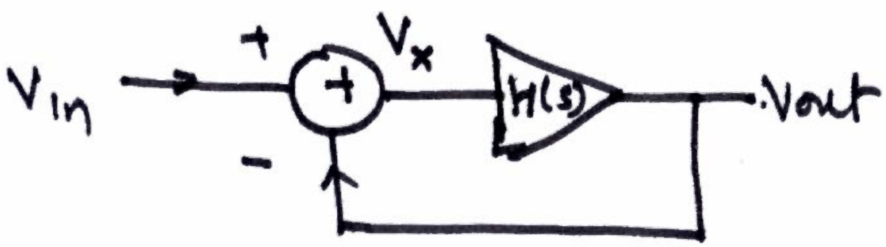
OSCILLATORS

$$A_{CL}(s) = \frac{H(s)}{1+H(s)}$$ Unity Gain Feedback

If $s = j\omega_0$ and $H(j\omega_0) = -1$

then $A_{CL}(j\omega_0) = \infty$ at $\omega = \omega_0$

This condn is essentially condition for oscillations.



A typical Negative feedback system is shown here. Here

$$V_x = V_{out} + |H(j\omega_0)| V_{out} + |H(j\omega_0)|^2 V_{out} + \cdots$$
(Geometric Series)

If $|H(j\omega_0)| > 1$, then V_x is having a Diverging series.

while $|H(j\omega_0)| < 1$, then V_x has converging series representation, and its magnitude is Finite.

We write

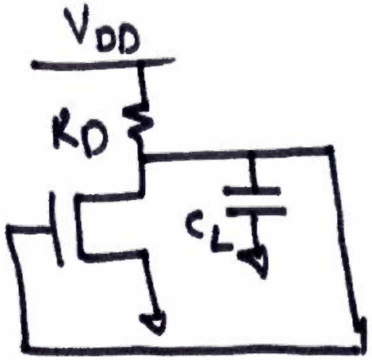
$$V_x = \frac{V_{out}}{1 - |H(j\omega_0)|} = \text{Finite} \left\{ \text{if } |H(j\omega_0)| < 1 \right\}$$

Barkhausen Criteria :

In a Negative feedback system, if Two conditions → $|H(j\omega_0)| \geq 1$ & $\angle H(j\omega_0) = 180^\circ$ are satisfied then the system will Oscillate

Ring Oscillator

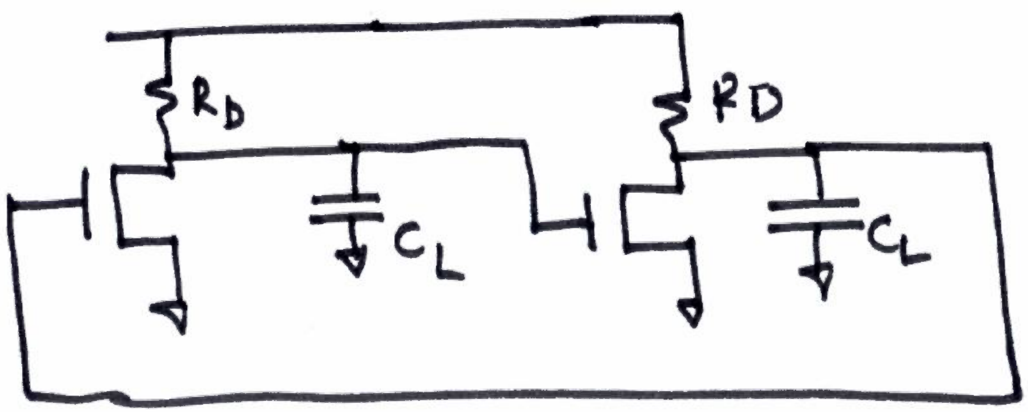
1.



Pole $p_1 = \frac{1}{R_D C_L}$

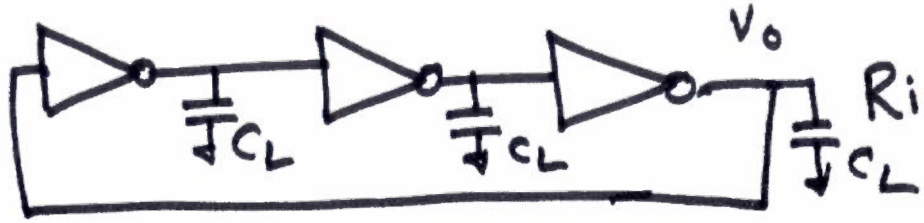
Phase shift at Pole is 45°
& total Phase shift is 90°,
plus 180° from Transistor = 270°
Hence Loop does not sustain oscillation.

2



2 Poles will give
total Phase shift of
180° + 180° = 360°. This
satisfies Oscillation Condn!
However one does not
observe Oscillations here.

The Feedback satisfies Barkhausan Criterion only at
$\omega_0 = 0$. The circuit therefore exhibits Latch Action.

3. 

Ring Oscillator.

$\omega_0 = \frac{1}{R_D C_L}$ and transfer function show Triple pole at ω_0.

or $$H(s) = \frac{-A_{V0}^3}{\left[1+\left(\frac{s}{\omega_0}\right)\right]^3}$$

We can see that for Oscillation to sustain

$A_{V0} = 2$ and $\omega_{osc} = \sqrt{3}\,\omega_0$

It means that each stage must contribute 60° from one RC Combination

Let us look into a Negative Feedback system with Triple Pole at P_1.

i We start with open loop Gain with Triple Pole

$$A_{OL}(s) = \frac{A_{VO}}{\left(1 - \frac{s}{P_1}\right)^3} \qquad \text{— (1)}$$

A_{VO} is a DC Gain (or Call 'Low' Frequency Gain)

ii If this OPEN Loop Amplifier gets a feedback with feedback Network Gain of β [Assume $\beta \neq F(f)$], then Closed Loop Gain is:

$$A_{CL}(s) = \frac{A_{OL}(s)}{1 + A_{OL}(s)\,\beta} \qquad \text{— (ii)}$$

CDEEP
IIT Bombay
EE 618 L 17 / Slide 05

or $$A_{CL}(s) = \frac{\dfrac{A_{vo}}{\left(1 - \frac{s}{p_1}\right)^3}}{1 + \dfrac{A_{vo}\beta}{\left(1 - \frac{s}{p_1}\right)^3}}$$

$$= \frac{A_{vo}}{\left(1 - \frac{s}{p_1}\right)^3 + A_{vo}\beta} \quad \text{— (iii)}$$

we define Loop Gain $T_o = A_{vo}\beta$ — (iv)

At any other Frequency

$$T(s) = T(j\omega) = \frac{T_o}{\left(1 - \frac{s}{p_1}\right)^3} = \frac{T_o}{\left(1 + j\frac{\omega}{|p_1|}\right)^3}$$

-- (v)

We observe Poles $A_{CL}(s)$ as are obtained from eq (iii), or to say

$$\left(1-\frac{s}{p_1}\right)^3 + T_0 = 0 \qquad -(vi)$$

or $$\left(1-\frac{s}{p_1}\right)^3 = -T_0$$

or $$1-\frac{s}{|p_1|} = \sqrt[3]{-T_0}$$

or $$s = |p_1|\left(1-\sqrt[3]{-T_0}\right)$$

Using Algebra we know if $x = \sqrt[3]{-y}$

then $$x^3+y=0$$

or $$x^3+(y^{1/3})^3=0$$

$$\text{or} \quad (x + y^{1/3})(x^2 + x y^{1/3} + y^{2/3}) = 0$$

$$\therefore \quad x_1 = -y^{1/3}$$

$$x_2 = -y^{1/3} e^{j\frac{\pi}{3}}$$

$$x_3 = -y^{1/3} e^{-j\pi/3}$$

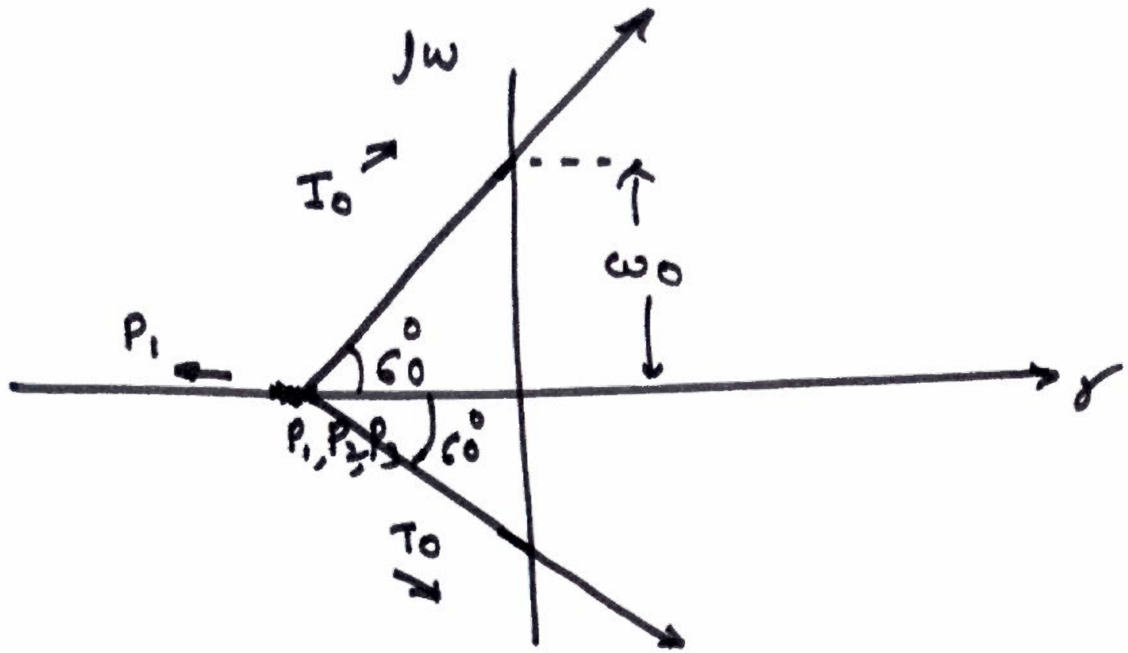
$\therefore$ Three roots of eq (vi) are

$$s_1 = P_1 \left(1 + \sqrt[3]{T_0} \right)$$

$$s_2 = P_1 \left(1 + \sqrt[3]{T_0}\, e^{j\pi/3} \right)$$

$$s_3 = P_1 \left(1 + \sqrt[3]{T_0}\, e^{-j\pi/3} \right)$$

The Root Locus looks like



At $\omega = \omega_0$, one observe locus intersects $j\omega$ axis.

clearly here Real value vanishes at $\omega = \omega_0$

or to say $1 - \text{Re}\left(\sqrt[3]{T_0}\, e^{j\pi/3}\right) = 0$ or $\sqrt[3]{T_0} \cos 60^\circ = 1$

$\therefore \boxed{T_0 = 8}$ at $\omega = \omega_0$

CDEEP
IIT Bombay

Clearly beyond $T_0 > 8$, the system will be unstable, and at $T_0 = 8$, it is critically stable.

From the 'R.Locus' figure

$$\tan 60^\circ = \frac{\omega_0}{|p_1|}$$

or $\omega_0 = \sqrt{3}\,|p_1| = 1.73205\,|p_1|$

Just above frequency ~~above~~ of ω_0, σ is v.small positive and imaginary part exists. This is like Positive feedback and System will show Growth response, as

$$\propto e^{\sigma t} \sin\omega_0 t \quad \text{(Growing Sinusoid)}$$

We can use Bode's plot too to get this result.

We have

$$T(s) = T(j\omega) = \frac{T_0}{\left(1 + j\frac{\omega}{|P_1|}\right)^3}$$

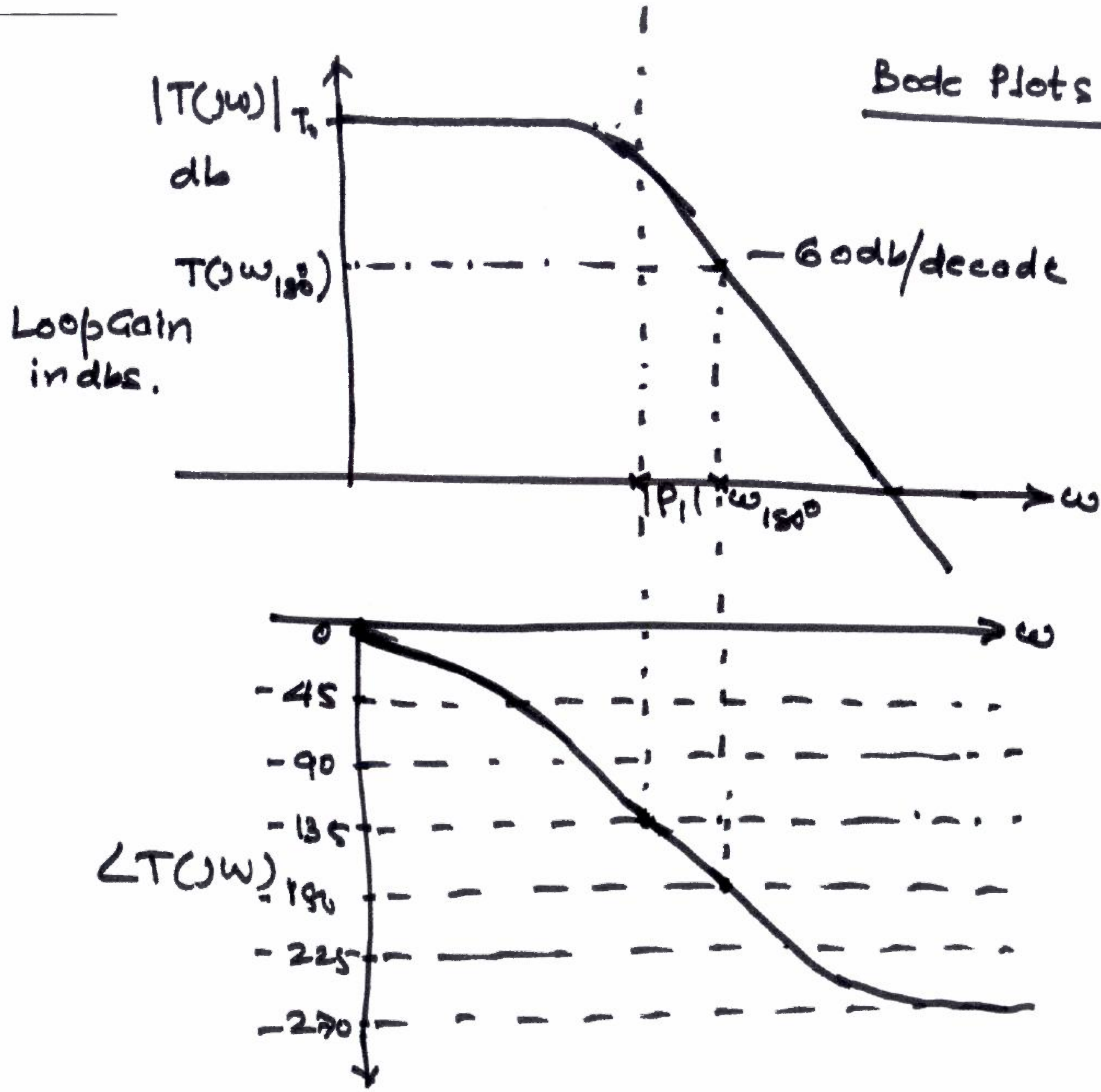
We plot $|T(j\omega)|$ and $\angle T(j\omega)$ as function of Frequency (Bode Plot)

(A) We observe that at Pole frequency $|P_1|$, the phase is -135^0. (Triple pole: Each pole 45^0/decade)

(B) And $T(j\omega_{180^0})$ is <u>Positive</u> at $\omega = \omega_{180^0}$

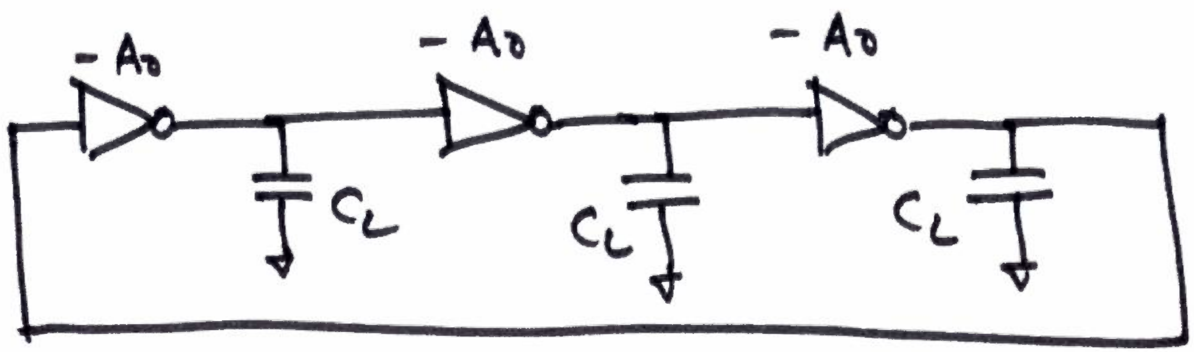
ω_{180^0} is the Frequency at which $\angle T(j\omega) = -180^0$

Bode Plots



|T(jω)|
db
T_0
Loop Gain in dbs.
T(jω180°)
-60db/decade
|P1|
ω180°
ω
0
-45
-90
-135
∠T(jω)
-180
-225
-270
ω

We return to Ring Oscillator case as



Here $\omega_0 = (r_{out} \cdot C_L)^{-1}$; A_0 is a Stage Gain

$$\therefore \quad H(s) = \frac{-A_0^3}{\left[1+\left(\frac{s}{\omega_0}\right)\right]^3}$$

is the net Transfer fn

ω_0 used here is $|\omega_0|$

Each stage should contribute 60° Phase, then

$$\omega_{osc} = \omega_0 \tan^{-1}\frac{\pi}{3} = \sqrt{3}\,\omega_0$$

And at $\omega = \omega_{osc}$, Loop Gain = 1

$\therefore \quad 1 = \dfrac{A_0^3}{8}$

or $\boxed{A_0 = 2}$

$\therefore$ Ring Oscillator thus will oscillate at frequency $= \dfrac{\omega_0 \cdot \sqrt{3}}{2\pi}$ if the Stage Gain is 2 and each stage gives $\dfrac{\pi}{3}$ phase shift.

Clearly from earlier discussion of Triple Pole case, we can say that

$$V_0 = a\, e^{\frac{(A_0-2)}{2}\omega_0 t} \cdot \cos\left(\frac{\sqrt{3}A_0}{2}\omega_0 t\right)$$

which means for $A_0 < 2$ — Damping

$A_0 > 2$ — Growth uncontrolled

& $A_0 = 2$ — Oscillation.

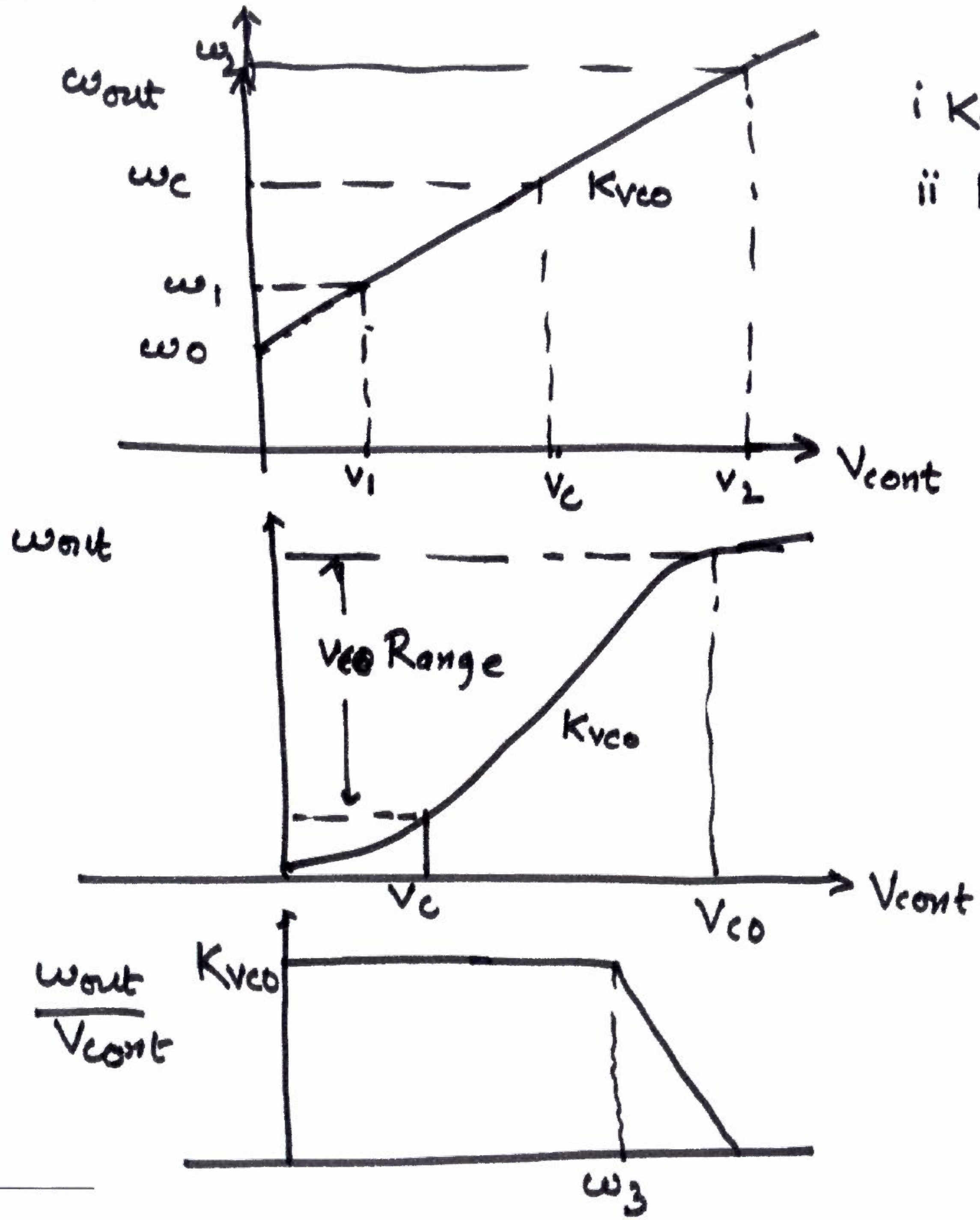
Most commonly used VCOscillators

1. Source Coupled – Multivibrators

2. Current Starved VCO.

The word VCO → Voltage Controlled Oscillators which essentially means tuning of Frequency of Oscillator using voltage control.

$$\omega_{out} = \omega_0 + K_{VCO} \cdot V_{cont}$$



i K_{vco} = VCO Gain

ii Range = $\omega_2 - \omega_1$

CDEEP
IIT Bombay

EE 618 L 27 / Slide 16

$$K_{vco} = \frac{2\pi(f_{max} - f_{min})}{V_{max} - V_{min}}$$

Typical Power ~~Dissipated~~ Used

= 1 to 10 mW

Generally VCO operates from 1 Mradian/sec to 10 M radian/sec

$K_{vco} \sim$ 0 to 5 Mradian/sec/V

Terminology for VCO

CDEEP
IIT Bombay

(i) Centre Frequency :→ Midrange Value

(ii) Tuning Range :— Decided by

(a) Variation of VCO centre frequency with Process and Temperature variation

(b) Frequency range needed for an Application

Major worry: Variation of Output Phase and Frequency as a result of Noise superposition on $V_{control}$.

Frequency-noise at Output $\propto K_{vco}$

Hence for this Noise to reduce, K_{vco} be smaller

However the Tuning Range $\propto K_{vco}$

$\therefore$ For larger Tuning Range, we need <u>Higher</u> K_{vco}.

(iii) Tuning Linearity

$$K_{vco} = \frac{\partial \omega_{out}}{\partial V_{DD}} = \frac{\partial \omega_{out}}{\partial C} \cdot \frac{\partial C}{\partial V_{DD}} \qquad C = f(V_{DD})$$

Which means K_{vco} is not constant.

(iv) Output Amplitude :– Larger Amplitude be desired.

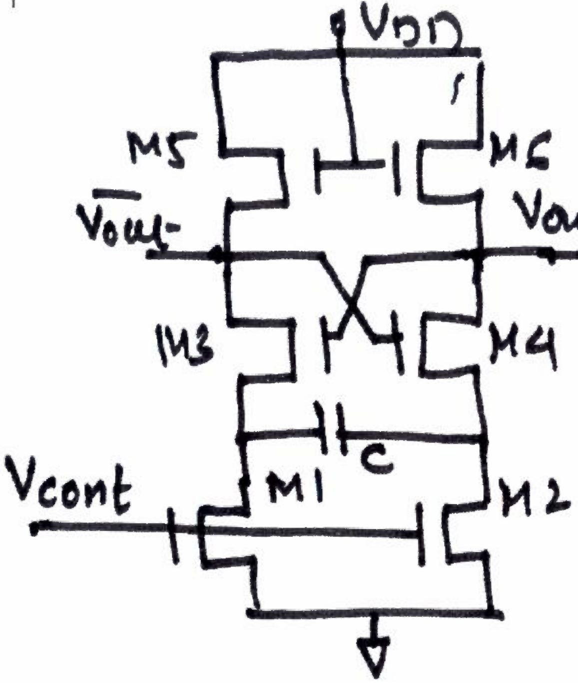
(v) Power Dissipation :– 1 to 10mW

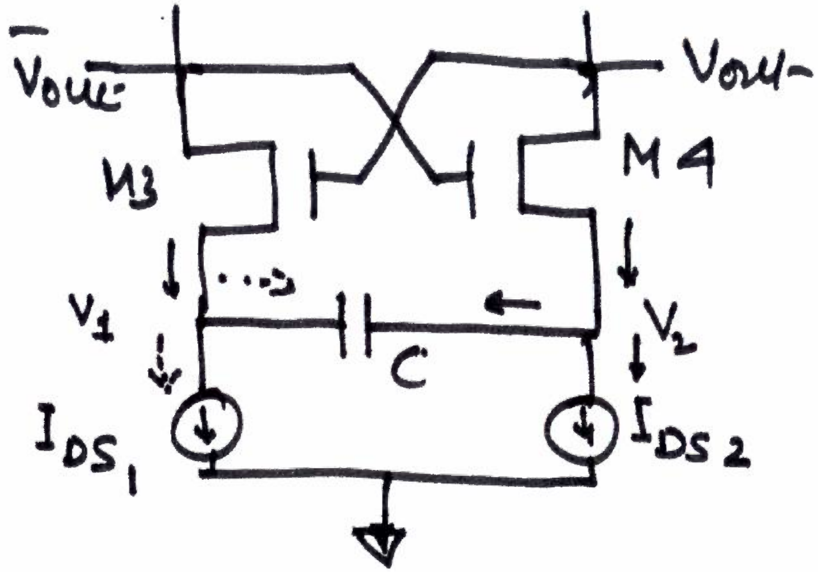
Trade-off: Power – Frequency – Noise

(vi) Signal Purity :

Even with a constant K_{vco} & V_{cont}, the output signal is not always Periodic (Perfect). One sees 'Noise' both in Phase & Frequency. These are called <u>Phase Noise</u> and <u>Jitter</u>

Source Coupled VCO





CDEEP
IIT Bombay

EE 618 L 27 / Slide 20

Here Vcont provides Bias to M1 & M2 such that

$I_{DS1} = I_{DS2}$ is current source biasing scheme

As M5 & M6 are in Saturation (n-channel), hence

$V_{out\,max}$ (or $\overline{V_{out}}_{max}$) $= V_{DD} - V_{Tn}$ where V_{Tn} is threshold of all N-channel devices