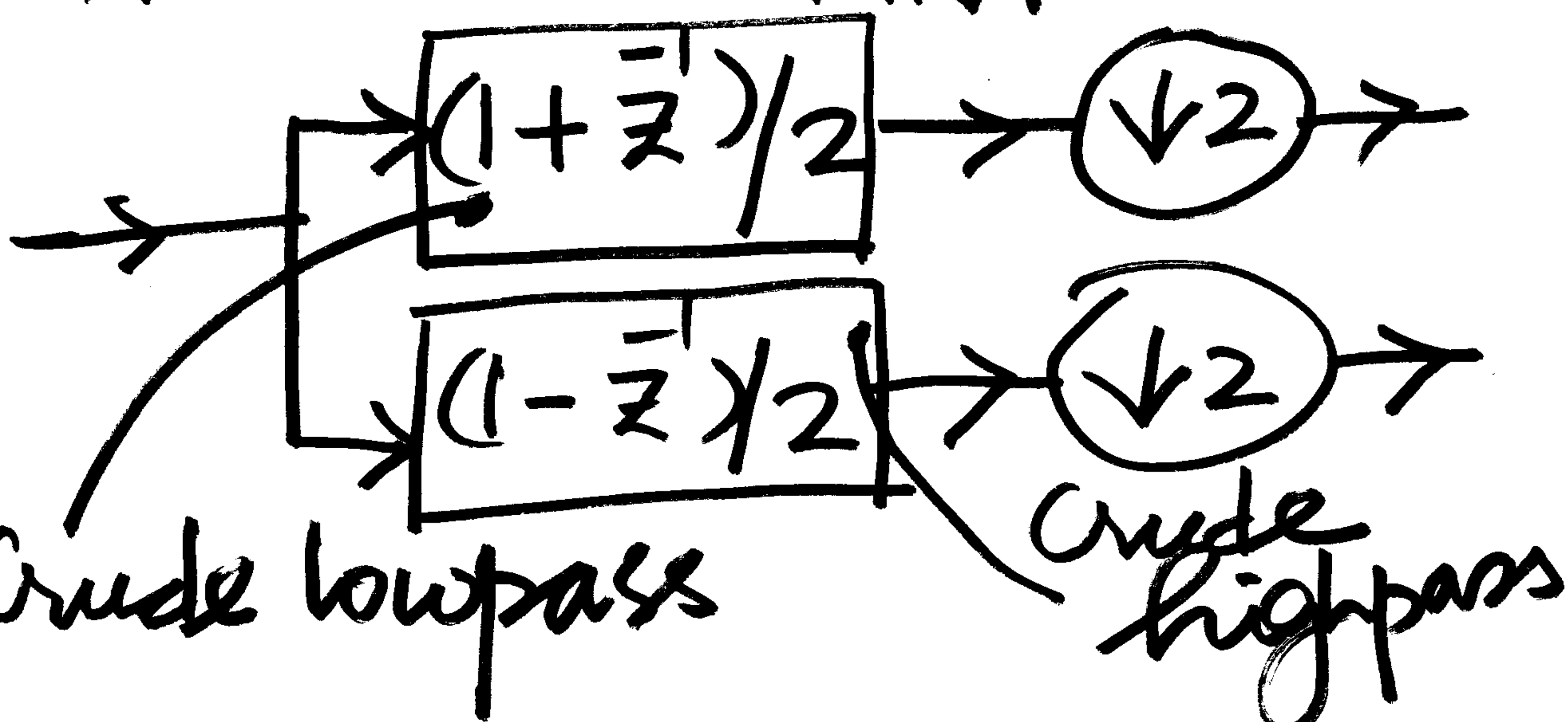
Lecture 8
Relating Φ, ψ and the Filters

Generic Analysis Filter Bank

(Two-band, because <u>dyadic</u>)

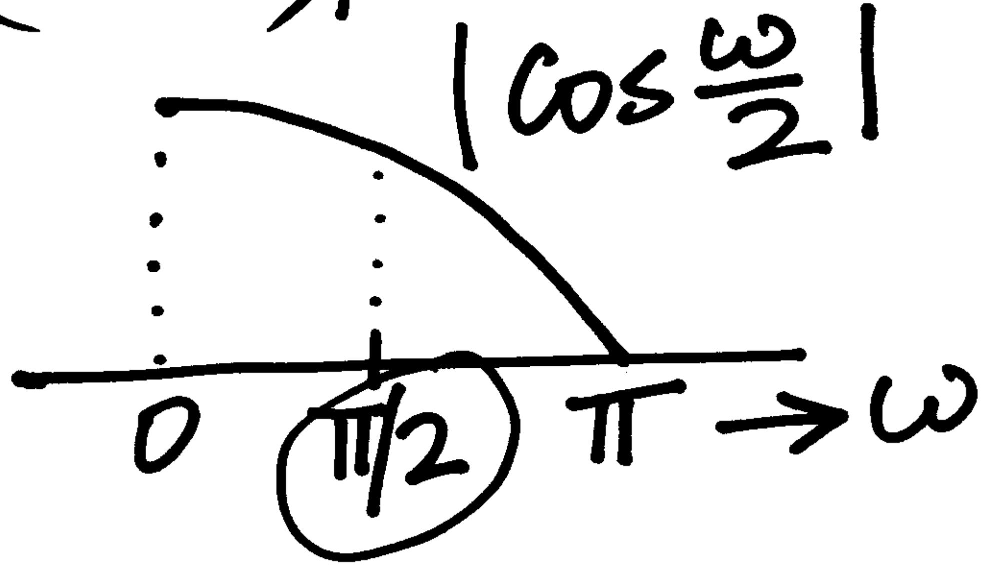
Two band Analysis Filter Bank:

In the Haar MRA:

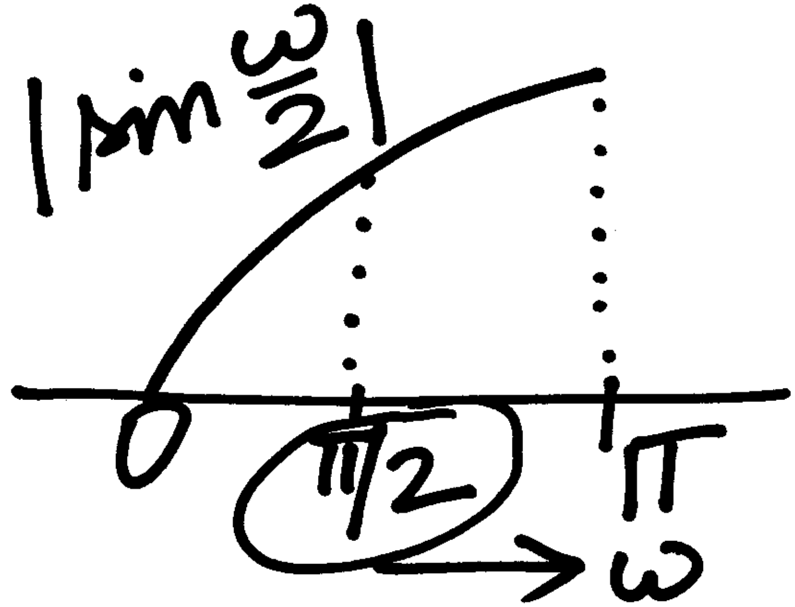


Actual Haar Analysis Filter Responses:

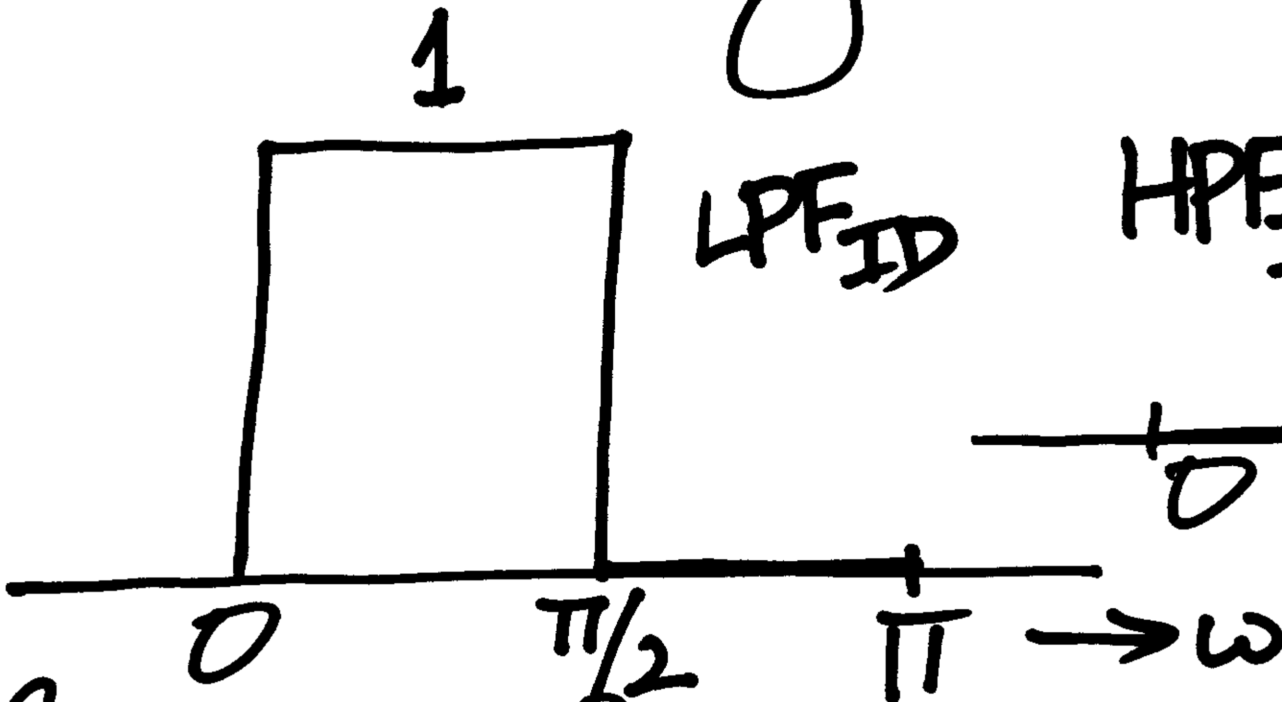
$(1+z^{-1})/2$



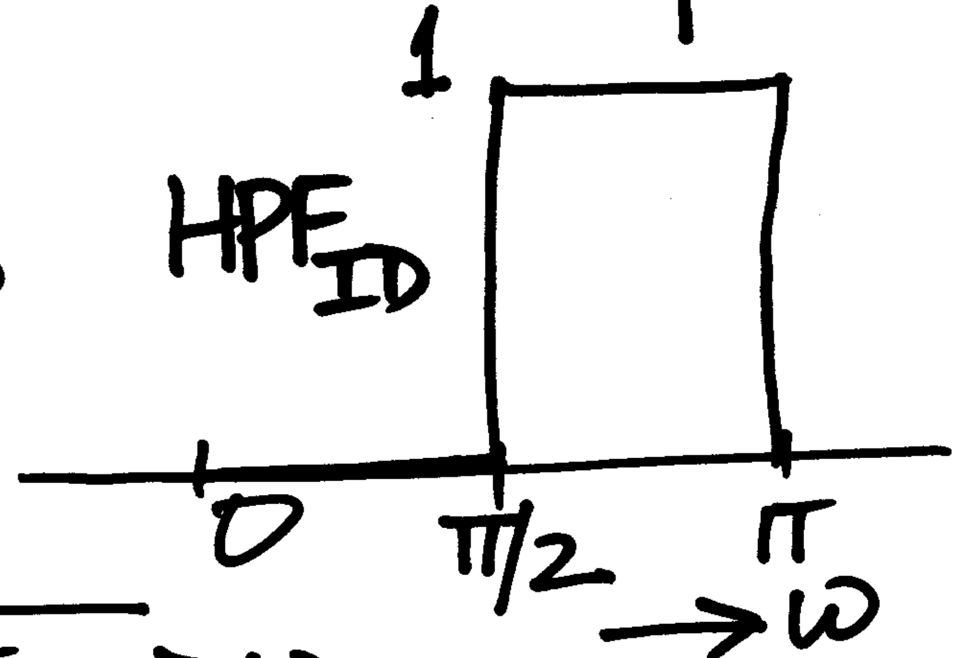
$(1-z^{-1})/2$



Ideal Analysis Filter Response:

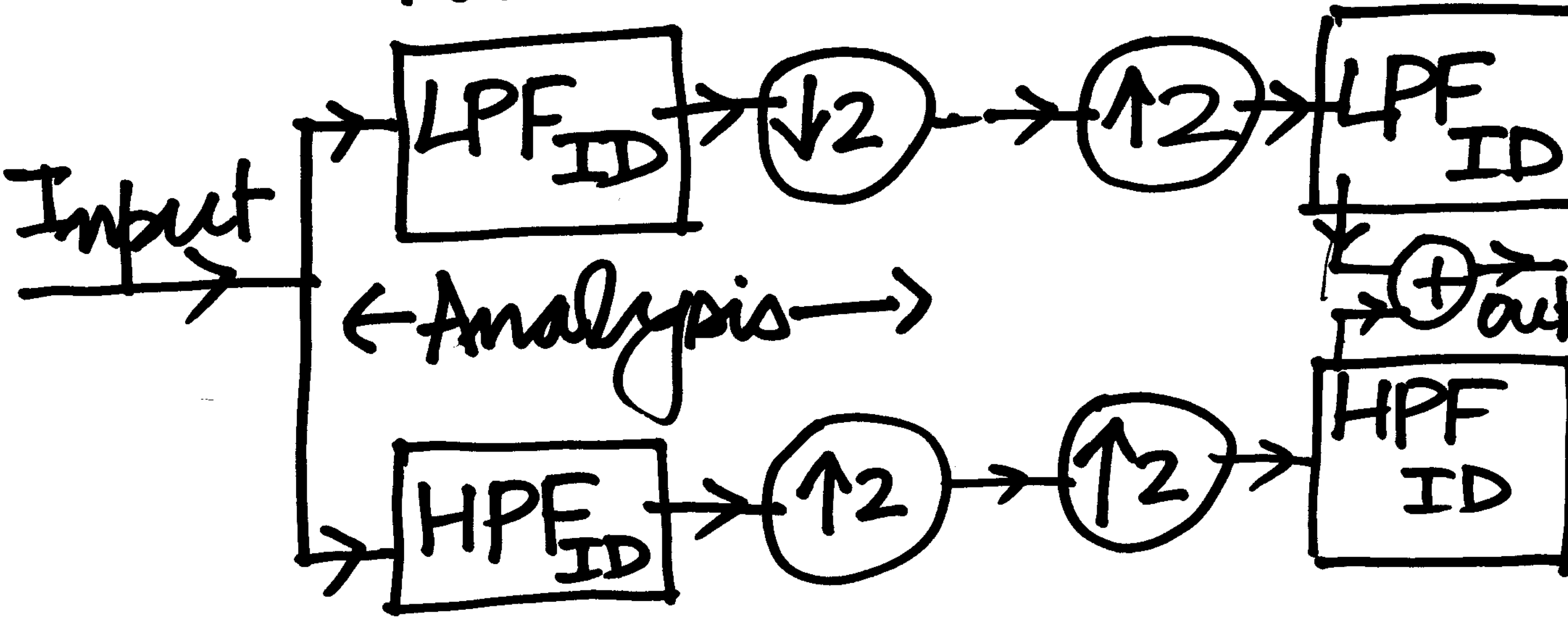


lowpass filter



Highpass filter

The Ideal Two Bank Filter Bank:



Why are the ideal filters unattainable?

1. The ideal filters are infinitely <u>noncausal</u>

Frequency response $\longrightarrow$ Impulse response

Inverse Discrete Time Fourier Transform

Ideal frequency response

$H_{ideal}(\omega)$

Impulse response = Inverse DTFT of $H_{ideal}(\omega)$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} H_{ideal}(\omega) \cdot e^{j\omega n} \, d\omega .$$

For the ideal lowpass filter this would be:

$$h[n] = \begin{cases} \dfrac{\sin \frac{\pi}{2} n}{\pi n}, & n \neq 0 \\ \frac{1}{2}, & n = 0. \end{cases}$$

Exercise:

Calculate the ideal impulse response of HPF_{ID}!

Disqualifications of the ideal filter:

1. Infinite noncausality!

Disqualification 2:
The ideal filter is unstable!

$$\sum_{n} |h[n]| \text{ divergent}$$

Disqualification 3:
The ideal filter
is IRRATIONAL !!

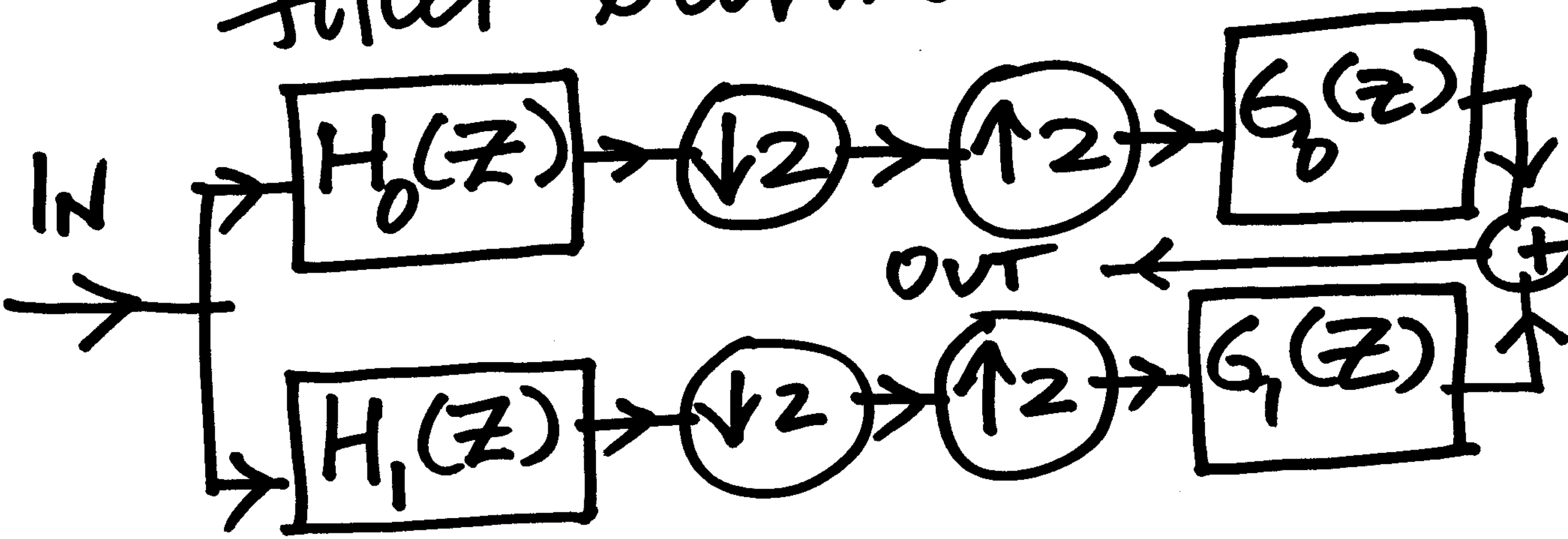
Example of an irrational system function

$e^{z^{-1}}$, $|z| > 0$ as ROC

$h[n] = \frac{1}{n!} u[n]$

$0! = 1$, $n! = n(n-1)!$, $\forall n \geq 1$

A realizable two-band filter bank:



provided

$H_0(z)$, $G_0(z)$,

$H_1(z)$, $G_1(z)$ are

all <u>rational</u> system functions!

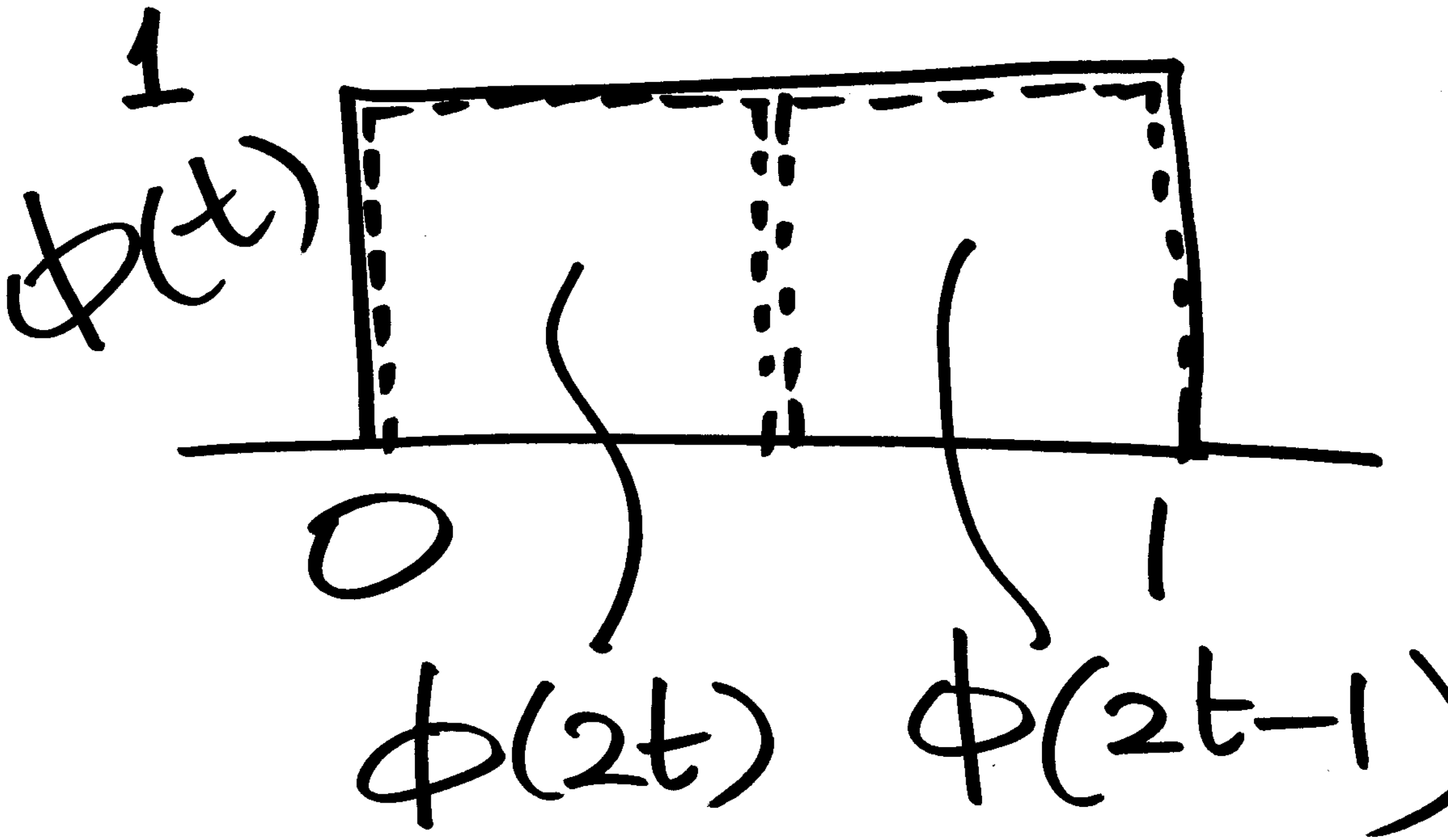
$H_0(z)$ and $G_0(z)$ aspire to be ideal lowpass discrete filter with cutoff $\frac{\pi}{2}$

$H_1(z)$ and $G_1(z)$ aspire to be highpass ideal filters with cutoff $\pi/2$!

Haar MRA:

$\phi(t) \in V_0 \subset V_1$

$\phi(t)$ should be expressible in terms of $\phi(2t-n)_{n\in\mathbb{Z}}$


1
ϕ(t)
0
1
ϕ(2t)
ϕ(2t-1)

$$\phi(t) = \phi(2t) + \phi(2t-1)$$

Dilation equation

Dilation equation
Coefficients for $\Phi(t)$

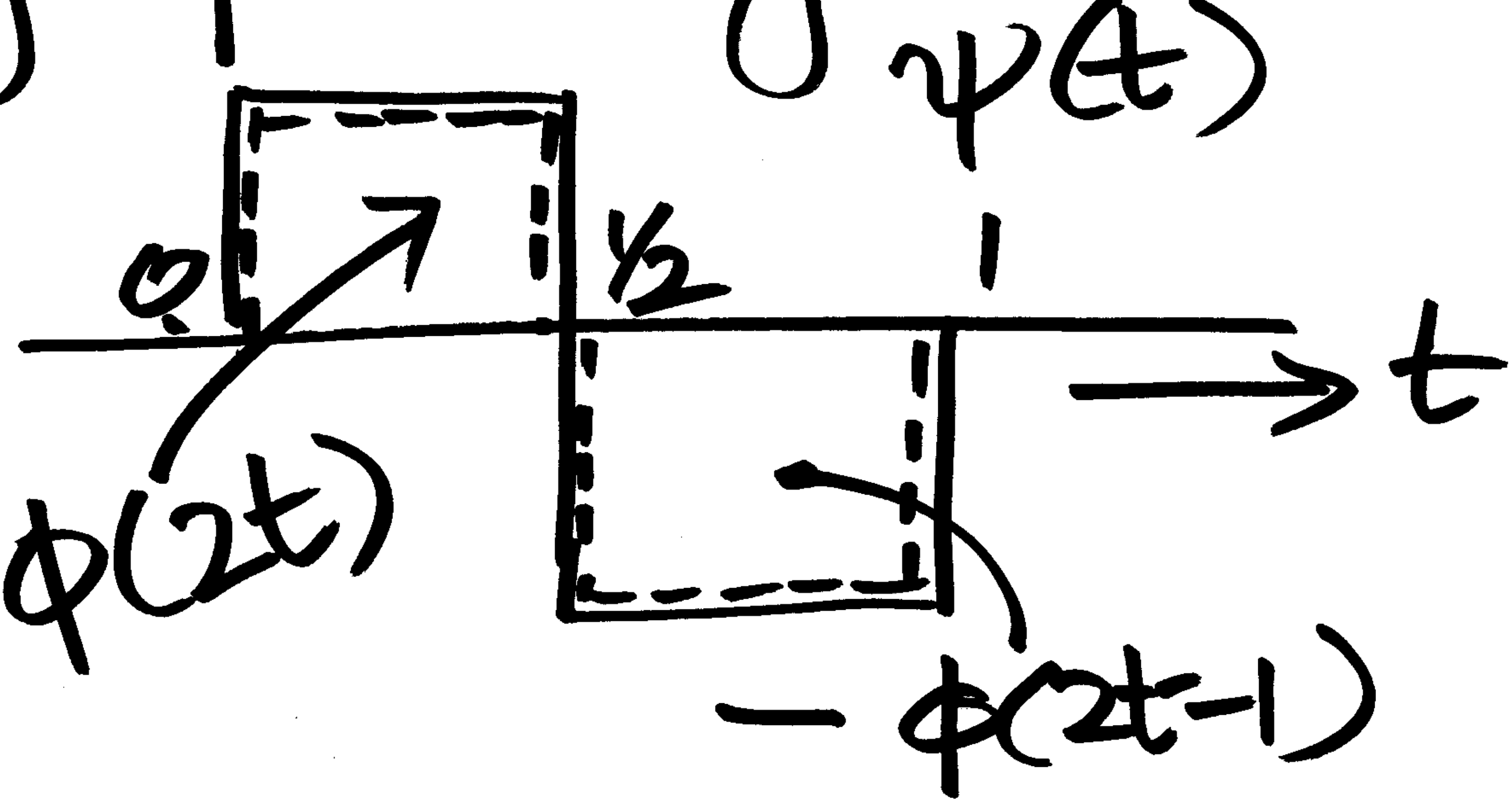
$1 \qquad 1$

$\uparrow_0$

$\psi(t)$ Haar wavelet

$\in V_1$ should be expressible in terms of basis $\{\phi(2t-n)\}_{n \in \mathbb{Z}}$

Graphically:



If $g[n]$ = impulse response of the highpass filter in the two band filter bank:

Coefficient sequence in this dilation equation for $\psi(t)$ =

$$\begin{array}{cc} 1 & -1 \\ \uparrow & \\ 0 & \end{array}$$

Dilation equation for $\psi(t)$:

$$\psi(t) = \phi(2t) - \phi(2t-1)$$

$h[n]$: impulse response of lowpass filter in two-band filter bank:

essentially: . . . .

$$\phi(t) = \sum_{n \in \mathbb{Z}} h[n]\,\phi(2t-n)$$

dilation equation for $\phi(\cdot)$

$$\psi(t) = \sum_{n \in \mathbb{Z}} g[n]\, \phi(2t-n)$$