Optical Solitons

GVD is balanced by SPM

Normalized parameters

$$U = \frac{A}{\sqrt{P_0}}, \quad \xi = \frac{z}{L_D}, \quad \tau = \frac{T}{T_0}$$

Loss is neglected

$$\frac{\partial U}{\partial \xi} - j\,\mathrm{sgn}(\beta_2)\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2} + jN^2|U|^2 U = 0$$

$$N = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} \rightarrow \text{Order of a soliton}$$

$N = 1 \rightarrow$ Fundamental soliton

Define $u = N U \rightarrow U = u/N$

Anomalous dispersion $\beta_2 < 0$, $sgn(\beta_2) = -1$

$$\frac{\partial u}{\partial \xi} + j \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + j |u|^2 u = 0$$

Assuming $u(\xi, \tau) = V(\tau) e^{j\phi(\xi,\tau)}$

$$\phi(\xi, \tau) = -\underbrace{K}_{\text{phase const}} \xi + \underbrace{\delta}_{\text{frequency shift}} \tau$$

$\delta = 0$

$$\phi(\xi, \tau) = -K \xi$$

$$\frac{d^2 V}{d\tau^2} = 2V(K - V^2) \qquad \times\ 2\frac{dV}{d\tau}$$

$$\underbrace{\int 2\frac{dV}{d\tau}\cdot\frac{d^2V}{d\tau^2}\,d\tau}_{I} = \int 2V(K-V^2)\,2\frac{dV}{d\tau}\,d\tau$$

$$I = \int 2\frac{dV}{d\tau}\frac{d^2V}{\partial\tau^2}\,d\tau$$

$$= 2\frac{dV}{d\tau}\cdot\frac{dV}{d\tau} - 2\int\frac{d^2V}{d\tau^2}\cdot\frac{dV}{d\tau}\,d\tau$$

$$= 2\left(\frac{dV}{d\tau}\right)^2 - I \Rightarrow I = \left(\frac{dV}{d\tau}\right)^2$$

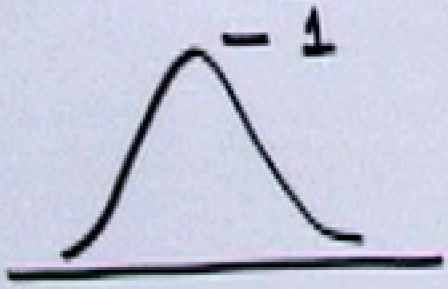
$$\left(\frac{dV}{d\tau}\right)^2 = \int 4V(K - V^2)\,dV$$

$$= 2KV^2 - V^4 + C$$

Apply B.C. $V = 0,\ dV/d\tau = 0 \quad \tau \to \infty$

$$\Rightarrow C = 0$$

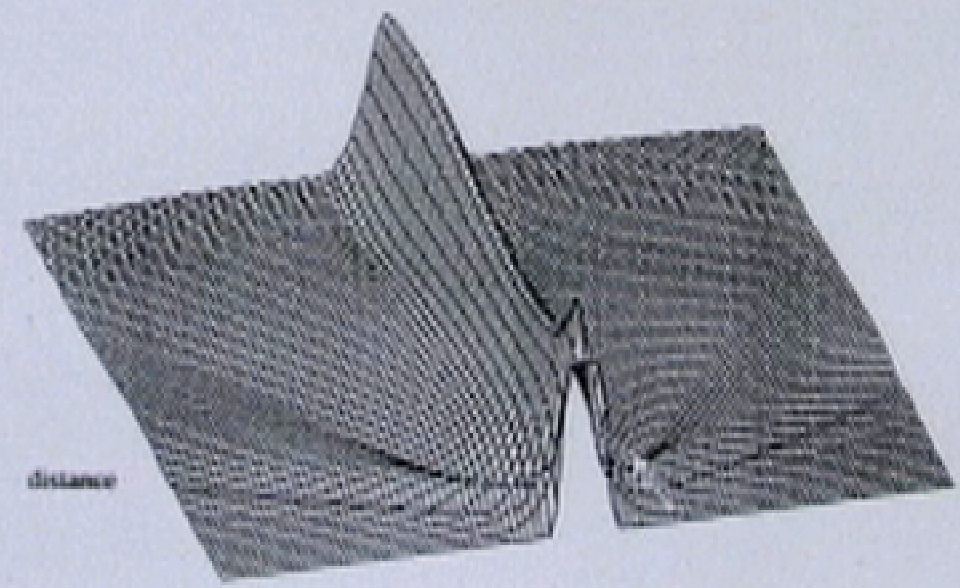
Normalization : $V = 1$ at $\tau = 0$



$dV/d\tau = 0 \leftarrow$ At peak

$$K = 1/2$$

$$\left(\frac{dV}{d\tau}\right)^2 = V^2 - V^4$$

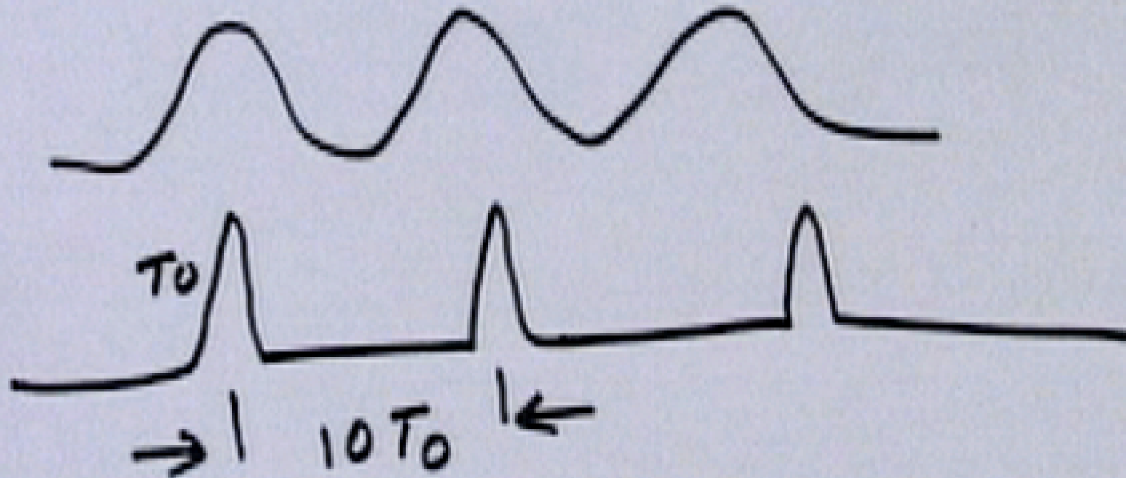


$$A(0,T) = \exp\left[-\frac{1}{2}\left(\frac{T}{T_0}\right)^{2m}\right]$$

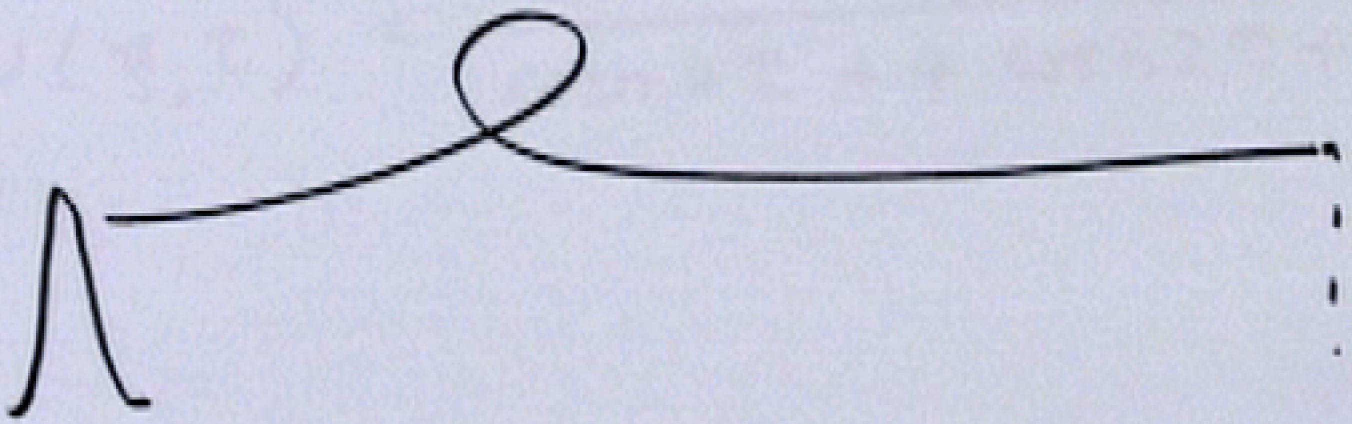
time

$$A(0,T) = \exp\left[-\frac{1}{2}\left(\frac{T}{T_0}\right)^{2m}\right]$$

Pulse shape	Energy retained in soliton (%)
gaussian (m=1)	99 %
supergaussian (m=3)	92 %
rectangular (m → ∞)	86 %



RZ data $\longrightarrow$ Duty cycle $\sim 10\%$



$L_{NL} < L_D$

$$N = \frac{L_D}{L_{NL}} > 1$$

$$\underline{N = 2}$$

$$u(\xi, \tau) = \frac{2\left\{\cosh 3\tau + 3\, e^{j4\xi} \cosh \tau\right\} e^{j\xi/2}}{\cosh 4\tau + 4\cosh 2\tau + 3\cos 4\xi}$$

Distributed Amplification

Raman Amplification

$$\frac{dV}{d\tau} = V\sqrt{1-V^2}$$

$$\int \frac{dV}{V\sqrt{1-V^2}} = \int d\tau$$

$$\text{sech}^{-1}(V) = \tau$$

$$V = \text{sech}(\tau)$$

$$u(\xi, \tau) = \underbrace{\text{sech}(\tau)\, e^{-j\xi/2}}_{\text{Fundamental Soliton}}$$