Frequency Response of Amplifiers

Concepts of Poles & Zeros.

A Transfer F^n $A_v(s) = A_{vo} \cdot \dfrac{\left(\frac{s}{\omega_z} + 1\right)}{\left(\frac{s}{\omega_1}+1\right)\left(\frac{s}{\omega_2}+1\right)}$

ω_z is called 'zero' frequency at which $A_v(s) \rightarrow 0$

ω_1, ω_2 - - are Poles (Pole frequencies) at which $A_v(s) \rightarrow \infty$

In a typical MOS Amplifier we have around Two 'Poles' and One 'zero'

Two Poles occur from Input Side & Output side

These can be termed as ω_{in} and ω_{out}

And ω_z is a 'zero' frequency occuring due to Feedback.

Ref. : Sedra, Smith & Chandorkar
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SINGLE POLE TRANSFER FUNCTION

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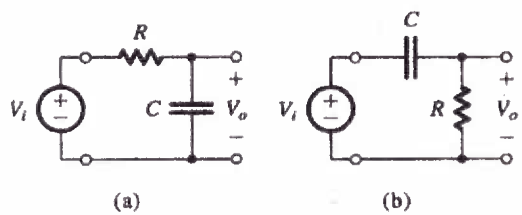

FIGURE 1.22 Two examples of STC networks: (a) a low-pass network and (b) a high-pass network.

TABLE 1.2 Frequency Response of STC Networks

	Low-Pass (LP)	High-Pass (HP)
Transfer Function $T(s)$	$\dfrac{K}{1+(s/\omega_0)}$	$\dfrac{Ks}{s+\omega_0}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\dfrac{K}{1+j(\omega/\omega_0)}$	$\dfrac{K}{1-j(\omega_0/\omega)}$
Magnitude Response $\lvert T(j\omega)\rvert$	$\dfrac{\lvert K\rvert}{\sqrt{1+(\omega/\omega_0)^2}}$	$\dfrac{\lvert K\rvert}{\sqrt{1+(\omega_0/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega = 0$ (dc)	K	0
Transmission at $\omega = \infty$	0	K
3-dB Frequency	$\omega_0 = 1/\tau$; $\tau \equiv$ time constant $\tau = CR$ or L/R	
Bode Plots	in Fig. 1.23	in Fig. 1.24

BODE PLOTS of STC

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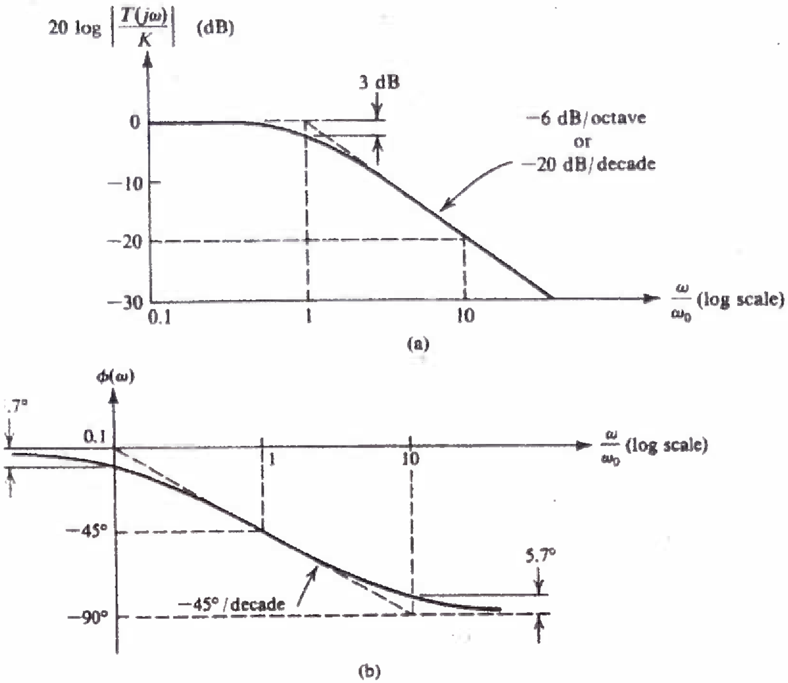


FIGURE 1.23 (a) Magnitude and (b) phase response of STC networks of the low-pass type.

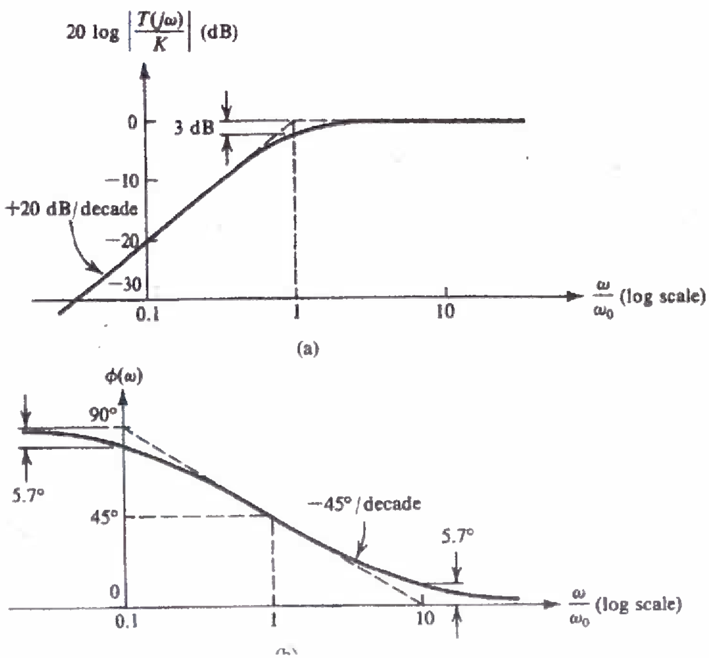
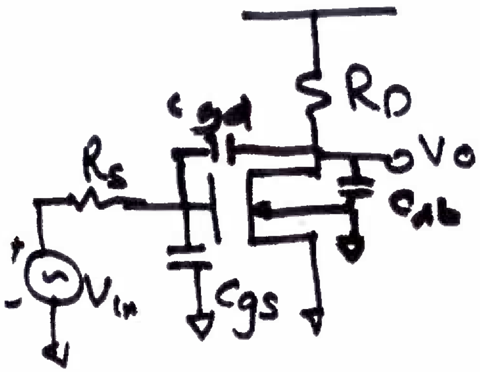
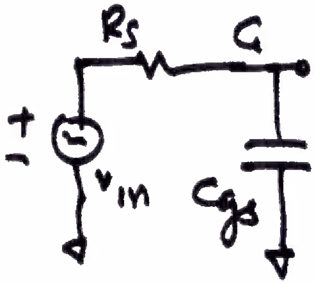


FIGURE 1.24 (a) Magnitude and (b) phase response of STC networks of the high-pass type.

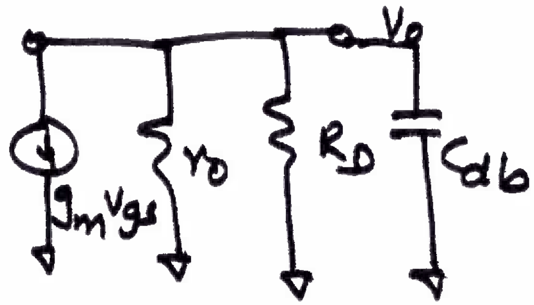
Input pole & Output Pole



$$\omega_{in} = \frac{1}{R_s C_{gs}}$$



input



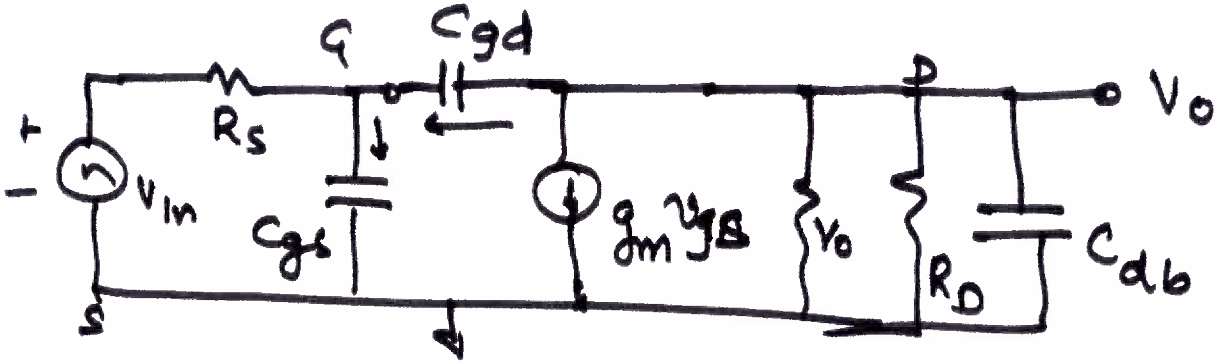
output

$r_o \| R_D \cong R_D$

& $$\omega_{out} = \frac{1}{R_D C_{db}}$$

However in equivalent ckt of Amplifier we also have capacitance C_{gd}

Then Eq. Ckt looks like



Assume
$r_o \| R_D = R_D$
Also C_{db} effect is neglected

$$v_{gs} = \frac{\frac{1}{C_{gs} \cdot s}}{\frac{1}{sC_{gs}} + R_s} v_{in} = \frac{v_{in}}{1 + s \cdot R_s C_{gs}}$$

C_{gs} may include C_{gb} as it may occure in Saturation.

Then
$$A_v'(s) = \frac{v_o(s)}{v_{gs}(s)} = -\left\{g_m R_D\right\}\left\{\frac{1 - \frac{C_{gd} \cdot s}{g_m}}{1 + s \cdot C_{gd} \cdot R_D}\right\}$$

$$A_v(s) = \frac{V_o(s)}{V_{in}(s)} = -(g_m R_D)\frac{(1 - s \cdot C_{gd}/g_m)}{(1 + s R_s C_{gs})(1 + s \cdot R_D C_{gd})}$$

Approximately looking at denomenator we see

$$D = g_m (1 + s R_s C_{gs})(1 + s R_D C_{gd})$$

$$\cong (1 + s R_s C_{gs})(g_m + s \cdot g_m R_D C_{gd})$$

$$= (1 + s R_s C_{gs})(g_m + (-A_{vo}) s \cdot C_{gd})$$

$\cong 1 + s \{ C_{gs} + (1$ [redacted] $A_{vo}) C_{gd} \} R_s$

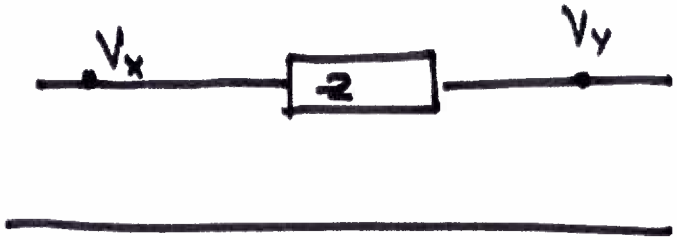
$$= 1 + R_s (C_{gs} + (1 + g_m R_D) C_{gd}) \cdot s$$

$$= 1 + R_s C_{in} \cdot s \qquad C_{in} = C_{gs} + (1 + g_m R_D) C_{gd}$$

$$\therefore \omega_{in} = \frac{1}{R_s C_{in}}$$

Similar Result we can attain by using Miller's Theorem:

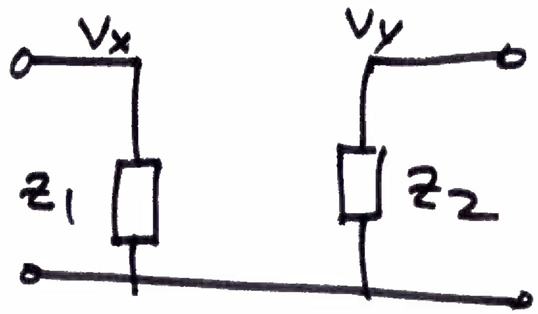
Given



where

$$A = \frac{V_y}{V_x} \quad (\text{Gain } F^n)$$

we can convert this to



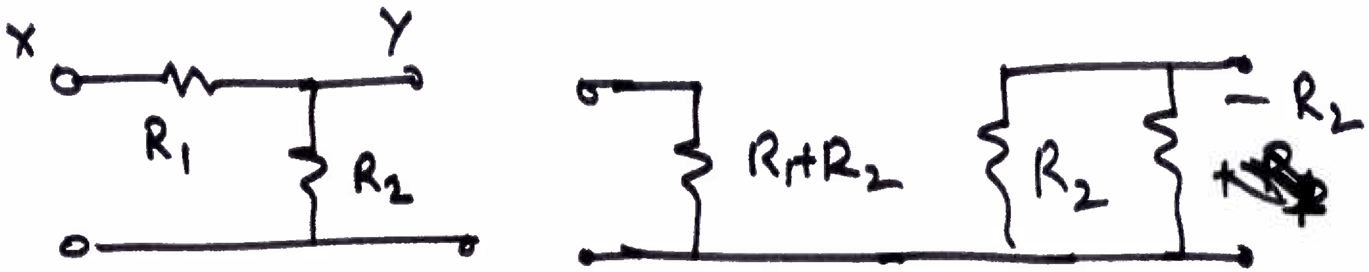
where

$$Z_1 = \frac{Z}{1-A}$$

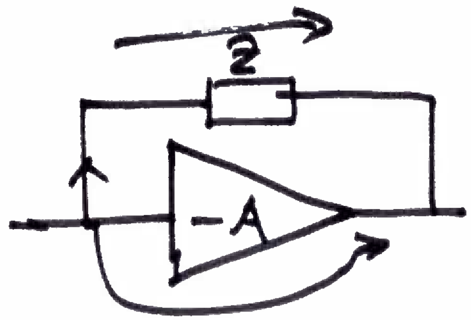
$$\& \quad Z_2 = \frac{A}{1-A} \cdot Z$$

If Z is capacitive C_{xy}

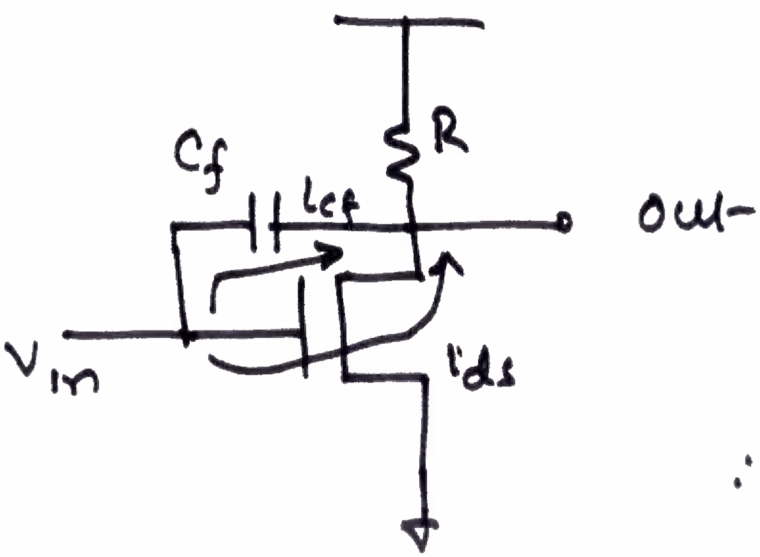
Limitations of Miller's Theorem



We must have Two paths from Input to Output for validity of Miller's theorem.



Continuing similar argument, we can see that a 'zero' of Transfer Function is only possible if we have two paths from Input to Output and at a frequency the value of current in two paths are equal in magnitude and 180^0 out of phase.

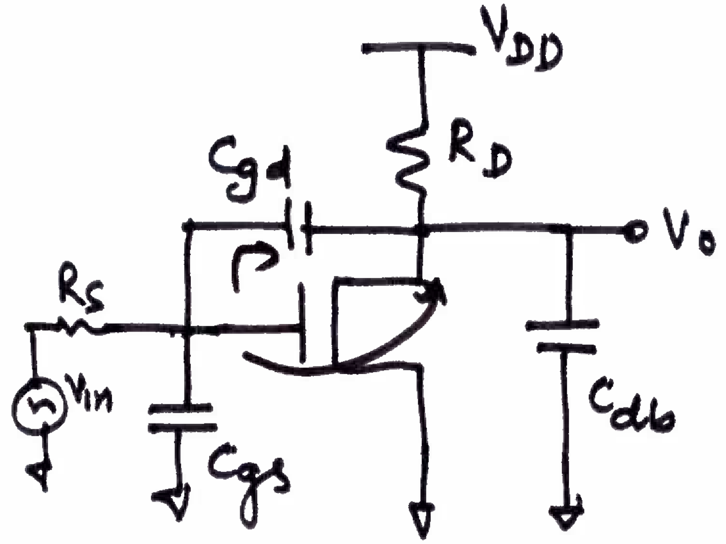


This is due to effect of "Feed Forward"

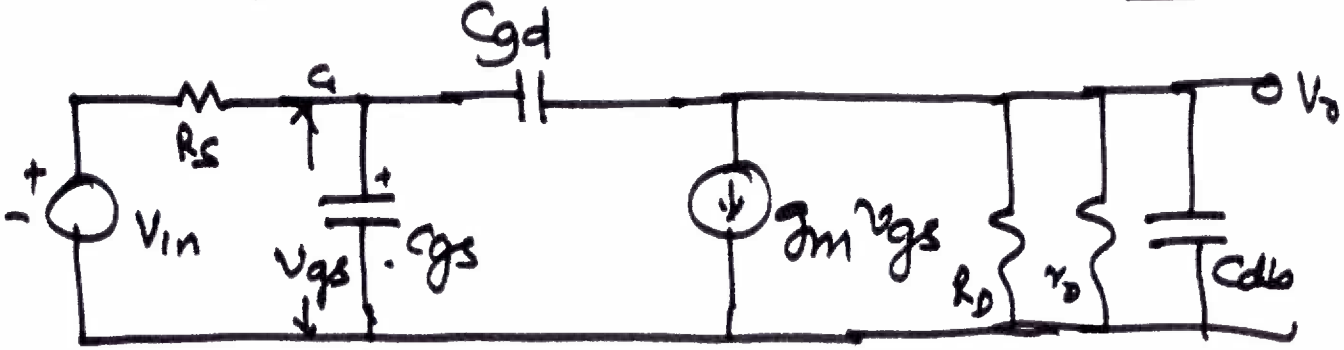
$$\boxed{i_{cf} = -i_{ds}}$$

$\therefore$ $i = 0$ $\therefore$ $V_{out} = 0$ $\longrightarrow$ 'zero' Occurance

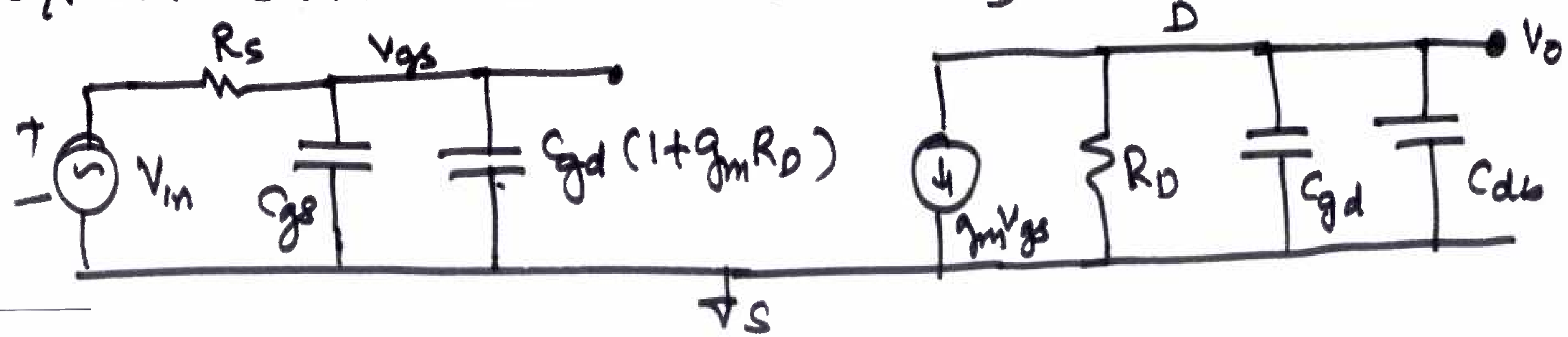
Returning to our problem of Frequency response of a CS Amplifier, whose HF model can be shown as



with eq ckt as —



Using DC Gain expression $A_{vo} = -g_m R_D$, we see that Eq. ckt could be shown as using Miller's theorem as :—



Then

$$\omega_{in} = \frac{1}{R_s\left[C_{gs} + (1+g_m R_D)\, C_{gd}\right]}$$

& $$\omega_{out} = \frac{1}{R_D\,(C_{gd} + C_{db})}$$

And Transfer F^n can be written as

$$A_v(s) = \frac{A_{vo}}{\left(1+\frac{s}{\omega_{in}}\right)\left(1+\frac{s}{\omega_{out}}\right)}, \quad A_{vo} = -g_m R_D$$

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If we solve complete eq. circuit with node analysis, we get

$$A_v(s) = -\frac{g_m R_D - C_{gd} \cdot s}{1 + R_s \cdot R_D \cdot C_{eq} \cdot s^2 + \left[R_s\{1+g_m R_D\}C_{gd} + R_s C_{gs} + R_D\{C_{gd}+C_{db}\}\right]s}$$

This leads to

$$\omega_{p1} = \frac{1}{R_s(1+g_m R_D)C_{gd} + R_s C_{gs} + R_D(C_{gd}+C_{db})}$$

&

$$\omega_{p2} = \frac{1}{\omega_{p1}} \cdot \frac{1}{R_s R_D (C_{gs}C_{gd} + C_{gs}C_{db} + C_{gd}C_{db})}$$

$$C_{eq} = (C_{gs}C_{gd} + C_{gs}C_{db} + C_{gd}C_{db})$$

If $R_D(C_{gd}+C_{db}) << R_S C_{gs} + R_S(1+g_m R_D)C_{gd}$

and $C_{gs} >> (1+g_m R_D)C_{gd} + \frac{R_D}{R_S}(C_{gd}+C_{db})$

Then $\omega_{p1} = \dfrac{1}{R_S\,[\,C_{gs}+(1+g_m R_D)C_{gd}}$

& $\omega_{p2} = \dfrac{1}{R_D(C_{gd}+C_{db})}$

If we see values of ω_{in} & ω_{out} as obtained by Miller's theorem use, we observe that

$\omega_{in} = \omega_{p1}$ & $\omega_{out} = \omega_{p2}$

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Hence in most cases we can find Dominant Pole, Non Dominant poles and even zeros using simple evaluations of ω_{in} and ω_{out}, which essentially are inversely related to Input Time constant $R_{in}C_{in}$ and $R_{out}C_{out}$, being output Time Constant.

This suggests a simple Technique of evaluation of Poles using a technique called "Zero-Value Time Constant" analysis. This is also at times refered as "Open Circuit Time Constant analysis".

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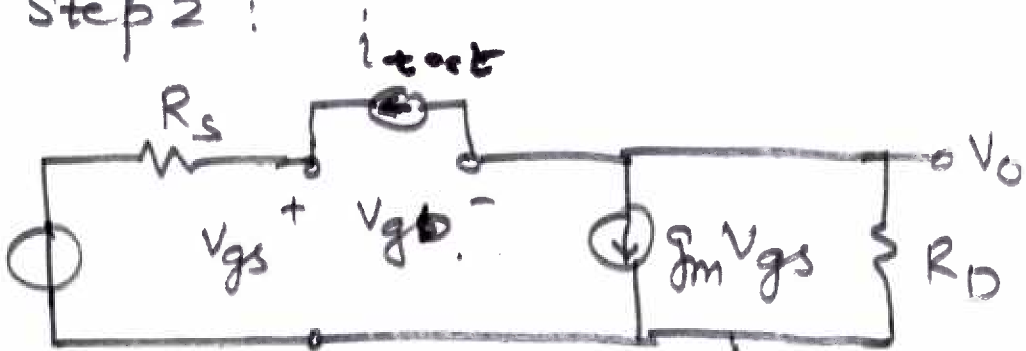
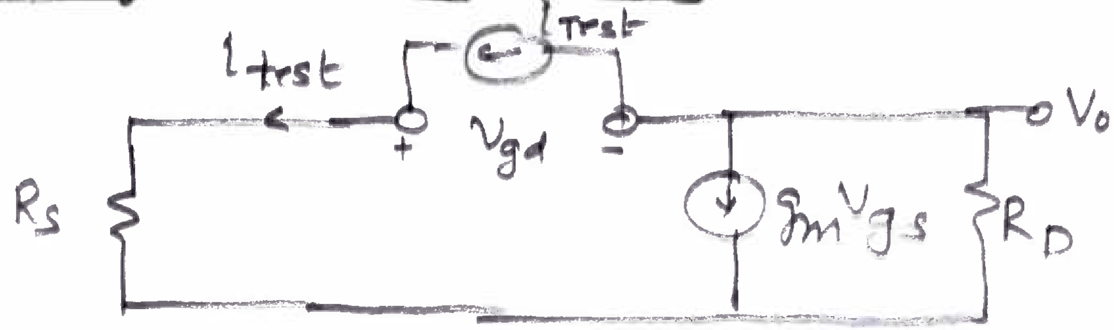
Step 2 :





$$R_{cgd} = \frac{v_{gd}}{i_{test}} = \frac{i_{test} \cdot R_s}{i_{test}} + \frac{R_D\,(g_m v_{gs} + i_{test})}{i_{test}}$$

$$R_{cgd} = R_s + \frac{g_m R_D\, i_{test} \cdot R_s}{i_{test}} + R_D$$

$$R_{cgd} = R_s + R_D + g_m R_s R_D$$

ZVTC Technique :

1. Remove all but One Capacitor.
   Short all independent Voltage Sources,
   open all independent Current Sources.

2. Calculate resistance (R1) seen by capacitor (C_1)
   and then evaluate Time constant $\tau_1 = R_1 C_1$.

3. Repeat this for all Capacitors and obtain
   $\tau_j = R_j \cdot C_j$

4. Sum all the Time-Constants and then we
   get $\omega_{-3db} = \dfrac{1}{\sum_{j=0}^{N} \tau_j}$ N is no. of Capacitors

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'ZVTC and SCTC Techniques'

— Limitations & Comments —

1. These do not lead to finding 'Zeros' of the systems

2. It is accurate enough if there exists a Dominant pole separated by larger frequency to next Non-dominant pole.

   In most Amplifiers which we use in Analog Systems, this condition is mostly met.

3. In these techniques, remove Coupling Capacitors from evaluation as they provide 'shorts' at higher frequency

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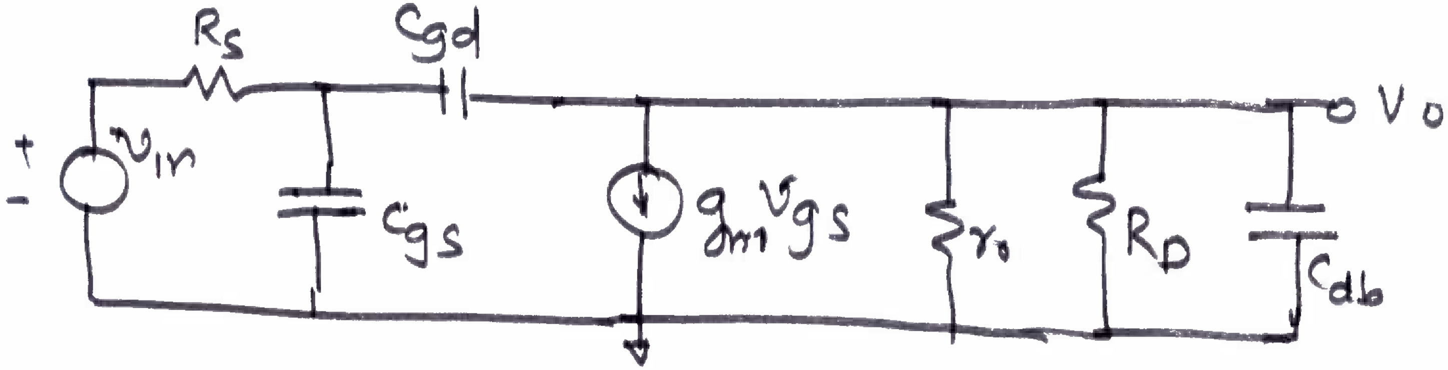
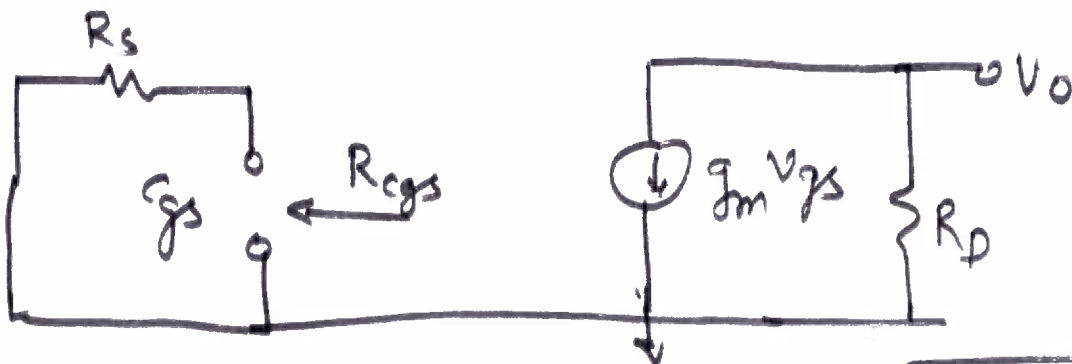
For the ckt. of CS Amplifier, we obtain ω_{-3db} now using ZVTC technique.



Step 1 : Time constant for C_{gs}. (Open C_{gd} & C_{db} arm for $C_{gd} = C_{db} = 0$)



Clearly $R_{cgs} = R_S$ $\therefore$ $\boxed{\tau_1 = R_s C_{gs}}$ — (1)

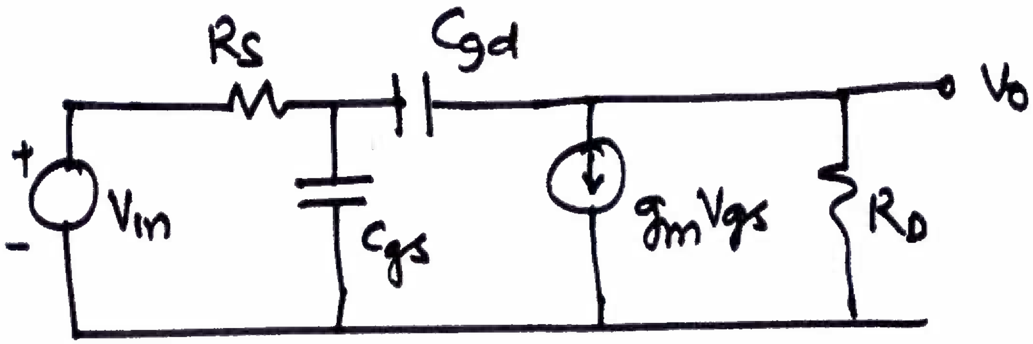
$$\therefore \omega_{-3db} = 2\pi f_{-3db}$$

$$= \frac{1}{R_S C_{gs} + R_D C_{db} + (R_S + R_D) C_{gd} + g_m R_S R_D C_{gd}}$$

Please pay attention to our earlier calculations of ω_{p1}. It seems that now we get $\boxed{\omega_{-3db} = \omega_{p1}}$ This is a dominant Pole.

Hence ZVTC technique is good enough approximation to get Dominant Pole. For Nondominant pole too, we use technique called 'Short Circuit Time Constant' technique.

Example:** We continue with same amplifier design and neglect C_{db} for this example



Solve for

$$\begin{cases} R_s = 10K = R_L \\ C_{gs} = 1\,pf \; ; \; C_{gd} = 20\,pf \\ g_m = 3\,mA/V \end{cases}$$

We have Two capacitors in the circuit, and hence we have TWO poles, & one of them will be Dominant Pole and other Nondominant pole.

(i) We use ZVTC technique to obtain Dominant Pole

& (ii) We use 'Short Circuit Time Constant' technique to get Non Dominant Pole.

**Ref: Gray, Myer et al.

Data : $R_s = 1K$, $R_D = 5K$

$I_{DS} = 1 mA$, $\beta'(W/L) = 100 mA/V^2$

$C_{gd} = 0.5 pf$, $C_{gb} = 0$

$C_{gs} = 5.0 pf$

Now $g_m = \sqrt{2 \times 100 \times 10^{-3} \times 10^{-3}} = 14.1 \ mA/V$

Using Miller Technique, the dominant pole is

$$\omega_{p1} = - \frac{1}{(1+g_m R_D) R_s C_{gd} + R_s C_{gs} + R_D C_{gd}}$$

$1 + g_m R_D = 1 + 10^{-3} \times 14.1 \times 5 \times 10^{3} = 71.5$

$(R_s + R_D) = 6K$; $(R_s + R_D) C_{gd} = 6 \times 10^{3} \times 0.5 \times 10^{-12}$
$= 3 \times 10^{-9}$

$R_s \cdot C_{gs} = 10^{3} \times 5 \times 10^{-12} = 5 \times 10^{-9}$

$$g_m \cdot R_D = 70.5 \quad ; \quad R_S R_D = 5\times10^{6}$$

$$R_S R_D C_{gd} = 5\times10^{6} \times 5\times10^{-13} = 2.5\times10^{-6}$$

$$g_m R_S R_D C_{gd} = 14.1\times10^{-3} \times 2.5\times10^{-6}$$

$$= 35.25\times10^{-9}$$

$$\therefore \quad \omega_{p1} = \frac{1}{3\times10^{-9} + 5\times10^{-9} + 35.25\times10^{-9}}$$

$$= \frac{10^{9}}{43.25\times10^{0}} = 23.12\times10^{6}$$

$$\therefore f_{p1} = \frac{1}{2\pi} \, 23.12\times10^{6} = 3.68 \text{ MHz}$$

$$\& \quad \omega_{p2} \simeq \frac{1}{R_D C_{gd}} = \frac{1}{5\times10^{3} \times 5\times10^{-13}} = \frac{10^{10}}{25} = 4\times10^{8}$$

$$f_{p2} = \frac{1}{2\pi} \cdot 4\times10^{8} = 63.6 \text{ MHz}$$

We compare these values with ZVTC & SCTC ('Short Ckt Time constant') technique based evaluations of f_{p1} & f_{p2}

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[1] ZVTC technique gives Dominant Pole:

Here $\tau_1 = R_s C_{gs} = 5 \times 10^{-9}$

$$\tau_2 = (R_S + R_D) C_{gd} + g_m R_S R_D C_{gd}$$

$$= 3 \times 10^{-9} + 35.25 \times 10^{-9}$$

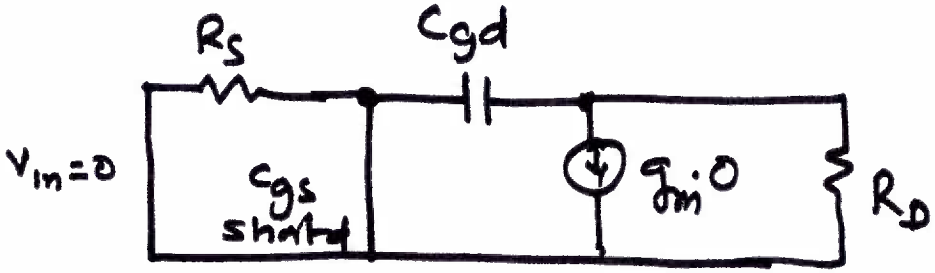
$$\therefore \quad \omega_{p1} = \frac{1}{\sum_j \tau_j} = \frac{1}{\tau_1 + \tau_2} = \frac{1}{43.25 \times 10^{-9}}$$

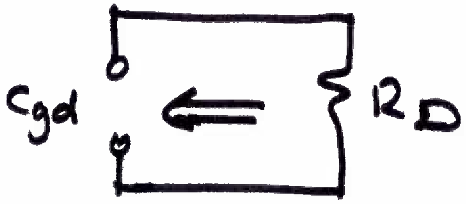
$$\therefore \quad \boxed{f_{p1} = 3.68 \text{ MHz}}$$ Dominant Pole

[2] Short Circuit Time Const. Technique :–

This gives Non dominant Pole.

(a) C_{gd} seen Resistance when C_{gs} is shorted

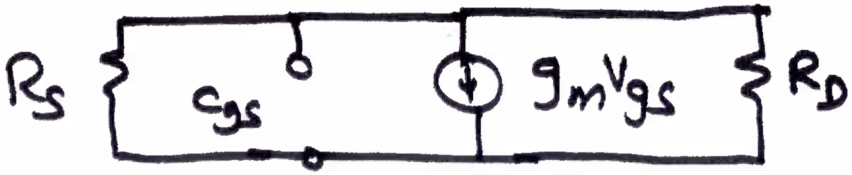




$\therefore \; R_{cgd} = R_D$

$\therefore \; \tau_{cgd} = R_D C_{gd} = 5\times 10^{3} \times 0.5\times 10^{-12} = 2.5 \times 10^{-9}$

(b) C_{gs} seen Resistance when C_{gd} is shorted



$\therefore \; R_{cgs} = (R_S \,||\, R_D \,||\, 1/g_m)$

$$\therefore \tau_{cgs} = R_{cgs} \cdot C_{gs}$$

$$= 65 \times 0.5 \times 10^{-12} \times 10 = 0.325 \times 10^{-9}$$

$$\tau_{sc} = 2.5 \times 10^{-9} + 0.325 \times 10^{-9}$$

$$= 2.825 \times 10^{-9}$$

$$\therefore \omega_{nondom} = \omega_{p2} = \frac{1}{2.825 \times 10^{-9}}$$

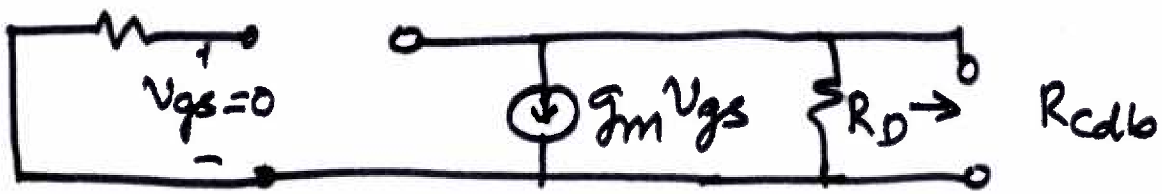
$$\therefore f_{p2} = \frac{10^{9}}{6.28 \times 2.825} = 56.3\ \text{MHz}$$

f_{p2} we obtained earlier with nodal analysis $= 63.6$ MHz

$$\therefore \quad \tau_2 = R_{cgd} \cdot C_{gd}$$

$$\therefore \quad \tau_2 = (R_S + R_D + g_m R_S R_D)\, C_{gd} \qquad \text{— (2)}$$

Step 3 : We get Time constant as seen by C_{db}



$$\therefore \quad R_{cdb} = R_D$$

$$\therefore \quad \tau_3 = R_D \cdot C_{db} \qquad \text{— (3)}$$

$$\therefore \quad \frac{1}{\tau_{-3db}} = \cancel{\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3}} = \frac{1}{\tau_1 + \tau_2 + \tau_3}$$