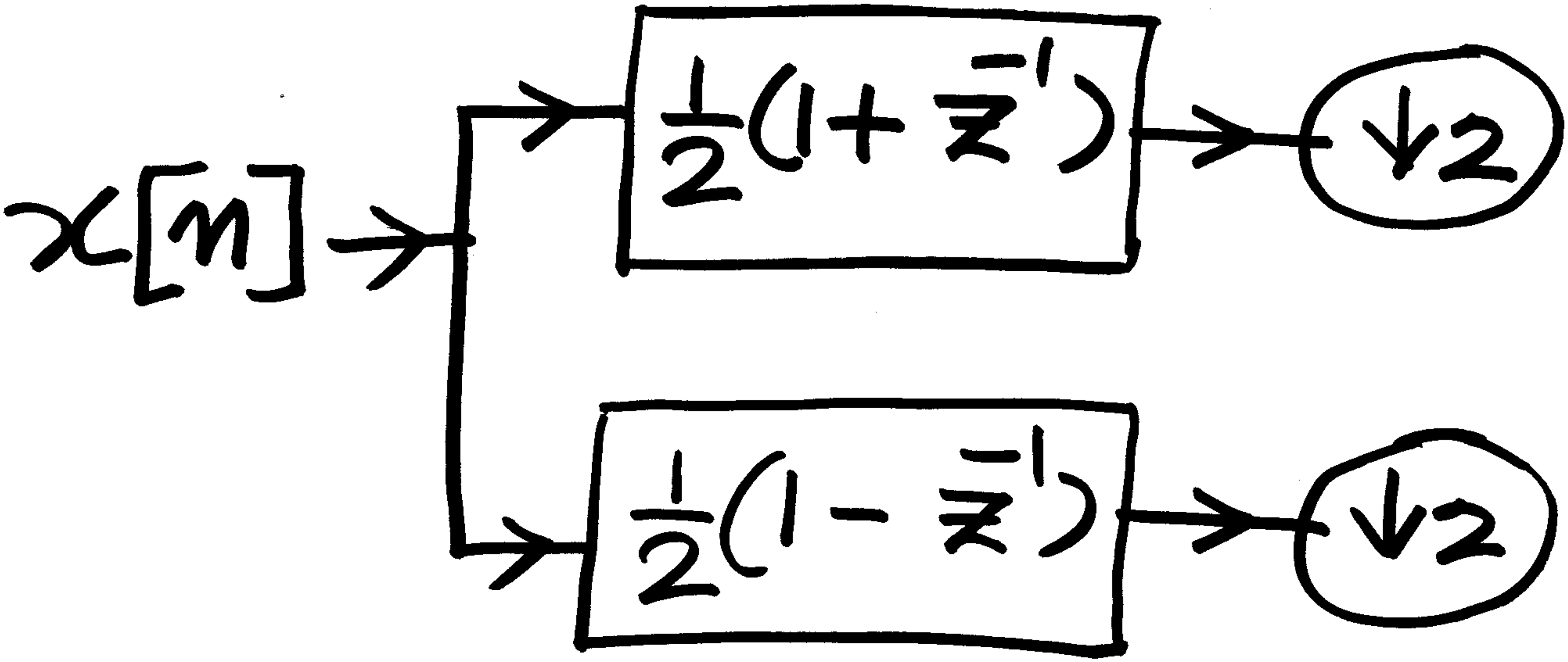
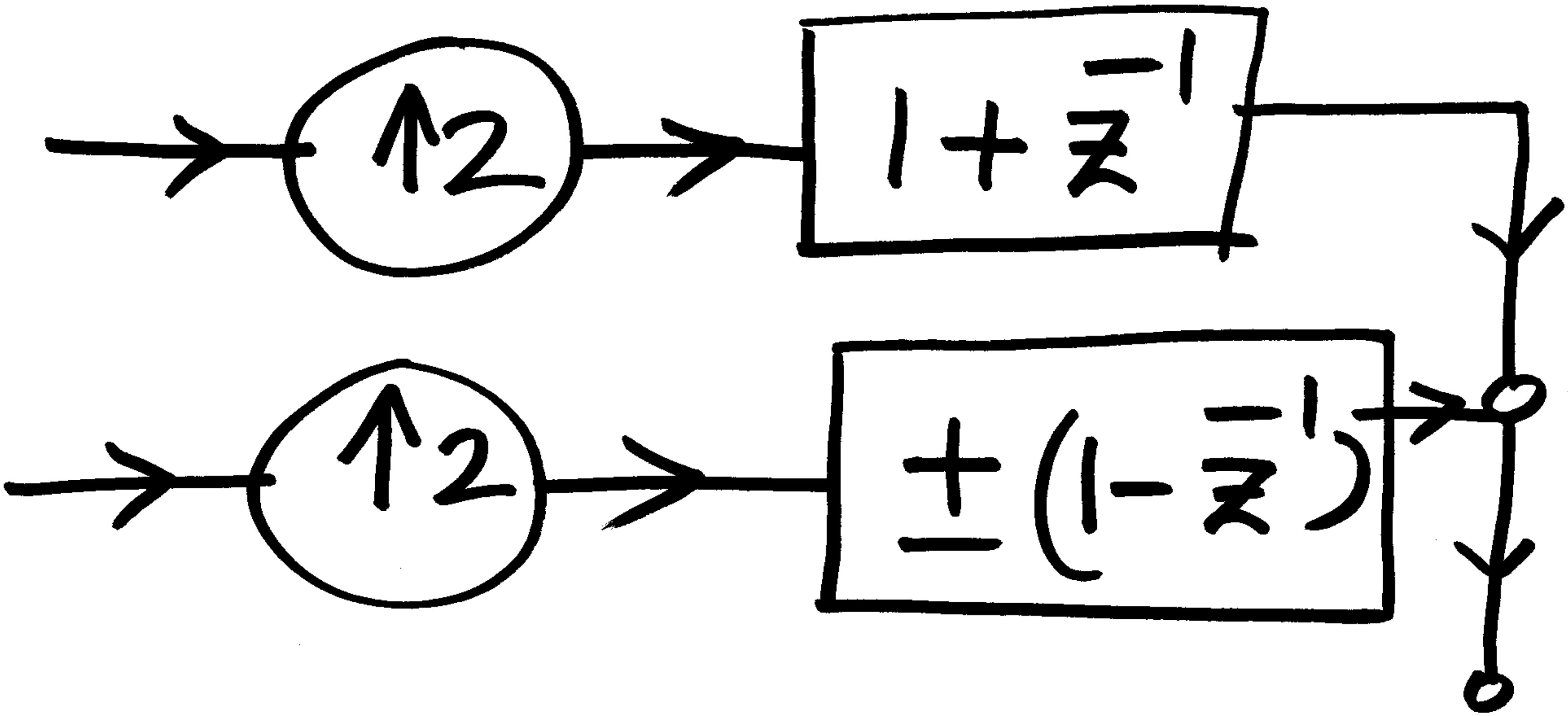
Date:- 18|1|10
Prof:- Gadre
Lec No. 7

LECTURE 7
HAAR FILTER BANK - ANALYSIS AND SYNTHESIS

HAAR ANALYSIS:



HAAR SYNTHESIS:



The frequency domain for the filter bank:

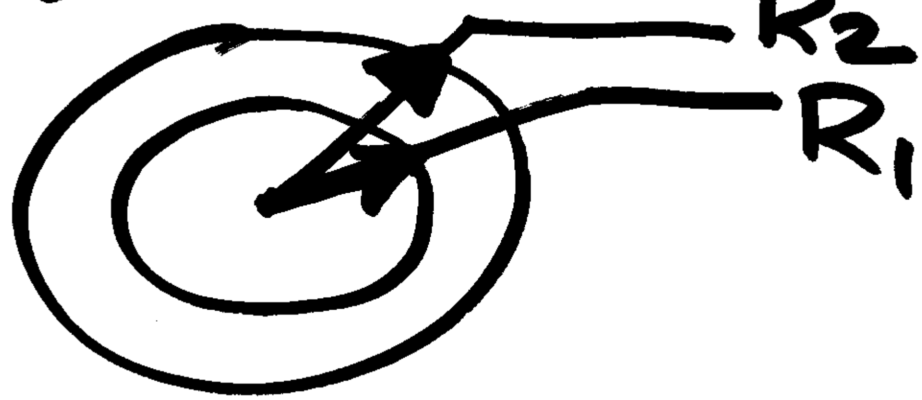
$$z = e^{j\omega}$$

$$x[n] \xrightarrow[\text{Z-transform}]{} $$

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n}$$

with $z \in$ Region of Convergence (R)

Typically R is between two circles centred at $z = 0$



Most generally

R_1 could be 0

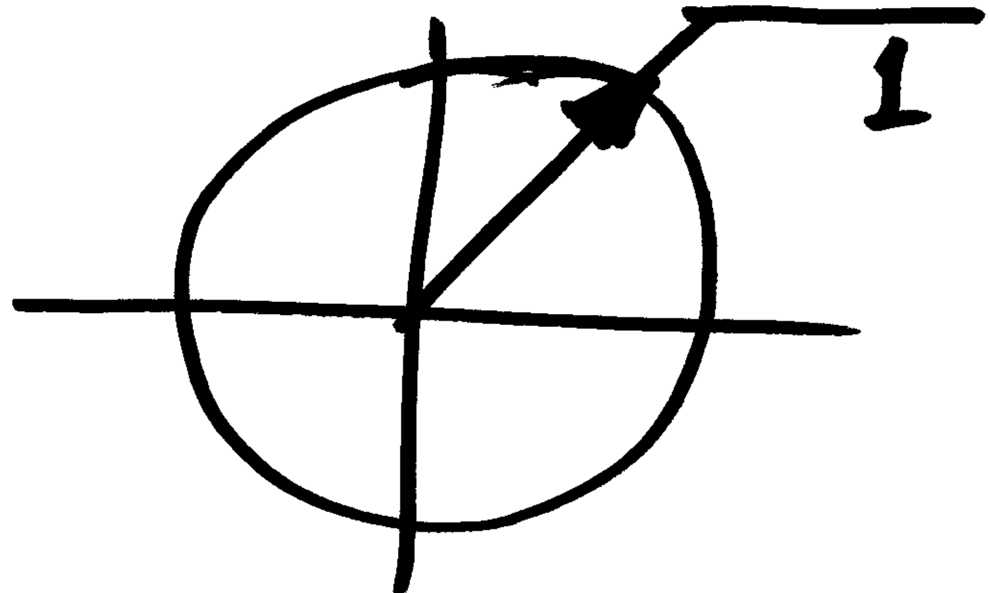
R_2 could be ∞

The boundary circles may be included/excluded

In particular if the unit circle

$|z| = 1$

$\in R$,



the system has a frequency response (if the underlying z-transform is a system function)

If we are talking about Z-transform of sequence, the sequence has a Discrete time Fourier Transform (DTFT)

We are well poised to look at the Haar filter bank with $|z| = 1$, $z = e^{j\omega}$

Analysis filters:

$$(1+z^{-1})/2$$

Frequency response: $z = e^{j\omega}$

$$= \left(1+e^{-j\omega}\right)/2$$

$$= \frac{e^{-j\frac{\omega}{2}}}{2}\underbrace{\left(e^{j\frac{\omega}{2}}+e^{-j\frac{\omega}{2}}\right)}_{2\cos\frac{\omega}{2}}$$

$$= \frac{1}{2} \cdot e^{-j\frac{\omega}{2}} \cdot 2 \cos\frac{\omega}{2}$$

$$= e^{-j\frac{\omega}{2}} \cdot \cos\frac{\omega}{2}$$

Magnitude $= \left|\cos\frac{\omega}{2}\right|$

$-\pi \leq \omega \leq \pi$

$\cos\frac{\omega}{2}$ real and ≥ 0

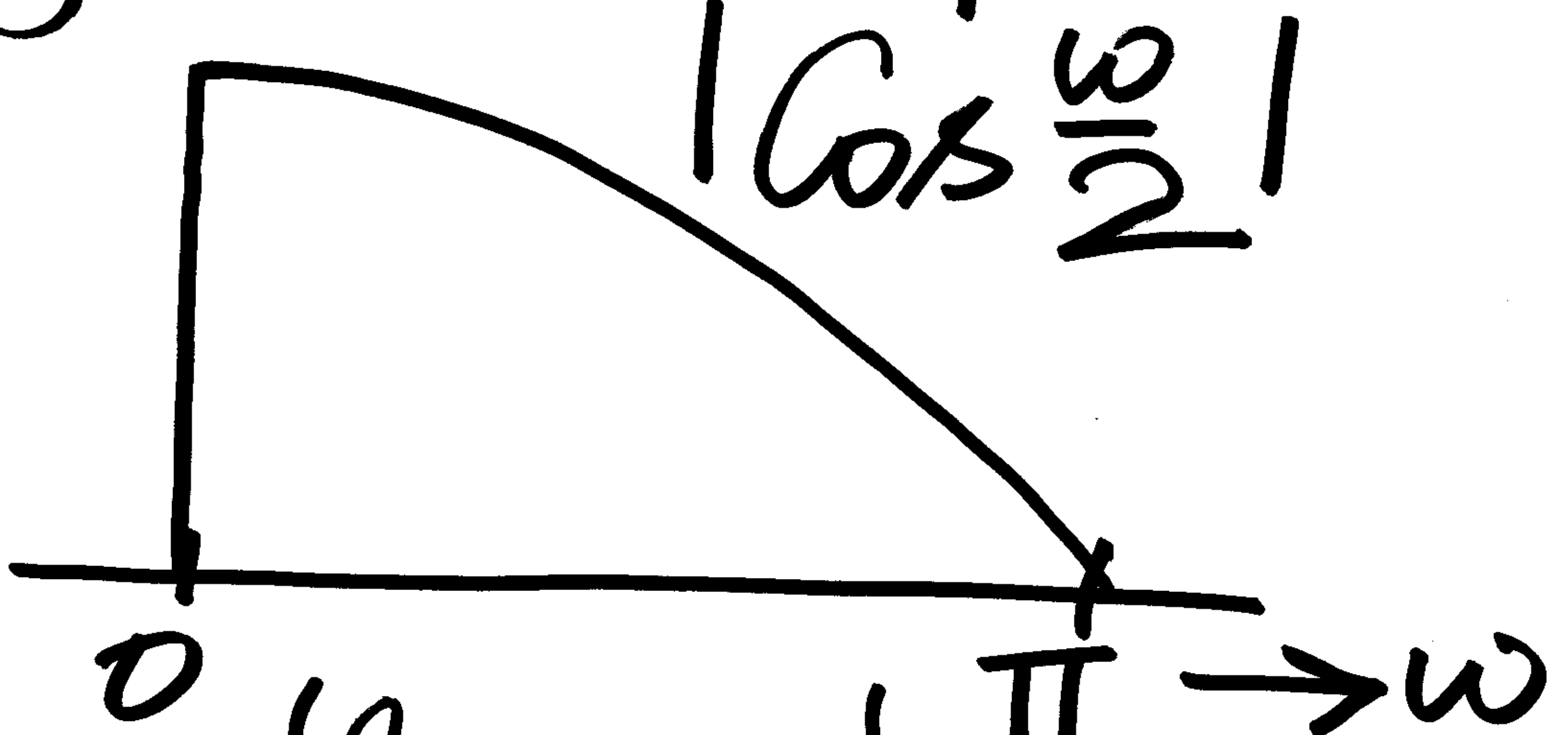
Phase contribution $= 0$
from it.

$$e^{-j\frac{\omega}{2}} \cdot \underbrace{\cos\frac{\omega}{2}}_{\text{no phase contribution}}$$

Phase contribution $= -\frac{\omega}{2}$

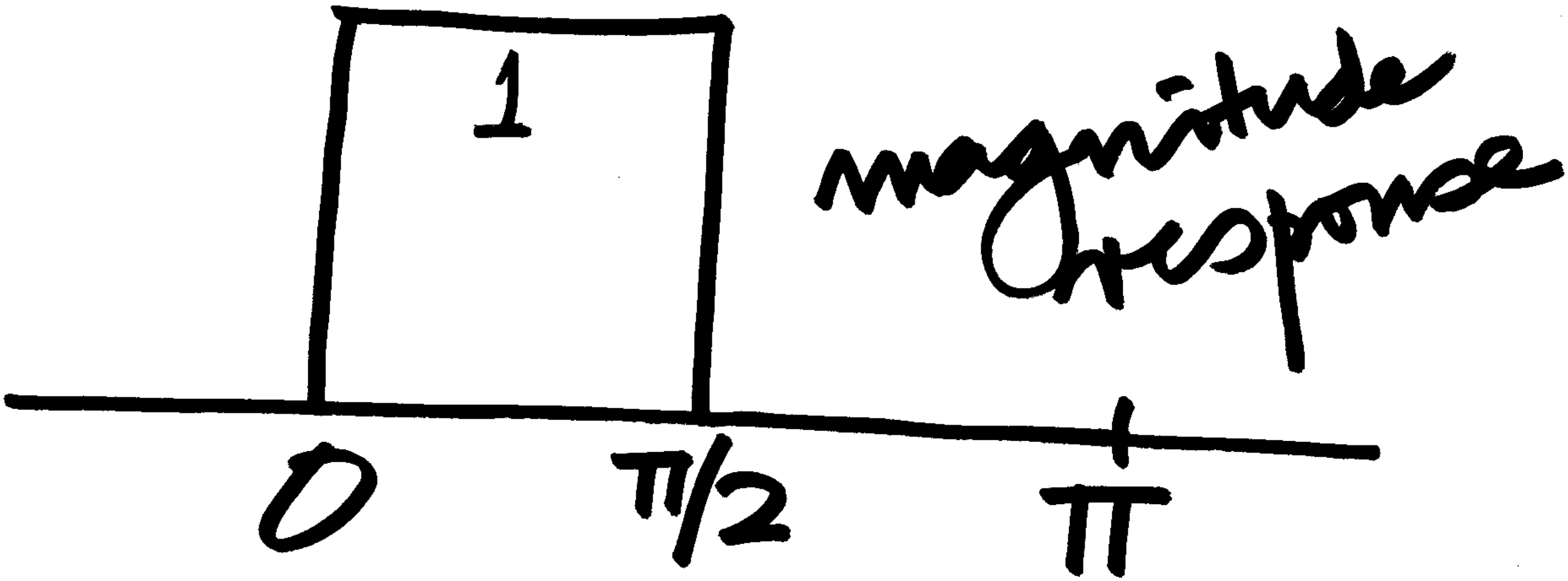
$-\pi \le \omega \le +\pi$

Magnitude response:

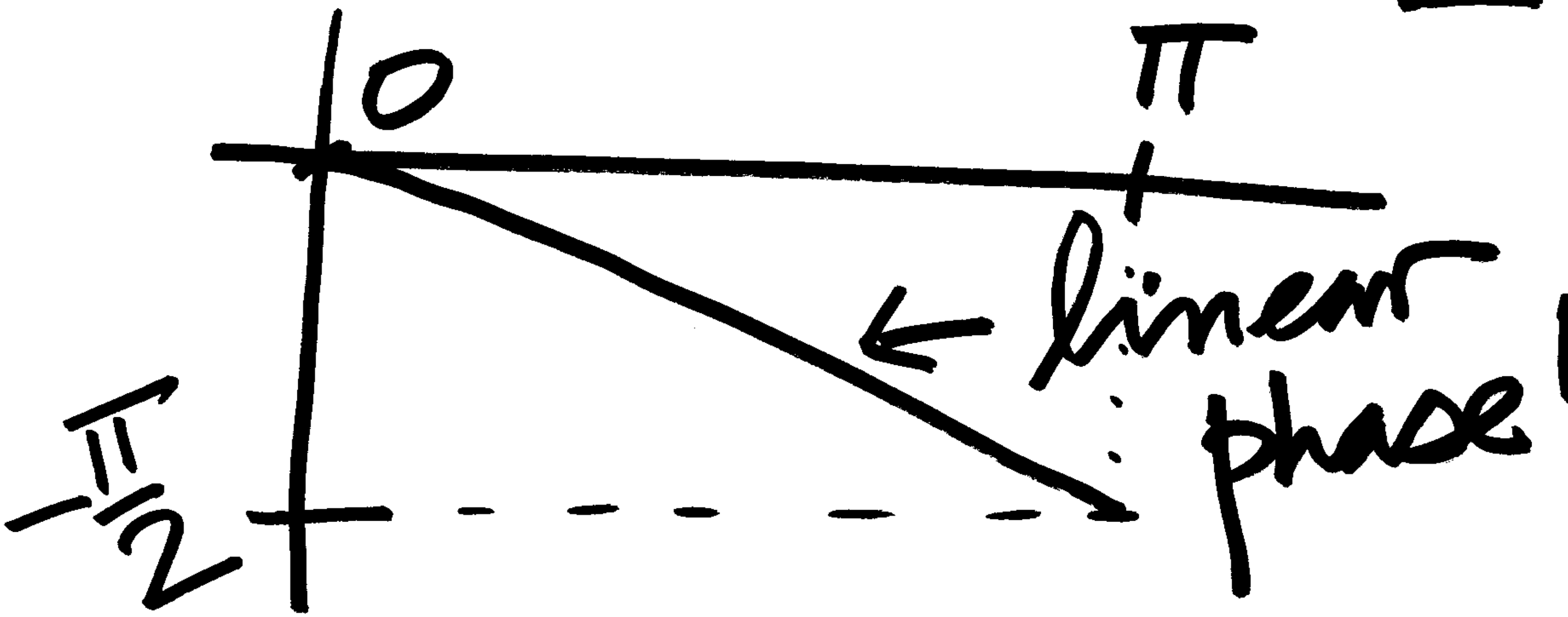


Crude 'lowpass' filter

Ideal lowpass discrete time filter, cutoff $\frac{\pi}{2}$



Phase response $= -\frac{\omega}{2}$



$$A_0 \cos(\Omega_0 t + \Phi_0) \longrightarrow \boxed{\text{Continuous time filter}} \longrightarrow ?$$

Frequency response $H(\Omega)$

$$|H(\Omega_0)| A_0 \cos\left\{\Omega_0 t + \phi_0 + \measuredangle H(\Omega_0)\right\}$$

$\measuredangle H(\Omega_0)$ essentially shifts sinusoid by - - -

$$\underbrace{|H(\Omega_0)|A_0}\cdots$$

$$\cdots \cos\left\{\phi_0+\Omega_0\left(t\right)+\frac{\measuredangle H(\Omega_0)}{\Omega_0}\right\}$$

$\Rightarrow t$ replaced by $t+\dfrac{\measuredangle H(\Omega_0)}{\Omega_0}$.

$\dfrac{\measuredangle H(\Omega_0)}{\Omega_0}$ is a time shift.

Phase response creates a time shift dependent on Ω_0.

Ideal: <u>no</u> $\measuredangle H(j\Omega)$!

Unachievable!
because of
causality

Linear phase means:

$$\frac{\measuredangle H(\Omega_0)}{\Omega_0} \text{ independent of } \Omega_0$$

$$= \tau_0$$

$$\Rightarrow \angle H(\Omega_0)$$

$$= \Omega_0 \tau_0$$

linear in Ω_0

"linear phase"

Second analysis filter

$$\frac{1}{2}\left(1 - z^{-1}\right)$$

Frequency domain

$$z = e^{j\omega}$$

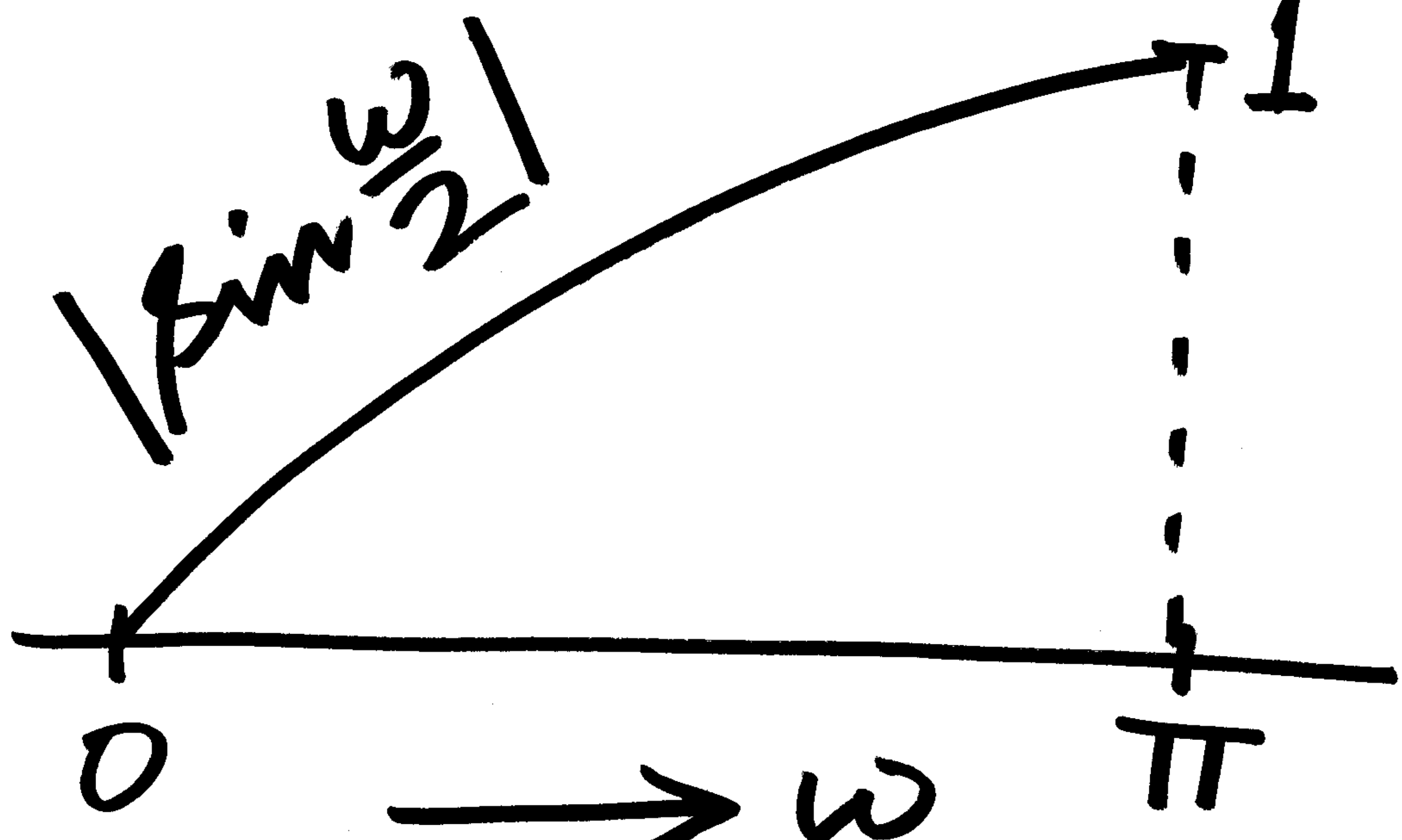
$$\frac{1}{2}\left(1-e^{-j\omega}\right)$$

$$= \frac{e^{-j\frac{\omega}{2}}}{2}\underbrace{\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)}_{2j\sin \omega/2}$$

$$= j \cdot e^{-j\frac{\omega}{2}} \sin\frac{\omega}{2}$$

Magnitude response $|\cdot|$ of this

$$= \left| \sin\frac{\omega}{2} \right|$$



Magnitude response

$\omega : 0 \longrightarrow \pi$

$$\underline{j}\,\underline{e^{-j\frac{\omega}{2}}}\,\sin\frac{\omega}{2}$$

$\sin\frac{\omega}{2} \geq 0$: no phase contribution

Phase response:

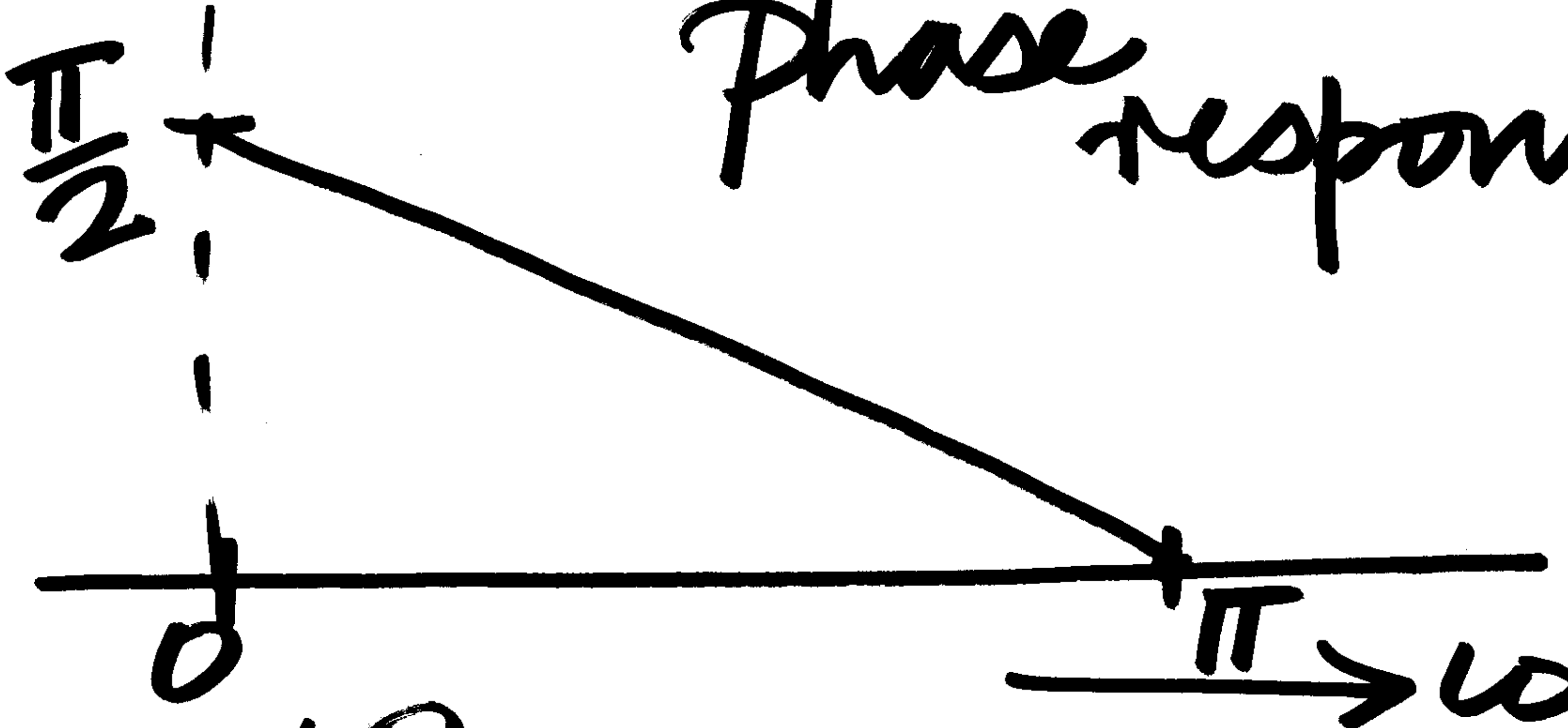
$0 \leq \omega \leq \pi$:

$$\frac{\pi}{2} - \frac{\omega}{2}$$

'j'

$$e^{-j\frac{\omega}{2}}$$

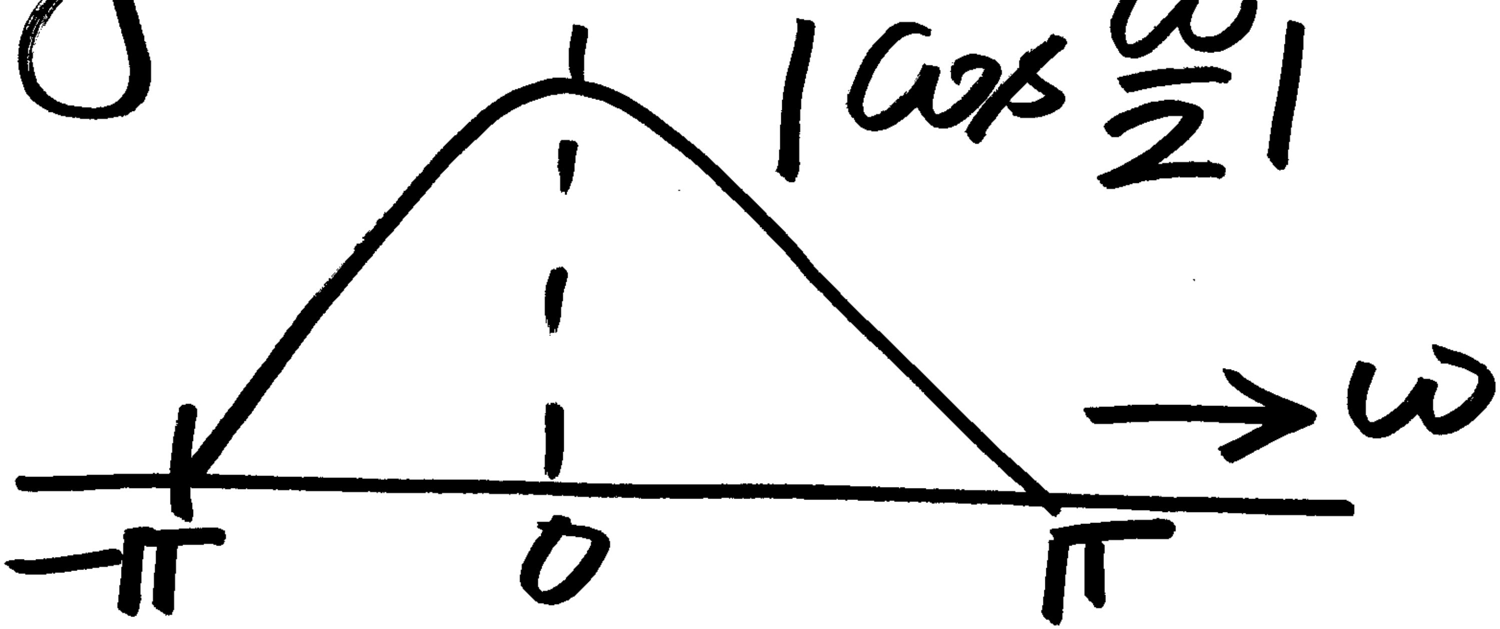
Phase response



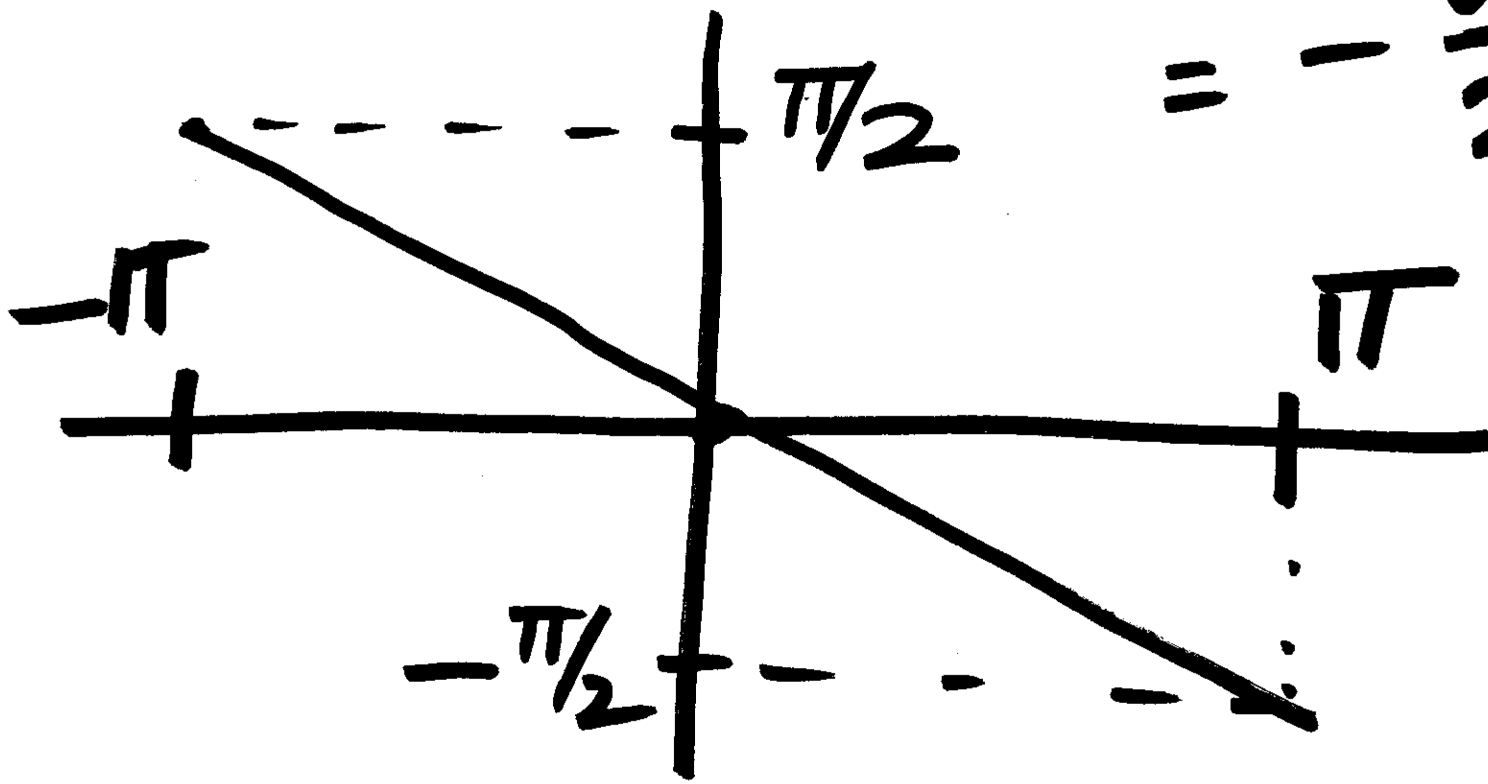
'pseudo' linear phase

$$\frac{1}{2}(1+z^{-1})$$

magnitude:



$\frac{1}{2}(1+z^{-1})$ Phase $= -\frac{\omega}{2}$



Sampled sinusoid ang freq ω $\rightarrow$ Discrete filter $\rightarrow$ Power emerges prop to $|H(\omega)|^2$

Frequency response $H(\omega)$ $\uparrow$ Discrete filter

Magnitude squared response:

$$\left|\cos\frac{\omega}{2}\right|^2 + \left|\sin\frac{\omega}{2}\right|^2 = 1 \; !$$

Frequency responses

$H_{upper}(\omega) \rightarrow$ upper branch

$H_{lower}(\omega) \rightarrow$ lower branch

$$H_{upper}(\omega) + H_{lower}(\omega) = 1$$

'MAGNITUDE COMPLEMENTARY'

$$|H_{upper}(\omega)|^2 + |H_{lower}(\omega)|^2 = 1$$

POWER COMPLEMENTARY!

If we consider the frequency responses together:

$$\frac{1}{2}(1+z^{-1}) + \frac{1}{2}(1-z^{-1})$$

$$= 1$$