Induced Polarization

$$P = \epsilon_0 \left\{ \chi^{(1)} \cdot \bar{E} + \chi^{(2)} : \bar{E}\bar{E} + \chi^{(3)} : \bar{E}\bar{E}\bar{E} + \cdots \right\}$$

Dominant term
(Dielectric const)

For SiO_2
is small

Non-linearity

$$\bar{n}(\omega, |E|^2) = \bar{n}(\omega) + n_2 |E|^2$$

Non-linearity coeff

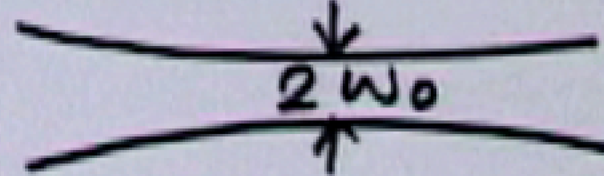
$$n_2 = \frac{3}{8n}\chi^{(3)} \approx 2.3 \times 10^{-22}\ m^2/V^2$$

Figure of Merit

$$\eta \sim I \cdot L_{eff}$$

Light Intensity W/m^2

Interaction length (m)



$$I = \frac{P}{\pi w_0^2}$$

spot radius.

$$L_{eff} = \frac{\pi w_0^2}{\lambda}$$

$$\eta_{bulk} = I \, L_{eff} = \frac{P}{\pi w_0^2} \cdot \frac{\pi w_0^2}{\lambda}$$

$$= \frac{P}{\lambda}$$

<u>Fiber</u> core radius a, Power Attn const α

$$I = \frac{P}{\pi a^2} \quad , \quad P(z) = P(0)\, e^{-\alpha z}$$

$$L_{eff} \approx 1/\alpha$$

$$\eta_{fiber} = \frac{P}{\pi a^2} \cdot \frac{1}{\alpha}$$

$$\frac{\eta_{fiber}}{\eta_{bulk}} = \frac{P/\pi a^2 \alpha}{P/\lambda} = \frac{\lambda}{\pi a^2 \alpha}$$

$a \sim 2\,\mu m$

$\alpha = 0 \cdot 2\ dB/km = 0 \cdot 2 \times 10^{-3}\ dB/m$

$\simeq 8 \times 10^{-5}\ /m$

$\lambda = 1550\ nm = 1 \cdot 55\,\mu m$

$= 10^{9}$

Maxwell's Equations.

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -\mu_0 \frac{\partial \bar{H}}{\partial t}$$

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} \qquad \bar{J} \equiv 0$$

$$\nabla \cdot \bar{D} = 0 \qquad \rho \equiv 0$$

$$\nabla \cdot \bar{B} = 0$$

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P} = \epsilon_0 \bar{E} + \epsilon_0 \chi^{(1)} E$$

$$= \epsilon_0 \underbrace{\{1 + \chi^{(1)}\}}_{\text{Dielectric const (complex)}} \bar{E} \leftarrow \text{Linear case}$$

$$\nabla \times \nabla \times \bar{E} = -\nabla \times \left\{ \mu_0 \frac{\partial \bar{H}}{\partial t} \right\} = -\mu_0 \frac{\partial}{\partial t}\left\{ \nabla \times \bar{H} \right\}$$

$$= -\mu_0 \frac{\partial}{\partial t}\left\{ \frac{\partial \bar{D}}{\partial t} \right\} = -\mu_0 \frac{\partial^2}{\partial t^2}\left\{ \epsilon_0 \bar{E} + \bar{P} \right\}$$

$$= -\mu_0 \frac{\partial^2 (\epsilon_0 \bar{E})}{\partial t^2} - \mu_0 \frac{\partial^2 \bar{P}}{\partial t^2}$$

$$\bar{P} = \epsilon_0 \left\{ \underbrace{\chi^{(1)} \cdot \bar{E}}_{P_L} + \underbrace{\chi^{(3)} \vdots \bar{E}\bar{E}\bar{E}}_{P_{NL}} \right\}$$

$$\nabla \left(\cancelto{0}{\nabla \cdot \bar{E}} \right) - \nabla^2 \bar{E} = -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} - \mu_0 \left\{ \frac{\partial^2 P_L}{\partial t^2} + \frac{\partial^2 P_{NL}}{\partial t^2} \right\}$$

Wave Equation

$$\nabla^2 \bar{E} - \frac{1}{c^2}\frac{\partial^2 \bar{E}}{\partial t^2} = \mu_0 \frac{\partial^2 P_L}{\partial t^2} + \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$$

$$\bar{E} = E_0 e^{j\omega_0 t}$$

signal frequency.

(space, time)

$$\tilde{E}(\bar{r}, \omega - \omega_0) = \int_{-\infty}^{\infty} E(\bar{r}, t)\, e^{-j(\omega-\omega_0)t}\, dt$$

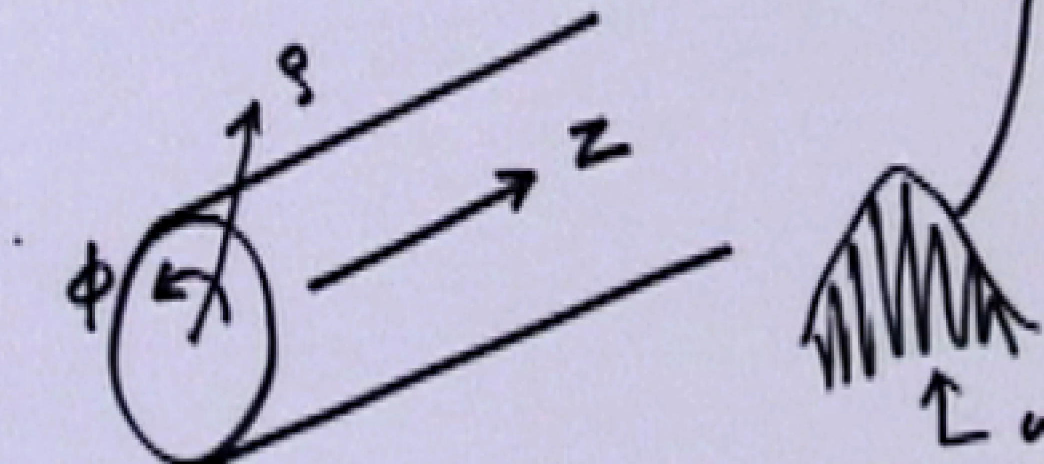
Fourier transform

$$\nabla^2 \tilde{E} + \epsilon(\omega) K_0^2 \tilde{E} = 0$$

$$\epsilon(\omega) = 1 + \chi^{(1)}(\omega) + \epsilon_{NL}$$

complex

$$\tilde{E}(\bar{r}, \omega-\omega_0) = \underbrace{F(\rho, \phi)}\,\tilde{A}(z, \omega-\omega_0)\,e^{-j\beta_0 z}$$



$$\nabla_\perp^2 \bar{F} + \{\epsilon(\omega)k_0^2 - \tilde{\beta}^2\}F = 0$$

$$-2j\beta_0 \frac{\partial \tilde{A}}{\partial z} + (\tilde{\beta}^2 - \beta_0^2)\tilde{A} = 0$$

$\frac{\partial^2 \tilde{A}}{\partial z^2}$ ← negligible.