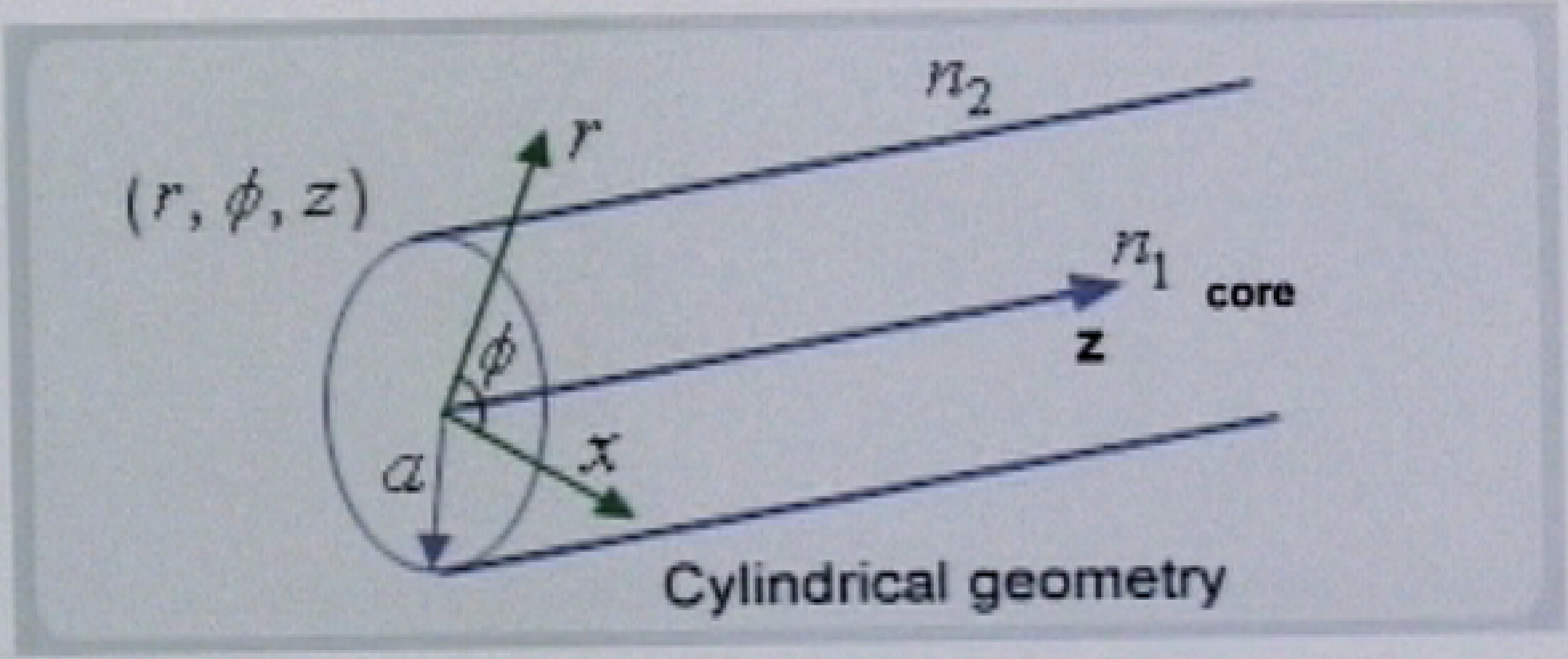
Wave Propagation in Optical Fibre



$$\varepsilon_1 = \varepsilon_0 n_1^2$$

$$\varepsilon_2 = \varepsilon_0 n_2^2$$

ε_0 = free space permittivity

$\mu = \mu_0$ = free space permeability

Inside Core $(r < a)$

Electric field:

$$E_{z1} = AJ_{\nu}(ur)e^{j\nu\phi - j\beta z + j\omega t}$$

Magnetic field:

$$H_{z1} = BJ_{\nu}\left(ur\right)e^{j\nu\phi - j\beta z + j\omega t}$$

$$u = \sqrt{\beta_1^2 - \beta^2}$$

$$\beta_1^2 = \omega^2 \mu\varepsilon_1 = \omega^2 \mu\varepsilon_0 n_1^2 = \beta_0^2\, n_1^2$$

In Cladding $(r > a)$

Electric field:

$$E_{z2} = CK_{\nu}(wr)e^{j\nu\phi - j\beta z + j\omega t}$$

Magnetic field:

$$H_{z2} = DK_{\nu}(wr)e^{j\nu\phi - j\beta z + j\omega t}$$

$$w = \sqrt{\beta^2 - \beta_2^2} \qquad \text{Real}$$

$$\beta_2^2 = \omega^2 \mu\varepsilon_2 = \omega^2 \mu\varepsilon_0 n_2^2 = \beta_0^2 n_2^2$$

$$\beta_0 n_2 = \beta_2 < \beta < \beta_1 = \beta_0 n_1$$

$$\frac{\beta}{\beta_0} = n_{eff}$$

$$n_2 < n_{eff} < n_1$$

Field components inside core and cladding

Core

$$E_{r1}, E_{\phi 1}, E_{z1}$$

$$H_{r1}, H_{\phi 1}, H_{z1}$$

Cladding:

$$E_{r2}, E_{\phi 2}, E_{z2}$$

$$H_{r2}, H_{\phi 2}, H_{z2}$$

Boundary conditions $r = a$

1. Tangential components of electric field is continuous.

2. Tangential components of magnetic field is continuous, since there are no surface currents.

$$E_{\phi 1} = E_{\phi 2}$$

$$E_{z1} = E_{z2}$$

$$H_{\phi 1} = H_{\phi 2}$$

$$H_{z1} = H_{z2}$$

Characteristic Equation

$$\left\{\frac{J_\nu'(ua)}{uJ_\nu(ua)}+\frac{K_\nu'(wa)}{wK_\nu(wa)}\right\}\left\{\beta_1^2\frac{J_\nu'(ua)}{uJ_\nu(ua)}+\beta_2^2\frac{K_\nu'(wa)}{wK_\nu(wa)}\right\}$$

$$=\frac{\beta^2\nu}{a}\left\{\frac{1}{u^2}+\frac{1}{w^2}\right\}^2$$

Hybrid Mode

$$J_\nu'(x) \equiv \frac{\partial}{\partial x} J_\nu(x)$$

$$K_\nu'(x) \equiv \frac{\partial}{\partial x} K_\nu(x)$$

$$u^2 = \omega^2 \mu \in_1 - \beta^2$$

$$w^2 = \beta^2 - \omega^2 \mu \in_2$$

For $v = 0$

$$\left\{ \frac{J_0'(ua)}{uJ_0(ua)} + \frac{K_0'(wa)}{wK_0(wa)} \right\} \left\{ \beta_1^2 \frac{J_0'(ua)}{uJ_0(ua)} + \beta_2^2 \frac{K_0'(wa)}{wK_0(wa)} \right\} = 0$$

$$\left\{ \frac{J_0'(ua)}{uJ_0(ua)} + \frac{K_0'(wa)}{wK_0(wa)} \right\} = 0$$

$$\frac{J_1(ua)}{uJ_0(ua)} + \frac{K_1(wa)}{wK_0(wa)} = 0 \text{ For TE mode}$$

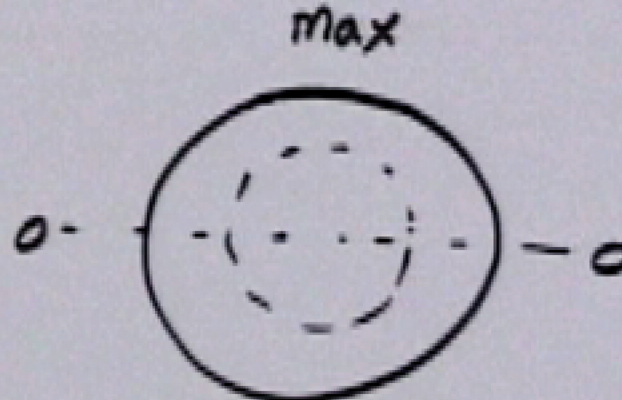
$$J_0'(x) = -J_1(x)$$

$$e^{j\nu\phi}$$

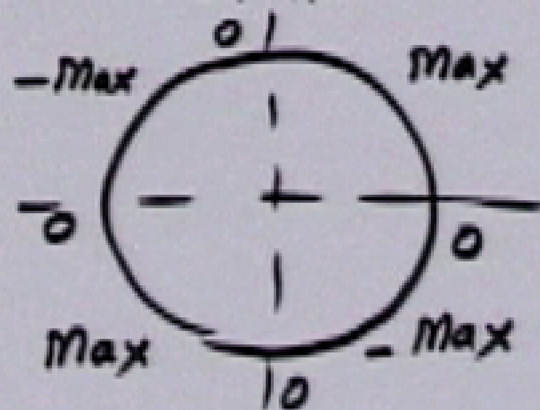
Meridional Ray $\nu = 0$ circularly symmetric field.

Skew rays

$\nu = 1$



$\nu = 2$



$$u^2 = \omega^2 \mu \in_1 - \beta^2$$

$$w^2 = \beta^2 - \omega^2 \mu \in_2$$

For $v = 0$

$$\overbrace{\left\{ \frac{J_0'(ua)}{uJ_0(ua)} + \frac{K_0'(wa)}{wK_0(wa)} \right\}}^{TE} \overbrace{\left\{ \beta_1^2 \frac{J_0'(ua)}{uJ_0(ua)} + \beta_2^2 \frac{K_0'(wa)}{wK_0(wa)} \right\}}^{TM} = 0$$

$$\left\{ \frac{J_0'(ua)}{uJ_0(ua)} + \frac{K_0'(wa)}{wK_0(wa)} \right\} = 0$$

$$\frac{J_1(ua)}{uJ_0(ua)} + \frac{K_1(wa)}{wK_0(wa)} = 0 \quad \text{For TE mode}$$

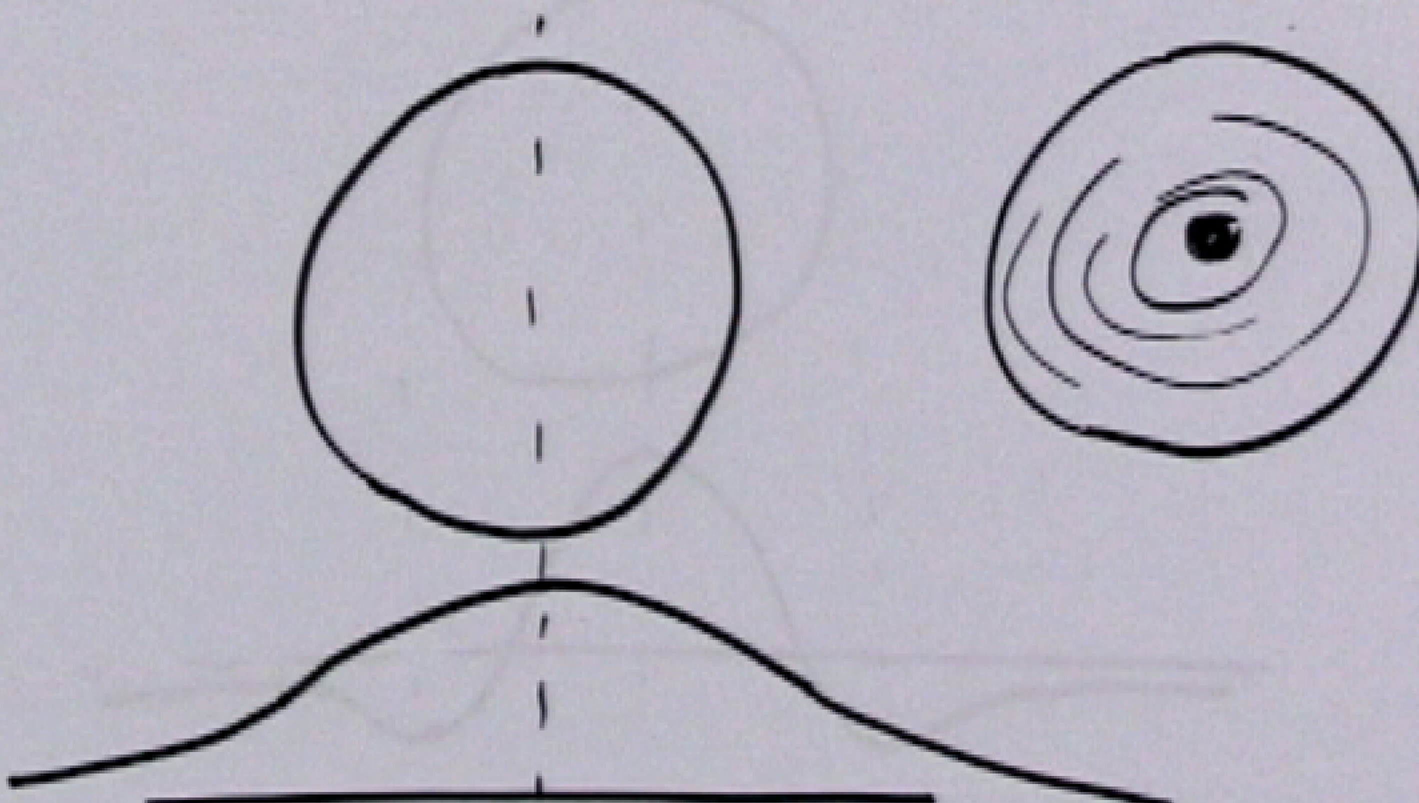
$$J_0'(x) = -J_1(x)$$

$$TE_{0m} \Rightarrow TE_{01}, TE_{02}, \ldots$$

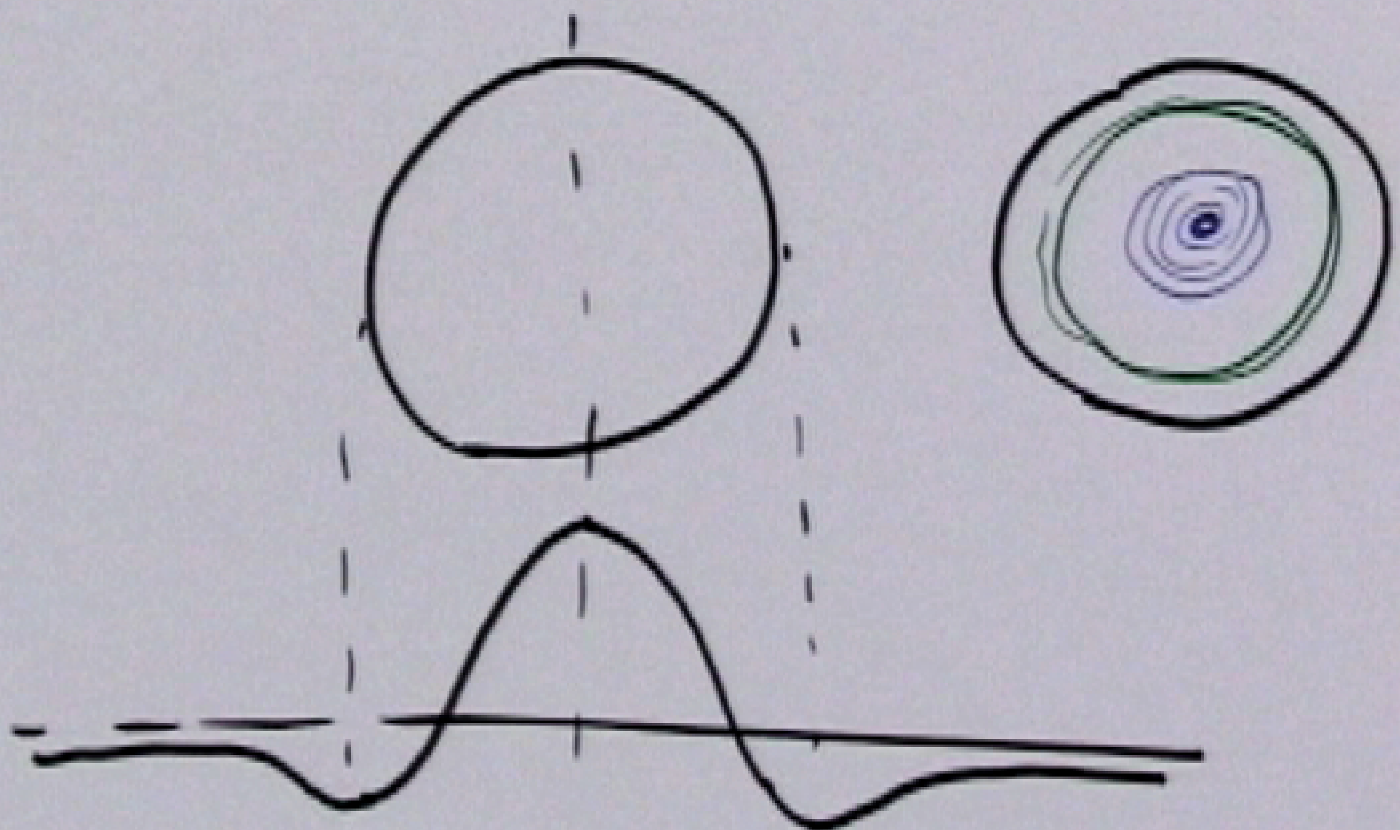
$$TM_{0m} \Rightarrow TM_{01}, TM_{02}, \ldots$$

$$HE_{\nu m}$$

TE_{01}



TE_{02}



$$u^2 = \omega^2 \mu\varepsilon_1 - \beta^2$$

$$w^2 = -\omega^2 \mu\varepsilon_2 + \beta^2$$

$$\rightarrow u^2 + w^2 = \omega^2 \mu\varepsilon_1 - \omega^2 \mu\varepsilon_2$$

$$= \beta_1^2 - \beta_2^2$$

$$a^2\left(u^2 + w^2\right) = a^2(\beta_1^2 - \beta_2^2)$$

$$a^2\left(u^2 + w^2\right) = a^2(\beta_1^2 - \beta_2^2) = V^2$$

$$\mathbf{V}^2 = a^2(\beta_0^2 n_1^2 - \beta_0^2 n_2^2)$$

$$\mathbf{V} = a\beta_0\sqrt{n_1^2 - n_2^2} = \frac{a\omega}{c}\sqrt{n_1^2 - n_2^2}$$

V-number of an optical fiber

$$V = \frac{\omega a}{c}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda}(NA)$$

Normalized Propagation constant β

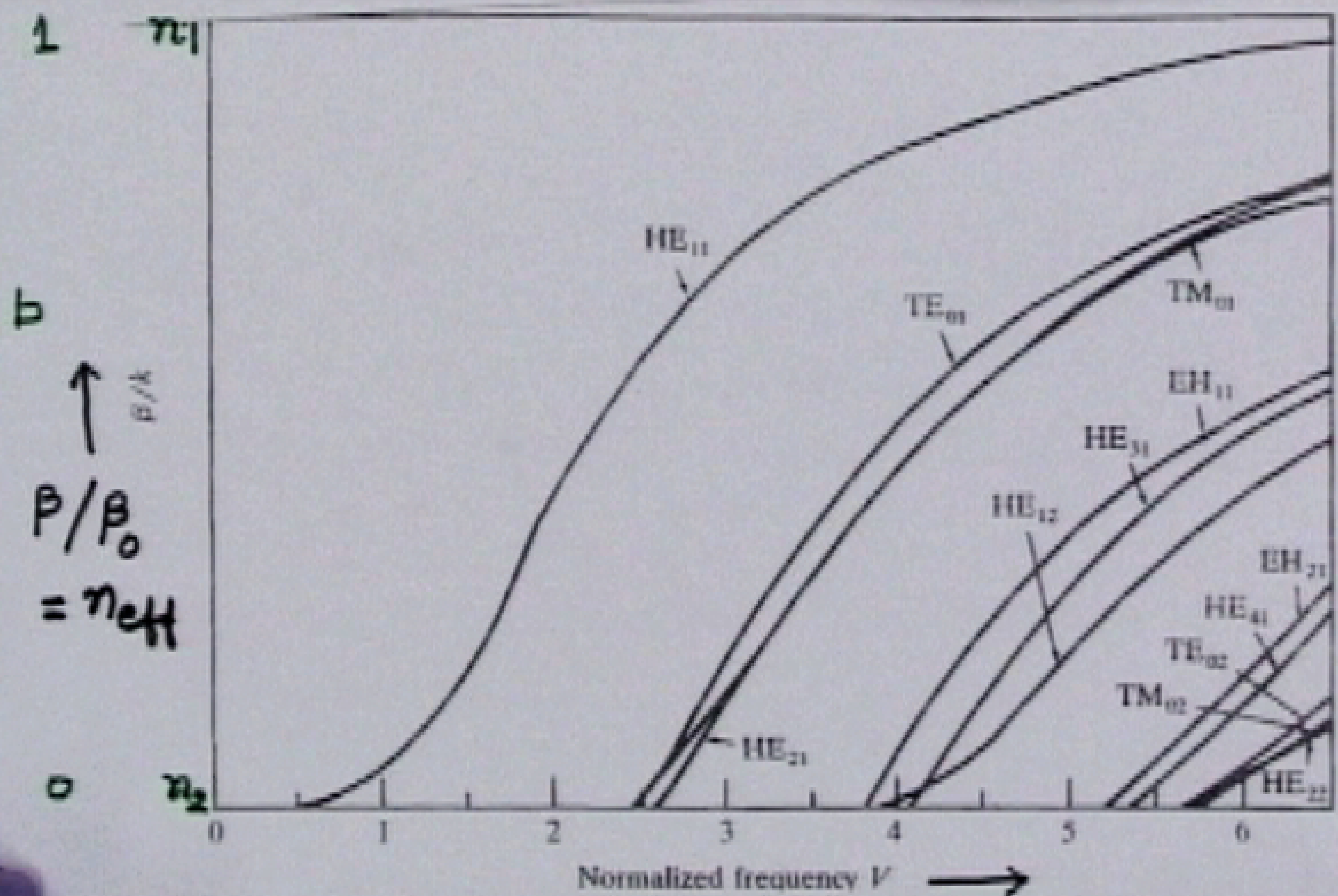
$$b = \frac{n_{eff}^2 - n_2^2}{n_1^2 - n_2^2} \approx \frac{n_{eff} - n_2}{n_1 - n_2}$$

Effective index of propagation $n_{eff} \equiv \frac{\beta}{(2\pi / \lambda)}$

0 < b < 1

$$\beta_2 < \beta < \beta_1$$

Cut-off $\beta \rightarrow \beta_2$


1
n_1
b
β/β_0
$= n_{eff}$
0
n_2
β/k
HE_{11}
TE_{01}
TM_{01}
EH_{11}
HE_{31}
HE_{12}
EH_{21}
HE_{41}
TE_{02}
TM_{02}
HE_{21}
HE_{22}
0
1
2
3
4
5
6
Normalized frequency V