As $\frac{g_m}{I_{DS}}$ is our Design Parameter, lets see the Short channel effect in this perspective

For Degenerate Source resistance case

$$\frac{g_m}{I_{DS}} = \frac{1}{1+g_m R_{sx}} \cdot \frac{2}{V_{ov}}$$

or $$\frac{1}{V_{ov}} = \frac{(1+g_m R_{sx})}{2} \frac{g_m}{I_{DS}}$$ or $$\frac{1}{(1+g_m R_{sx}) V_{ov}} = \frac{g_m}{2 I_{DS}}$$

Substituting this in $\frac{I_{ODS}}{I_{SS}}$ expressions for Long & Short channel cases.

Long channel : $$\frac{I_{ODS}}{I_{SS}} = \left[\frac{1}{2}\frac{g_m}{I_{DS}}V_{id}\right] - \frac{1}{8}\left[\frac{1}{2}\frac{g_m}{I_{DS}}V_{id}\right]^3$$





Short Channel case

$$\frac{I_{ODS}}{I_{SS}} = \left[\left(\frac{g_m}{2I_{DS}}\cdot V_{id}\right) - \frac{1}{8}\left(\frac{1}{1+g_m R_{SX}}\right)\left(\frac{1}{2}\frac{g_m}{I_{DS}}\cdot V_{id}\right)^3\right]$$

So even with this design parameter $\left(\frac{g_m}{I_{DS}}\right)$ Linearity $\left(\frac{I_{ODS}}{I_{SS}}\right)$ definitely is better for short channel case than Long channel one.

However this was found only with one effect.

# Current Sources & Sinks



In CMOS Analog IC, current source/sink (cs) acts like a basic Building Block.

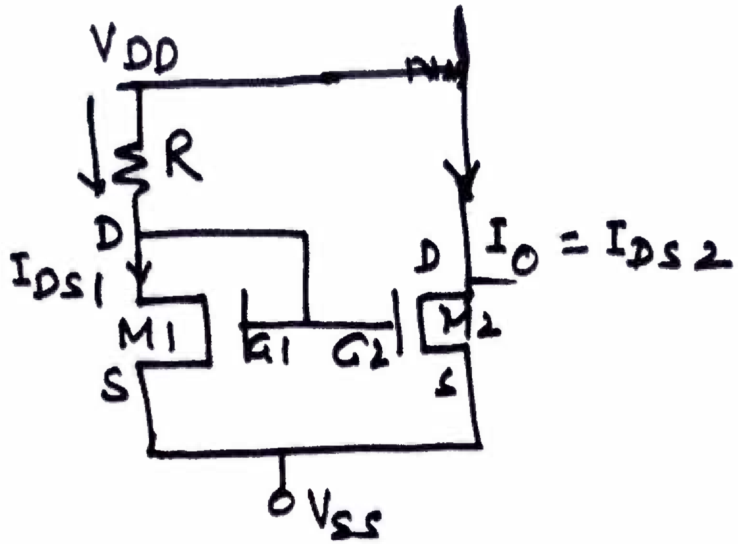
(i) Major requirement for Good CS is, it's Output Impedance be very high (Ideal $R_{out} = \infty$).

(ii) To keep devices in Saturation, output swing be limited

→ The current Mirror try to satisfy above needs but may face limits.

(iii) $V_{min}$ — Drop across Current Source

# Current Mirror for $I_0$ Current Source




For M1

(i) Current through R is same as $I_{DS_1}$ of $M_1$

(ii) $V_{DS1} = V_{GS1}$

Clearly M1 is always in Saturation.

(iii) Further $V_{GS1} = V_{GS2} = V_{DS1}$

Hence $I_{DS1} = I_{DS2}$ (If $\frac{W_2}{L_2} = \frac{W_1}{L_1}$)

(iv) $I_{DS1} = \dfrac{V_{DD} - V_{GS1} - V_{SS}}{R} = \dfrac{\beta'}{2}\left(\dfrac{W_1}{L_1}\right)(V_{GS1} - V_{T1})^2(1+\lambda V_{DS1})$

&

$I_{DS2} = \dfrac{\beta'}{2}\left(\dfrac{W_2}{L_2}\right)(V_{GS2} - V_{T2})^2(1+\lambda V_{DS2})$

But $I_{DS2} = I_0$

$$\therefore \quad \frac{I_{DS2}}{I_{DS1}} = \frac{I_0}{I_{DS1}} = \frac{W_2/L_2}{(W_1/L_1)} \frac{(1+\lambda V_{DS2})(V_{GS_2}-V_{T2})^2}{(1+\lambda V_{DS1})(V_{GS1}-V_{T1})^2} \frac{\beta_2'}{\beta_1'}$$

If $V_{T2} = V_{T1}$, $\beta_2' = \beta_1'$

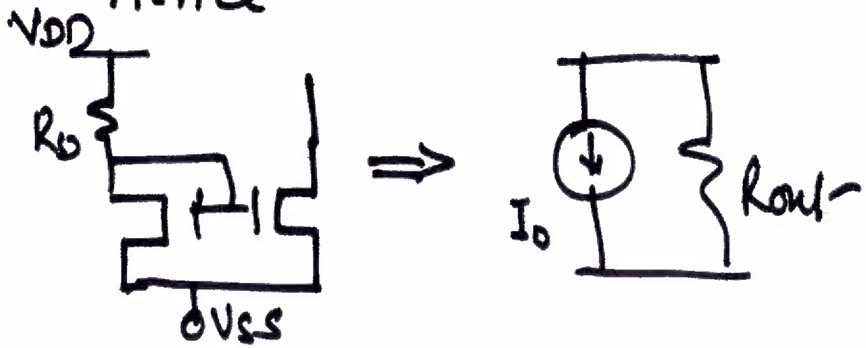
Further If $\lambda$ is very small, then

$$\frac{I_0}{I_{DS1}} = \frac{(W_2/L_2)}{(W_1/L_1)}$$

$$\text{or} \quad I_0 = \frac{(W/L)_2}{(W/L)_1} \cdot \frac{V_{DD}-V_{GS1}-V_{SS}}{R}$$

$$R_{out} = r_{o2} = \frac{1}{\lambda I_{DS_2}} = \frac{1}{\lambda I_o} = \frac{V_A}{I_0} \qquad (V_A \Rightarrow \text{Early Voltage})$$

Hence



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We take $V_{ov} = 200\ mV$, $V_T = 0.6\ V$, $\lambda = 0.06$

$V_{DD} = 1.5\ V = -V_{ss}$, $I_{DS1} = 10\ \mu A$

Then $V_{GS1} = 0.6 + 0.2 = 0.8\ V = V_{GS2}$

$$R = \frac{3 - 0.8}{I_{DS1}} = \frac{2.2}{10\ \mu A} = 220\ K$$

Now $I_0 = 10\ \mu A = \frac{\beta'}{2}\left(\frac{W}{L}\right)_2 (V_{GS_2} - V_{T2})^2$

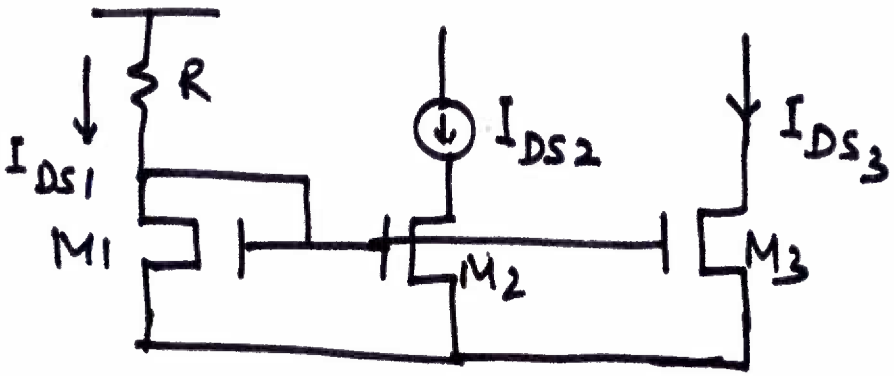
$$10^{-5} = \frac{110 \times 10^{-6}}{2}\left(\frac{W}{L}\right)_2 (0.8 - 0.6)^2 = 11 \times 10^{-6}\left(\frac{W}{L}\right)_2 \times 0.2$$

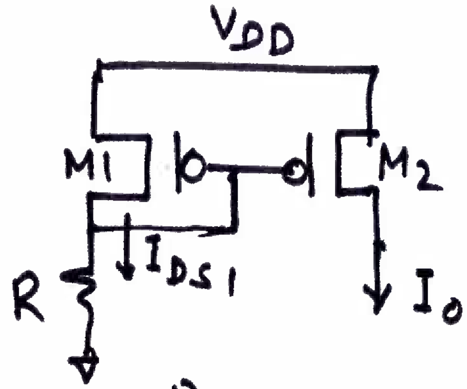
$$\frac{10}{2.2} = (W/L)_2 \quad \text{or} \quad \left(\frac{W}{L}\right)_2 = \frac{50}{11} = 4.54 \simeq 5\ \mu m/\mu m$$

$$R_{out} = \frac{1}{\lambda I_0} = \frac{1}{0.06 \times 10^{-5}} = 1.666\ M\Omega$$

If $L = 0.25\mu$ than $W = 1.135\ \mu m \cong 1.25\mu$

Now $V_{DS2} \geq V_{GS_2} - V_T$ for Saturation. Here $V_{DS_2} \geq 200\ mV$



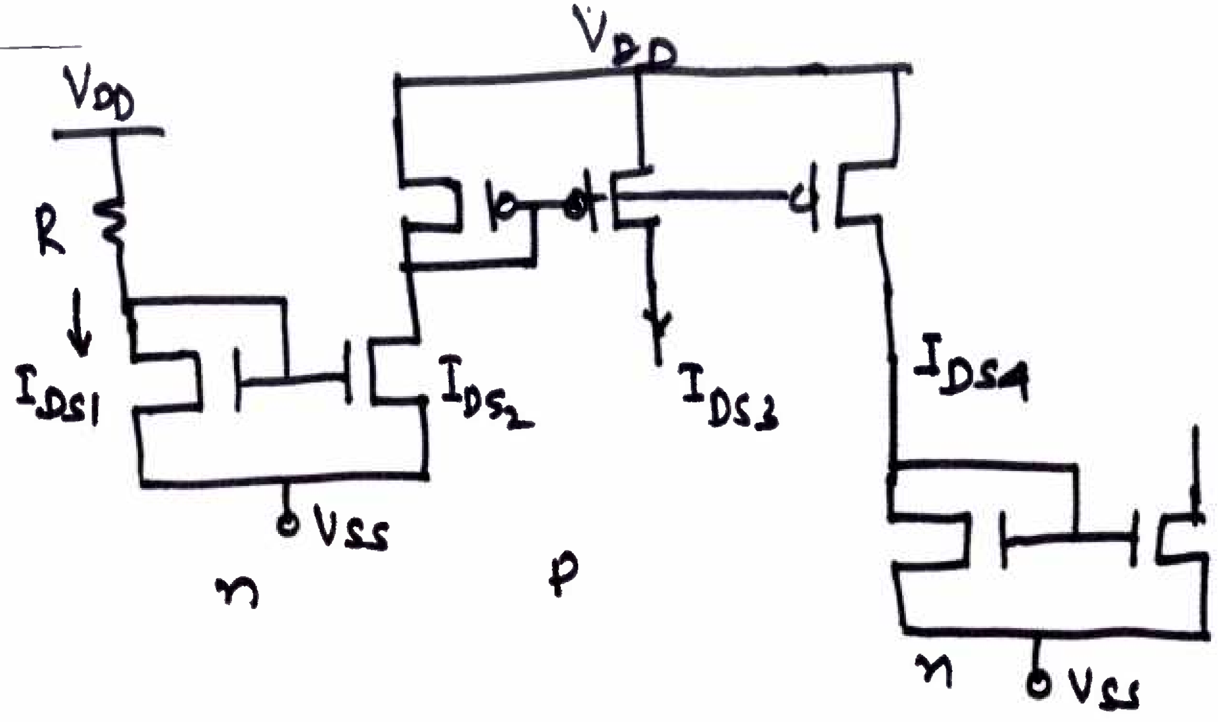

PMOS- CM

If $\frac{W_1}{L_1} = 10$, $\frac{W_2}{L_2} = 20$ and $\frac{W_3}{L_3} = 40$, then

$$\frac{I_{DS2}}{I_{DS1}} = \frac{20}{10} = 2 \quad \text{and} \quad \frac{I_{DS3}}{I_{DS1}} = \frac{40}{10} = 4$$

If $I_{DS1} = 10\,\mu A$, then $I_{DS2} = 20\,\mu A$ and $I_{DS3} = 40\,\mu A$

This is the principle of Current Mirror.






Typical Source/Sink Combination of Current Mirror

# Matching Accuracies

M1 & M2 transistors in a Current Mirror are not identical, in some parameters due to Process Variabilities.

We have $V_{GS1} = V_{GS2} = V_{GS}$

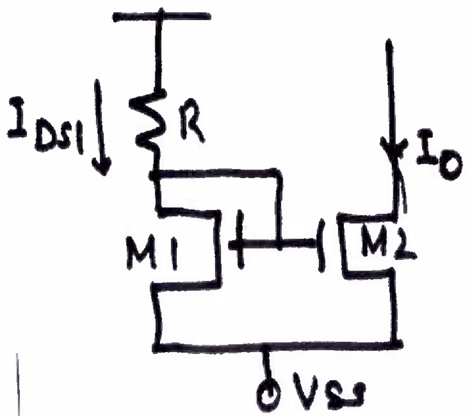
& Define $\Delta V_T = V_{T2} - V_{T1}$

$\Delta \beta' = \beta_2' - \beta_1'$

Let us assume $V_T$ variation is $\pm \Delta V_T/2$ and

$\beta'$ variation is $\pm \dfrac{\Delta \beta'}{2}$

$\lambda$ varies by $\pm \dfrac{\Delta \lambda}{2}$



Then
$$\beta_1' = \beta' - \Delta\beta'/2$$
$$\beta_2' = \beta' + \Delta\beta'/2$$
$$V_{T1} = V_T - \frac{\Delta V_T}{2}$$
$$V_{T2} = V_T + \frac{\Delta V_T}{2}$$
$$\lambda_1 = \lambda - \frac{\Delta\lambda}{2}$$
$$\lambda_2 = \lambda + \frac{\Delta\lambda}{2}$$

Hence
$$\frac{I_o}{I_{DS1}} = \frac{(\beta' + \Delta\beta'/2)(V_{GS} - V_T - 0.5\,\Delta V_T)^2\left[1 + \lambda V_{DS} + \frac{\Delta\lambda}{2} V_{DS}\right]}{(\beta' - \frac{\Delta\beta'}{2})(V_{GS} - V_T + 0.5\,\Delta V_T)^2\left[1 + \lambda V_{DS} - \frac{\Delta\lambda}{2} V_{DS}\right]}$$
$$= \frac{\beta'\{2(V_{GS}-V_T)^2\}\{1+\lambda V_{DS}\}\left(1 + \frac{\Delta\beta'}{2\beta'}\right)\left(1 - \frac{\Delta V_T}{4(V_{GS}-V_T)}\right)^2\left(1 + \frac{\Delta\lambda V_{DS}}{1+\lambda V_{DS}}\right)}{\beta'\{2(V_{GS}-V_T)^2\{1+\lambda V_{DS}\}\left(1 - \frac{\Delta\beta'}{2\beta'}\right)\left(1 + \frac{\Delta V_T}{4(V_{GS}-V_T)}\right)^2\left(1 - \frac{\Delta\lambda V_{DS}}{1+\lambda V_{DS}}\right)}$$

or

$$= \frac{\left(1+\frac{\Delta\beta'}{2\beta'}\right)\left(1-\frac{\Delta V_T}{4(V_{GS}-V_T)}\right)^2\left(1+\frac{\Delta\lambda V_{DS}}{1+\lambda V_{DS}}\right)}{\left(1-\frac{\Delta\beta'}{2\beta'}\right)\left(1+\frac{\Delta V_T}{4(V_{GS}-V_T)}\right)^2\left(1-\frac{\Delta\lambda V_{DS}}{1+\lambda V_{DS}}\right)}$$

We know

$$\left(1-\frac{\Delta x}{x}\right)^{-1} = 1+\frac{\Delta x}{x} \quad \text{if } \Delta x << x$$

Then

$$\frac{I_O}{I_{DS1}} = \left(1+\frac{\Delta\beta'}{2\beta'}\right)^2\left[1-\frac{\Delta V_T}{4(V_{GS}-V_T)}\right]^4\left[1+\frac{\Delta\lambda V_{DS}}{1+\lambda V_{DS}}\right]^2$$

$$= \left[1+\frac{\Delta\beta'^2}{4\beta'^2}+\frac{\Delta\beta'}{\beta'}\right]\left[1+\frac{\Delta V_T^2}{4(V_{GS}-V_T)^2}-\frac{\Delta V_T}{2(V_{GS}-V_T)}\right]^2\left[1+\frac{\Delta\lambda^2 V_{DS}^2}{(1+\lambda V_{DS})^2}+\frac{2\Delta\lambda V_{DS}}{1+\lambda V_{DS}}\right]$$

Since $\Delta$ terms are small, hence $\Delta^2$ terms are even smaller & hence neglected

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EE 618 L 13 / Slide 12

$$\frac{I_0}{I_{DS1}} = \left(1+\frac{\Delta\beta'}{\beta'}\right)\left(1-\frac{\Delta V_T}{2(V_{GS}-V_T)}\right)^{2}\left(1+\frac{2\Delta\lambda V_{DS}}{1+\lambda V_{DS}}\right)$$

$$= \left(1+\frac{\Delta\beta'}{\beta'}\right)\left[1-\frac{2\Delta V_T}{\cancel{2}(V_{GS}-V_T)}\right]\left(1+\frac{2\Delta\lambda}{\lambda}\right)$$

We take

$$\underline{1+\frac{2\Delta\lambda}{\lambda} \simeq 1}$$

$$= 1+\frac{\Delta\beta'}{\beta'} - \left(1+\frac{\Delta\beta'}{\beta'}\right)\left(\frac{\Delta V_T}{V_{GS}-V_T}\right)$$

$$= 1+\frac{\Delta\beta'}{\beta'} - \frac{2\Delta V_T}{(V_{GS}-V_T)} - \frac{\Delta\beta'}{\beta'}\,\frac{2\Delta V_T}{V_{GS}-V_T}$$

$$= 1+\frac{\Delta\beta'}{\beta'} - \frac{2\Delta V_T}{V_T}\cdot\frac{1}{\left(\frac{V_{GS}}{V_T}-1\right)} - \frac{\Delta\beta'}{\beta'}\cdot\frac{2\Delta V_T}{V_T}\cdot\frac{1}{\left(\frac{V_{GS}}{V_T}-1\right)}$$

Typically $\frac{I_0}{I_{DS1}} = 1 \pm 0.04$ i.e. 4% error if $\Delta f = \pm 5\%$