Self Phase Modulation

$$L_D >> L, \quad L \gtrsim L_{NL}$$

NLS: $$\frac{\partial U}{\partial z} = -j\,\frac{e^{-\alpha z}}{L_{NL}}\,|U|^2 U \qquad L_{NL} \equiv \frac{1}{\gamma P_0}$$

Let $$U = V\,e^{j\phi_{NL}}$$

$$\frac{\partial U}{\partial z} = \frac{\partial V}{\partial z}\,\cancel{e^{j\phi_{NL}}} + jV\,\cancel{e^{j\phi_{NL}}}\,\frac{\partial \phi_{NL}}{\partial z}$$

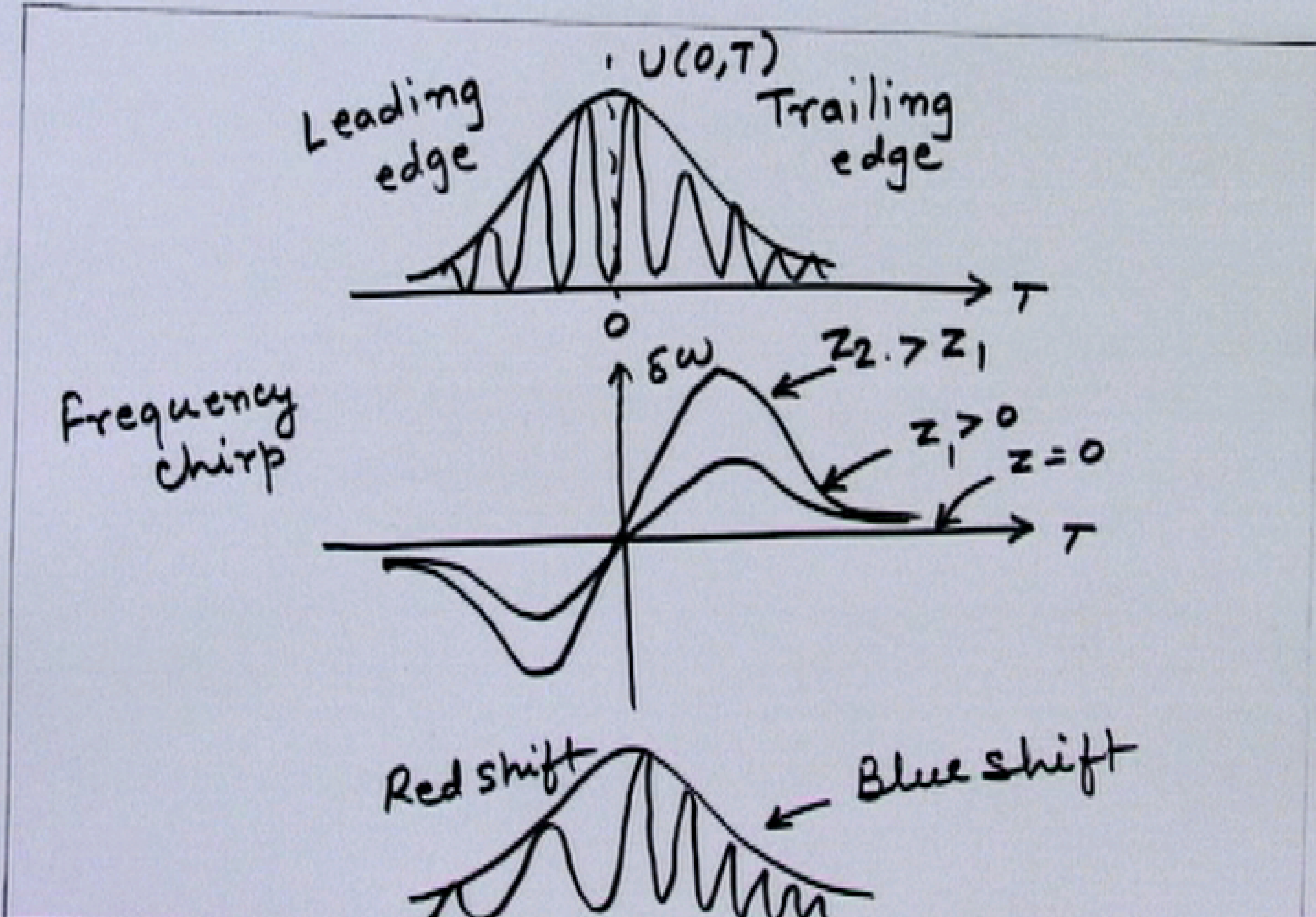
$$= -j\,\frac{e^{-\alpha z}}{L_{NL}}\,|V|^2\,V\,\cancel{e^{j\phi_{NL}}}$$

$$\underbrace{\frac{\partial V}{\partial Z} = 0}_{\text{Amplitude does not change}} \quad , \quad \underbrace{\frac{\partial \phi_{NL}}{\partial Z} = -\frac{e^{-\alpha Z}}{L_{NL}} |V|^2}_{\phi_{NL} \text{ changes with distance}}$$

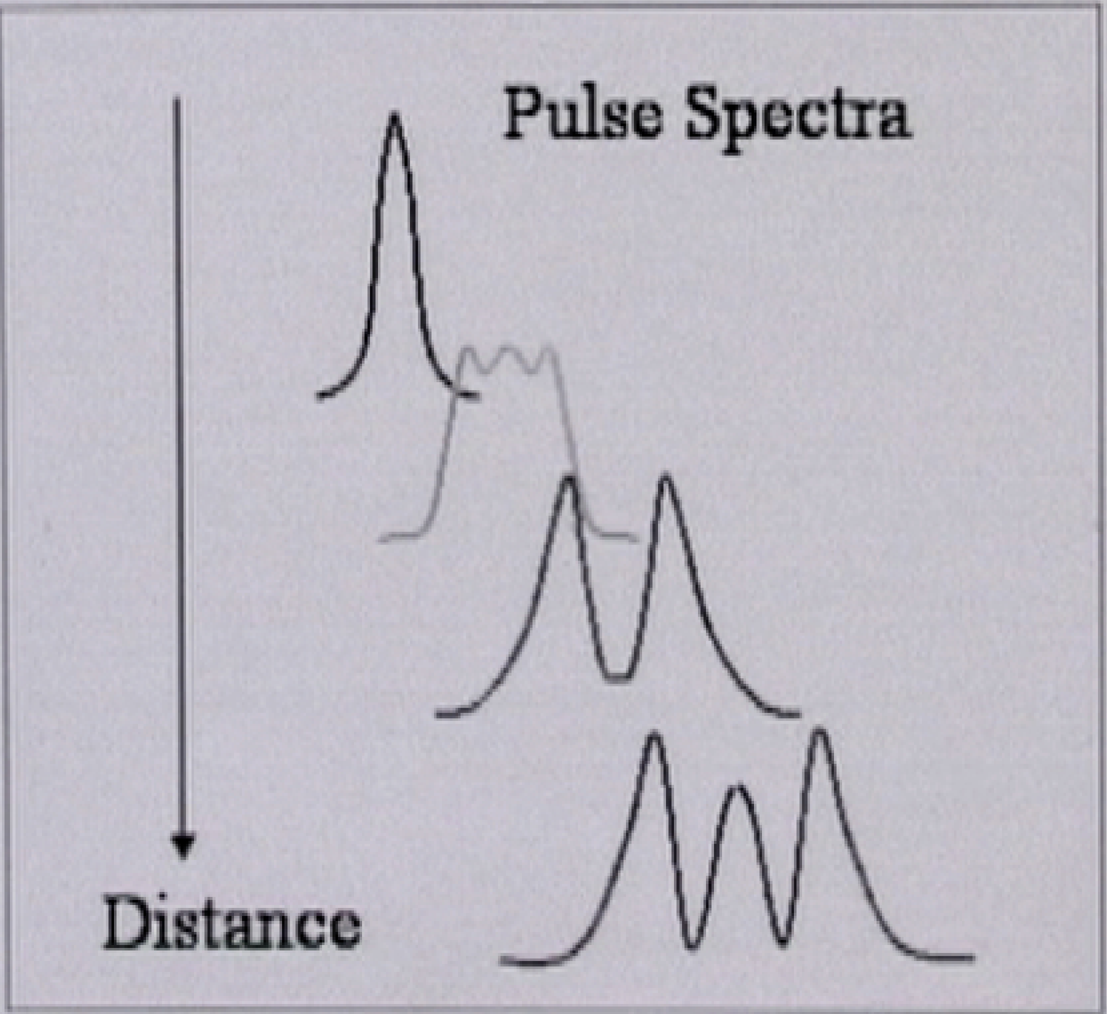
$$V = U(Z,T) = U(0,T)$$

$$\phi_{NL} = \int_0^L -\frac{e^{-\alpha Z}}{L_{NL}} |V|^2 \, dZ$$

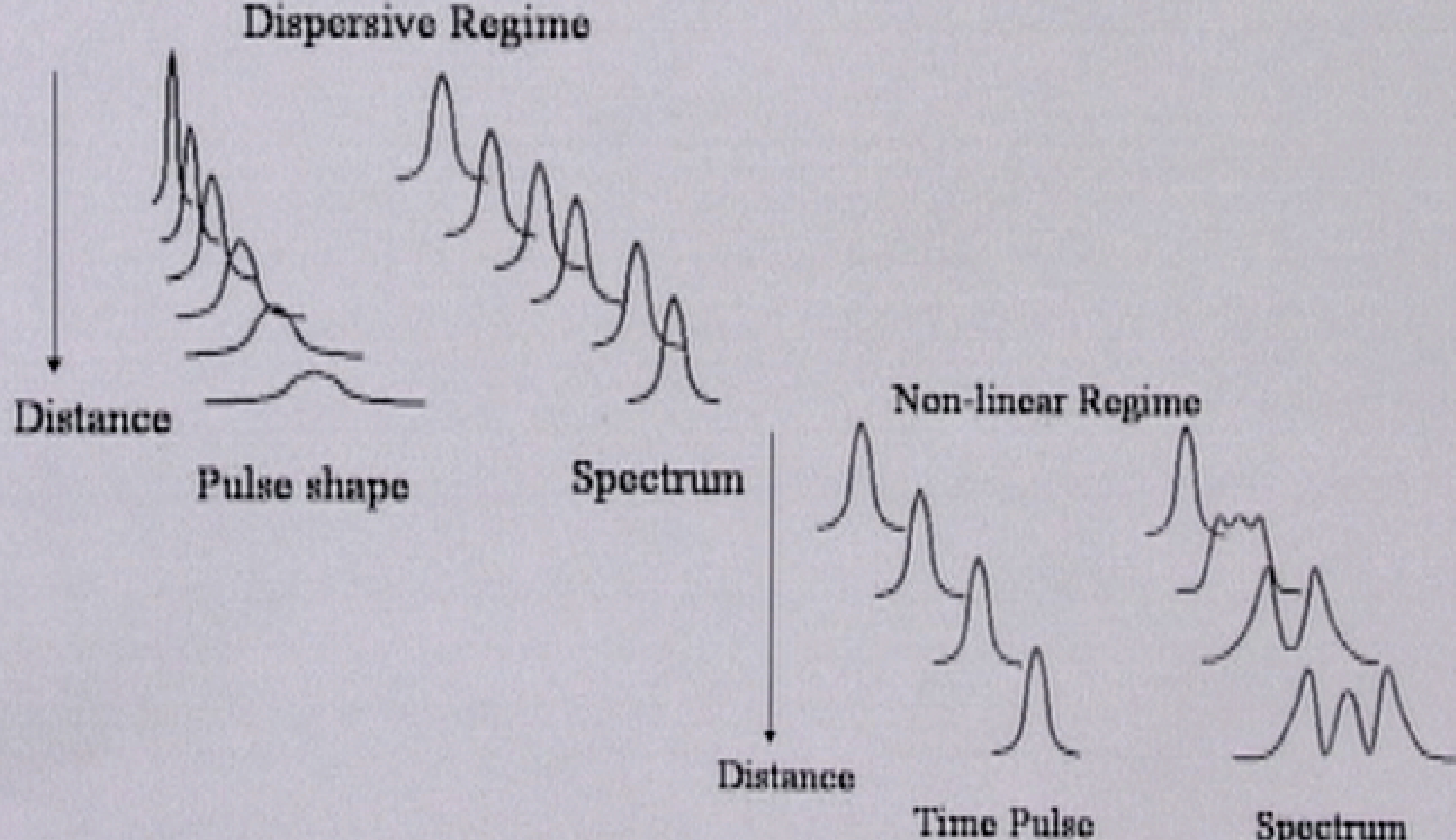
$$= -\frac{|U(0,T)|^2}{L_{NL}} \underbrace{\left\{ \frac{1 - e^{-\alpha Z}}{\alpha} \right\}}_{L_{eff}}$$


U(0,T)
Leading edge
Trailing edge
0
T
δω
z2 > z1
frequency chirp
z1 > 0
z = 0
T
Red shift
Blue shift

Spectral Broadening due to Non-linearity



Signal and spectrum

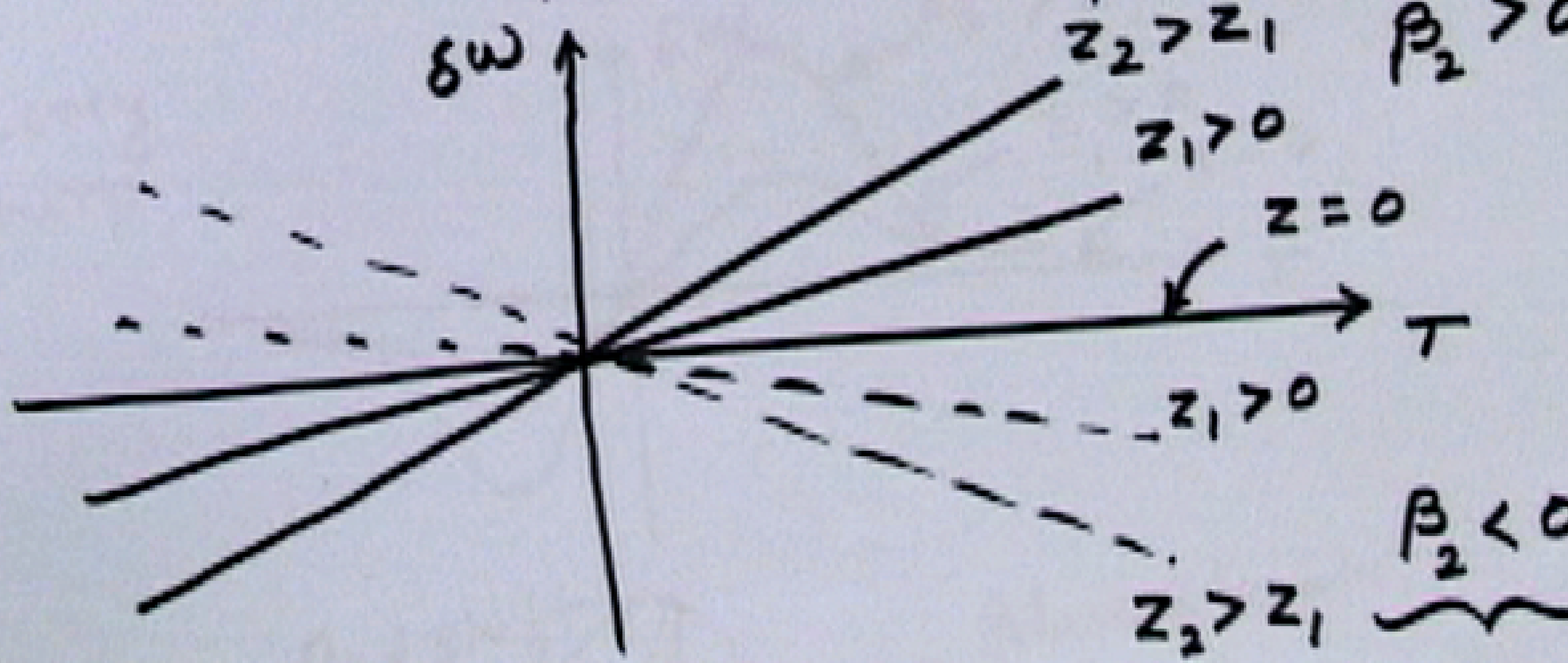


$\beta_2 > 0$ Normal dispersion

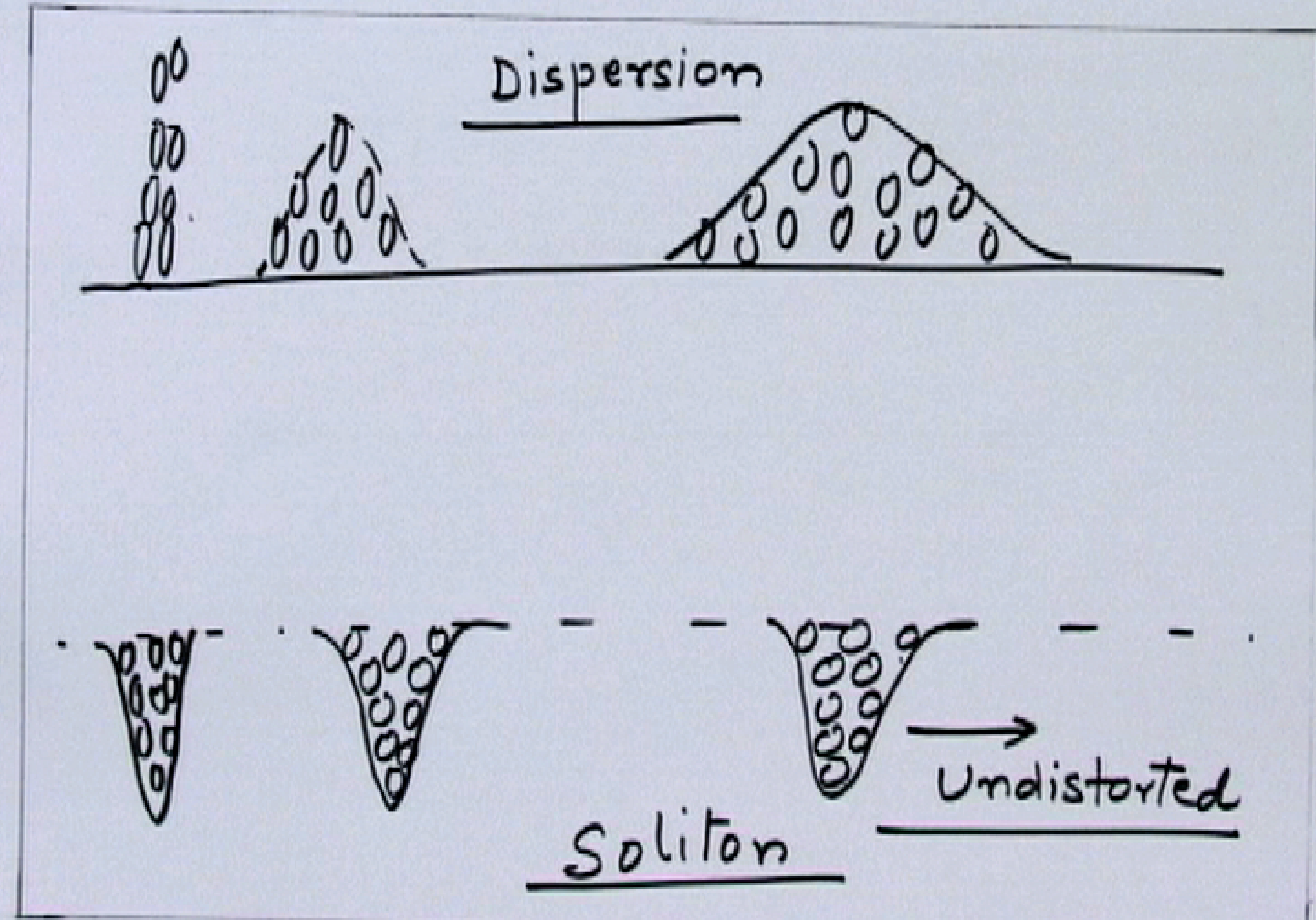
For $\lambda < 1300$ nm

$\beta_2 < 0$ $\lambda > 1300$ nm

Anomalous dispersion



Anomalous dispersion 1550 nm


Dispersion
Undistorted
Soliton

Define $u = NU \rightarrow U = u/N$

Anomalous dispersion $\beta_2 < 0$, $sgn(\beta_2) = -1$

$$\frac{\partial u}{\partial \xi} + j\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + j|u|^2 u = 0$$

Assuming $u(\xi, \tau) = V(\tau)\, e^{j\phi(\xi,\tau)}$

$$\phi(\xi, \tau) = -\underbrace{K}_{\text{phase const}}\xi + \underbrace{\delta}_{\text{frequency shift}}\tau$$

$\delta = 0$

$$\phi(\xi, \tau) = -K\xi$$

$$\frac{d^2 V}{d\tau^2} = 2V(K - V^2) \qquad \times 2\frac{dV}{d\tau}$$

$$\int 2\frac{dV}{d\tau} \cdot \frac{d^2 V}{d\tau^2}\, d\tau = \int 2V(K - V^2)\, 2\frac{dV}{d\tau}\, d\tau$$