## Lecture 49 – In search of scaling coefficients

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#### Framework

- Gave us power to move up or down the ladder
- We can now indeed zoom-in or zoomout of any part of the signal
- This makes the entire analysis 'scalable'!!
- Scalability stems out of multi-resolution framework !



#### Framework

- Leads us to two questions
- I) How do we go about selecting the mother wavelet and scale of analysis?
- 2) What is the procedure to calculate scaling and wavelet coefficients?



#### How Fourier Works!!











#### Application

- Detecting hidden jump discontinuity
- Consider function

$$g(t) = \begin{cases} t, 0 \le t < \frac{1}{2} \\ t - 1, \frac{1}{2} \le t < 1 \end{cases}$$

• Clear jump at *t*=0.5



#### Application

- Detecting hidden jump discontinuity
- Let's integrate

$$h(t) = \int g(t)dt = \begin{cases} \frac{t^2}{2}, 0 \le t < \frac{1}{2} \\ \frac{t^2}{2} - t + \frac{1}{2}, \frac{1}{2} \le t < 1 \end{cases}$$

• Cusp jump at *t*=0.5



#### Application

- Detecting hidden jump discontinuity
- Let's integrate again

$$f(t) = \int h(t)dt = \begin{cases} \frac{t^3}{6}, & 0 \le t < \frac{1}{2} \\ \frac{t^3}{6} - \frac{t^2}{2} + \frac{t}{2} - \frac{1}{8}, & \frac{1}{2} \le t < 1 \end{cases}$$

• Appears smooth to eye



#### Coefficients

- Who gives us coefficients of scaling equation?
- Haar

$$\psi(t) = \begin{cases} 1, & 0 \le x < \frac{1}{2} \\ -1, & \frac{1}{2} \le x < 1 \\ 0, & \text{otherwise} \end{cases}$$
$$\phi(t) = \begin{cases} 1, & 0 \le x < 1 \\ 0, & \text{otherwise} \end{cases}$$



• We can thinks of using three guiding theorems !

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- Theorem I:

For the scaling equation  $\phi(x) = \sum_{k} h_k \sqrt{2}\phi(2x-k)$ , with non-vanishing coefficients  $\{h_k\}_{k=N}^M$  only for  $N \le k \le M$ , its  $\phi(x)$  is with a compact support contained in interval [N, M]

- We can thinks of using three guiding theorems !
- Theorem 2:

If the scaling function  $\phi(x)$  has compact support on  $0 \le x \le N - 1$  and if,  $\{\phi(x - k)\}$  are linearly independent, then  $h_n = h(n) = 0$ , for n<0 and n > N - 1. Hence N is the length of the sequence.

- We can thinks of using three guiding theorems !
- Theorem 3:

If the scaling coefficieents  $\{h_k\}$ satisfy the condition for existence and orthogonality of  $\phi(x)$ , then

$$\varphi(x) = \sum_{k} g_k \sqrt{2} \phi(2x - k)$$

where,  $g_k = \pm (-1)^k h_{N-k}$ 

and,  $\int_{-\infty}^{\infty} \varphi(x-l)\phi(x-k)dx = \delta_{l,k} = 0, l \neq k$ 

# Properties of scaling coefficients 1. $\sum h_k = \sqrt{2}$ 2. $\sum h_{2k} = \frac{1}{\sqrt{2}}$ 3. $\sum h_{2k+1} = \frac{1}{\sqrt{2}}$

#### Properties of scaling coefficients

4. 
$$\sum |h_k|^2 = 1$$

5. 
$$\sum h_{k-2l}h_k = \delta_{l,0}$$

6. 
$$\sum 2h_{k-2l}h_{k-2j} = \delta_{l,j}$$



#### Thank You! Questions ??