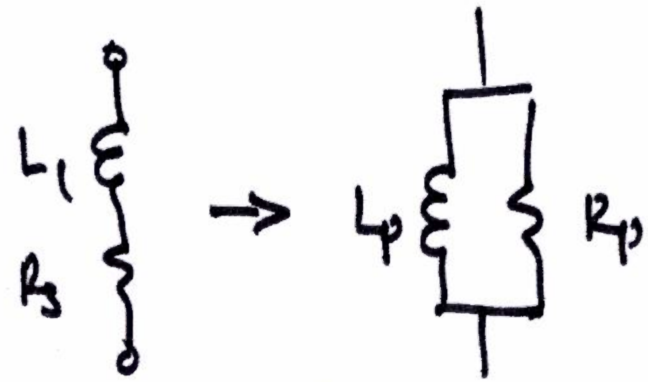
In Ideal LC Circuit at Oscillation frequency the Z_{Tc} (Z for Tuned Circuit) is Infinite.

But in non Ideal case as above $Z_{TC}(j\omega) \neq \infty$ at any frequency. Which essentiall means Circuit has finite Q

Tank circuit representation in Parallel form.



Let us keep $C_1 = C_p$, then



or

$$R_s + j\omega L_1 = \frac{R_p L_p S}{R_p + L_p S} = \frac{j\omega R_p L_p}{R_p + j\omega L_p} \quad -(1)$$

$$= \frac{j\omega R_p L_p (R_p - j\omega L_p)}{R_p^2 + \omega^2 L_p^2}$$

From ①

$$(R_s + j\omega L_1)(R_p + j\omega L_p) = j\omega R_p L_p$$

or $$R_s R_p - \omega^2 L_1 L_p + j\omega(L_p R_s + L_1 R_p)$$

$$= j\omega(R_p L_p) \quad \text{— (ii)}$$

Equating Real & Imaginary parts in (ii)

$$L_p R_s + L_1 R_p = R_p L_p \quad \text{— (iii)}$$

and $$R_s R_p - \omega^2 L_1 L_p = 0 \quad \text{— (iv)}$$

From (iv) $$\omega^2 L_1 L_p = R_s R_p$$

or $$L_p = \frac{R_s R_p}{\omega^2 L_1} \longrightarrow \text{(v)}$$

$$\text{or} \quad R_p = \frac{\omega^2 L_1 L_p}{R_s} \qquad \text{— (vi)}$$

Substituting R_p in (iii)

$$L_p R_s + L_1 \frac{\omega^2 L_1 L_p}{R_s} = \frac{\omega^2 L_1 L_p}{R_s} \cdot L_p$$

$$L_p R_s + \frac{\omega^2 L_1^2 L_p^2}{R_s \cdot L_p} = \frac{\omega^2 L_1^2 L_p^2}{R_s \cdot L_1}$$

$$L_p R_s + \frac{\omega^2 L_1^2 L_p^2}{R_s}\left(\frac{1}{L_p} - \frac{1}{L_1}\right) = 0$$

$$\text{or} \quad L_p^2 R_s^2 + \omega^2 L_1^2 L_p^2 = \frac{R_s L_p}{R_s L_1} \omega^2 L_1^2 L_p^2$$

$$1 + \frac{L_p^2 R_s^2}{\omega^2 L_1^2 L_p^2} = \frac{L_p}{L_1} \frac{\omega^2 L_1^2 L_p^2}{\omega^2 L_1^2 L_p^2}$$

or $$1 + \frac{R_s^2}{\omega^2 L_1^2} = \frac{L_p}{L_1}$$

We have $\boxed{Q = \frac{\omega L_1}{R_s}}$

or $$L_p = L_1 \left(1 + \frac{1}{Q^2}\right)$$

Since $Q >> 1$ (Typical Q of spiral inductor on Silicon is ≥ 3)

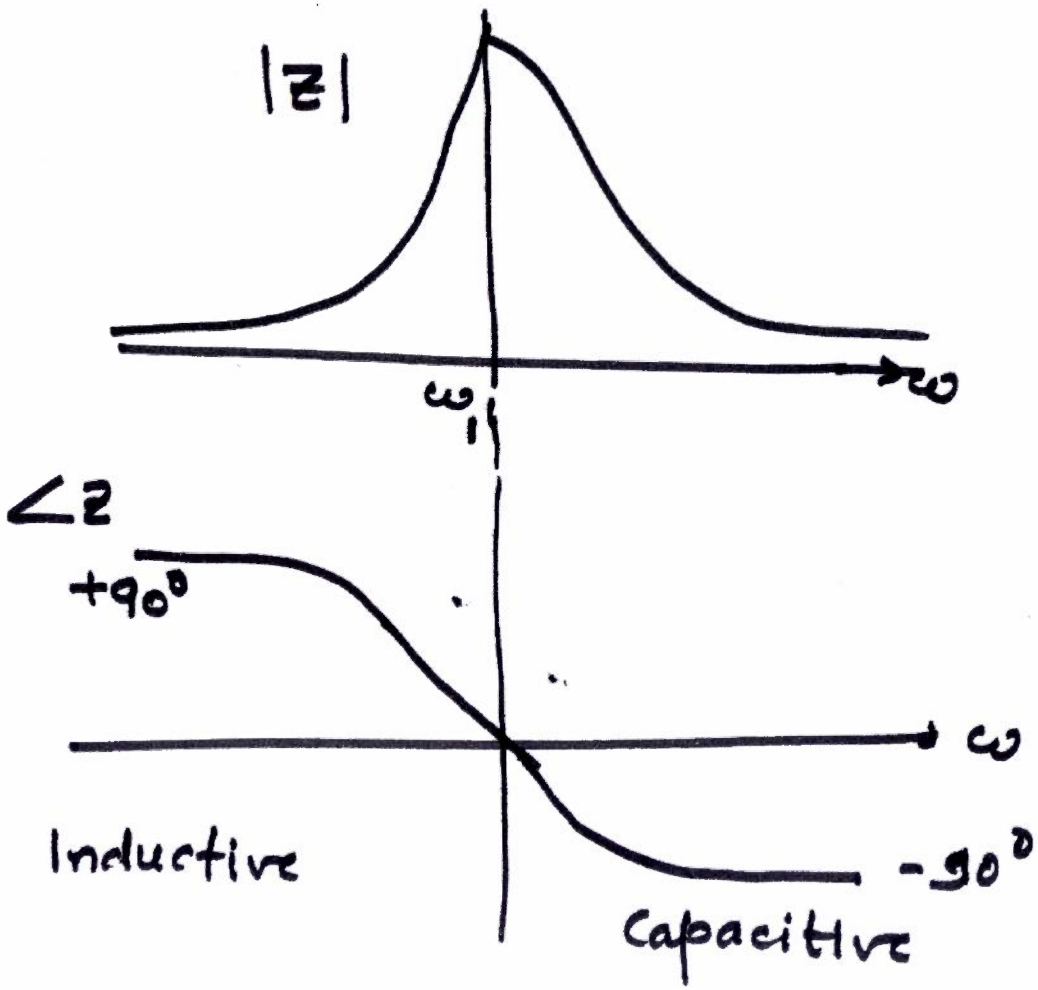
$$\therefore \boxed{L_p \cong L_1}$$

substituting this in eq. (vi)

$$R_p = \frac{\omega^2 L_1 L_p}{R_s} = \frac{\omega^2 L_1^2}{R_s} = \frac{\omega^2 L_1^2}{R_s^2} \cdot R_s = Q^2 R_s$$

$\boxed{R_p = Q^2 R_s}$ And $\boxed{C_p = C_1}$

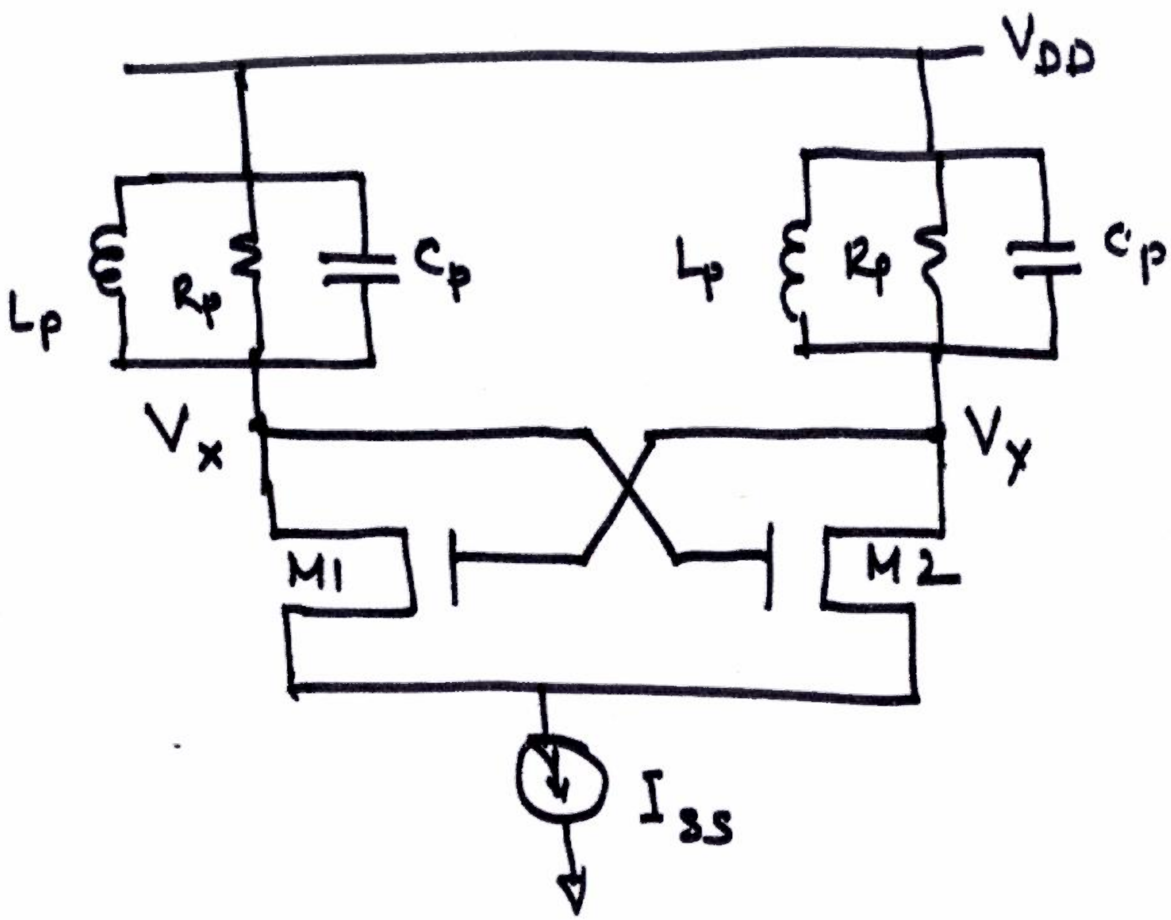
The Magnitude & Phase of Input Impedance Z of Parallel Tank Circuit is like:



$$\omega_1 = \frac{1}{\sqrt{LC}}$$

$$f_{os} = \frac{1}{2\pi\sqrt{L_1 C_1}} = \frac{1}{2\pi\sqrt{L_p C_p}}$$

Cross Coupled Oscillator



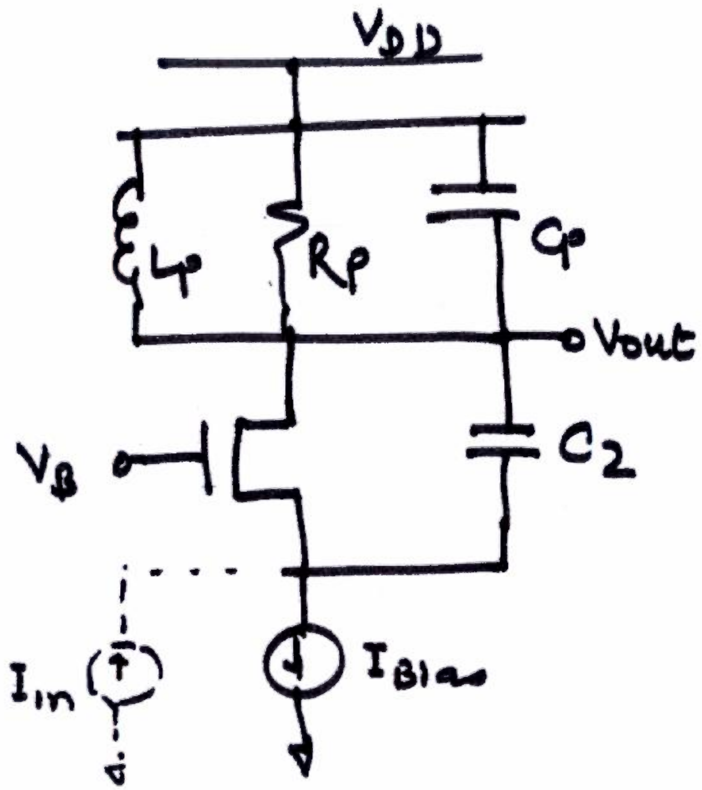
$$H(j\omega) = H_1(j\omega) \cdot H_2(j\omega)$$

$$H_0 = g_{m1} R_p \cdot g_{m2} \cdot R_p$$

If $H(0) \geq 1$

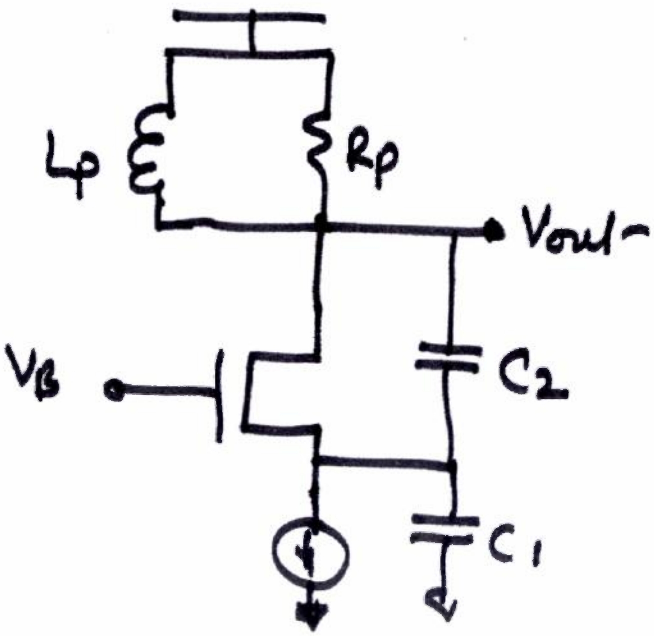
Then Oscillation will be Sustained.

Colpitts and Heartley Oscillators are example for Cross Coupled LC Oscillators

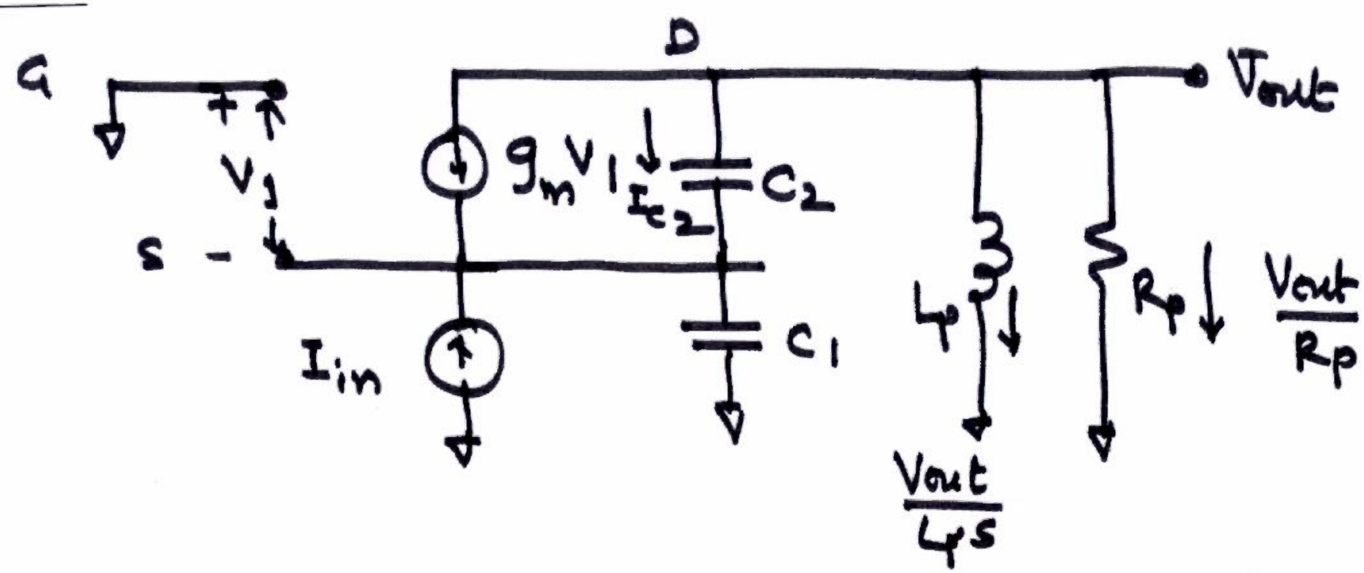


Principle :

$$\Rightarrow \frac{V_{out}}{I_n} = L_p s \parallel R_p \parallel \frac{1}{C_p s}$$



Colpitts Oscillator



$$\therefore \quad V_1 = -\left(I_{in} - \frac{V_{out}}{L_p s} - \frac{V_{out}}{R_p}\right) \cdot \frac{1}{C_1 s}$$

$$I_{C_2} = \left\{V_{out} - (-V_1)\right\} C_2 s = (V_{out} + V_1) C_2 s$$

$$\therefore \quad I_{C_2} = \left[V_{out} - \left(I_{in} - \frac{V_{out}}{L_p s} - \frac{V_{out}}{R_p}\right)\frac{1}{C_1 s}\right] C_2 s$$

Using Kirschoff's law at node V_{out} -

We have

$$g_m V_1 + I_{c_2} + \frac{V_{out}}{L_p s} + \frac{V_{out}}{R_p} = 0$$

Solving this equation we get

$$\frac{V_{out}}{I_{in}} = \frac{R_p L_p s (g_m + C_2 s)}{R_p C_1 C_2 L_p s^3 + (C_1 + C_2) L_p s^2 + [g_m L_p + R_p (C_1 + C_2)] s + g_m R_p}$$

For Oscillation $\frac{V_{out}}{I_{in}} \to \infty$ at oscillating frequency ω_R

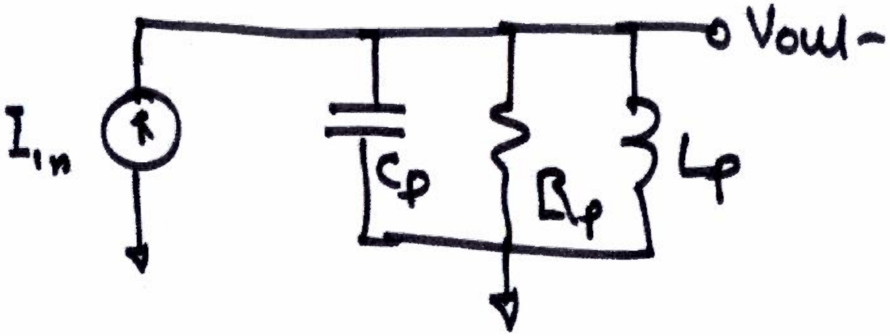
We obtain

$$\omega_R^2 = \frac{1}{L_p \frac{C_1 C_2}{C_1 + C_2}} \quad \& \quad g_m R_p = \frac{C_1}{C_2}\left(1 + \frac{C_2}{C_1}\right)^2$$

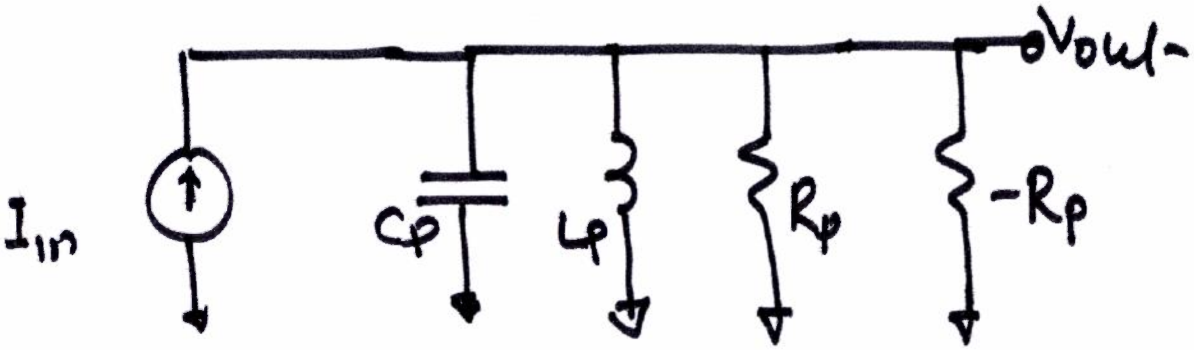
Minimum required DC Gain gives $g_m R_p \geq 4$ when $\frac{C_2}{C_1} = 1$

With C_p present $\omega_R^2 = 1 / \left[C_p + \frac{C_1 C_2}{C_1 + C_2}\right] L_p$

Single Port Oscillators.

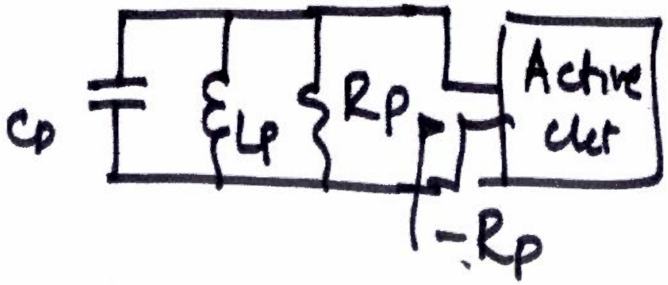


Oscillations die as R_p dissipates power

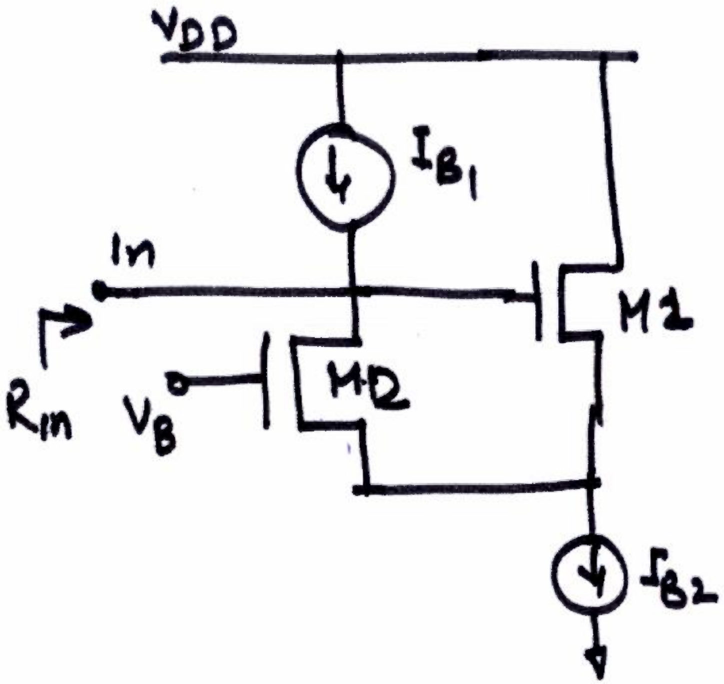


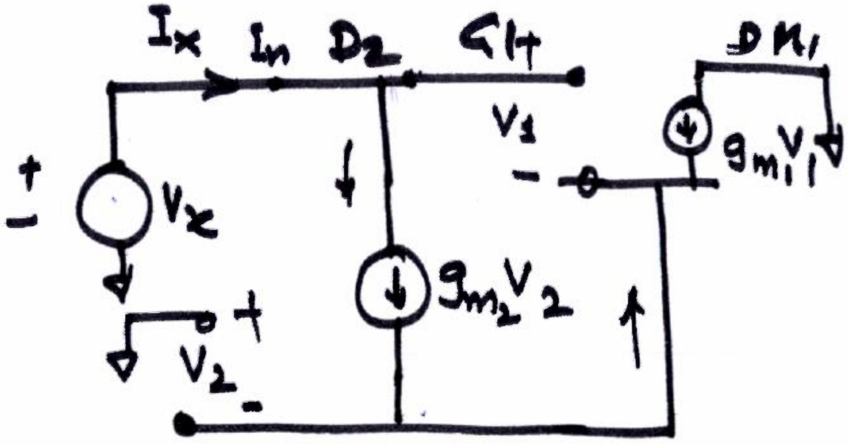
$$f_{osc} = \frac{1}{2\pi\sqrt{L_p C_p}}$$

-ve Resistance



Negative Impedance Generator





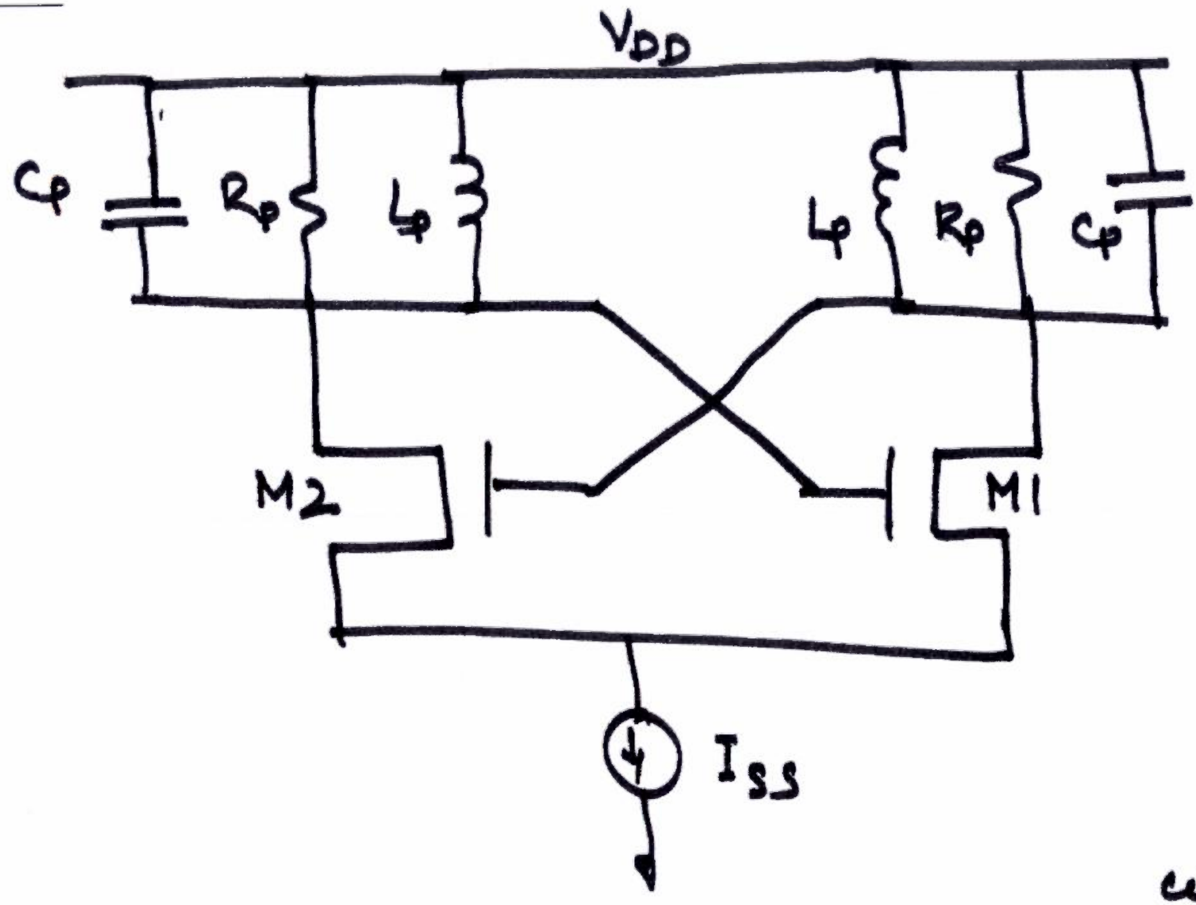
$$I_x = g_{m_2} V_2 = -g_{m_1} V_1$$

But $V_x = V_1 - V_2$

$$= \frac{I_x}{-g_{m1}} - \frac{I_x}{g_{m2}}$$

or $\dfrac{V_x}{I_x} = -\left(\dfrac{1}{g_{m1}} + \dfrac{1}{g_{m2}}\right) = -\dfrac{2}{g_m}$ if $g_{m1} = g_{m2} = g_m$

$\therefore R_{in} = -\,2/g_m$



Differential version
LC Oscillator
with -ve Resistance
Generator.