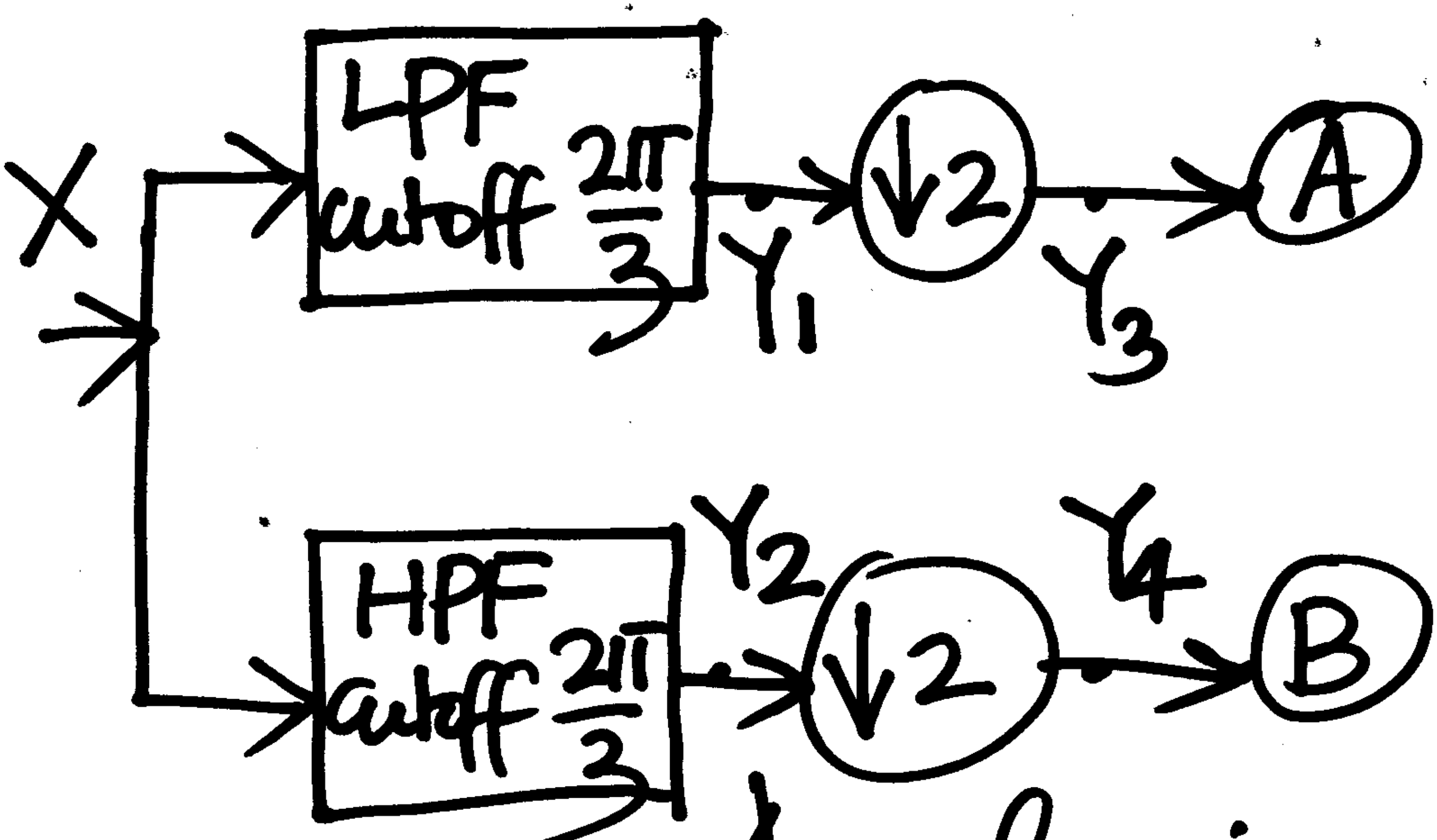
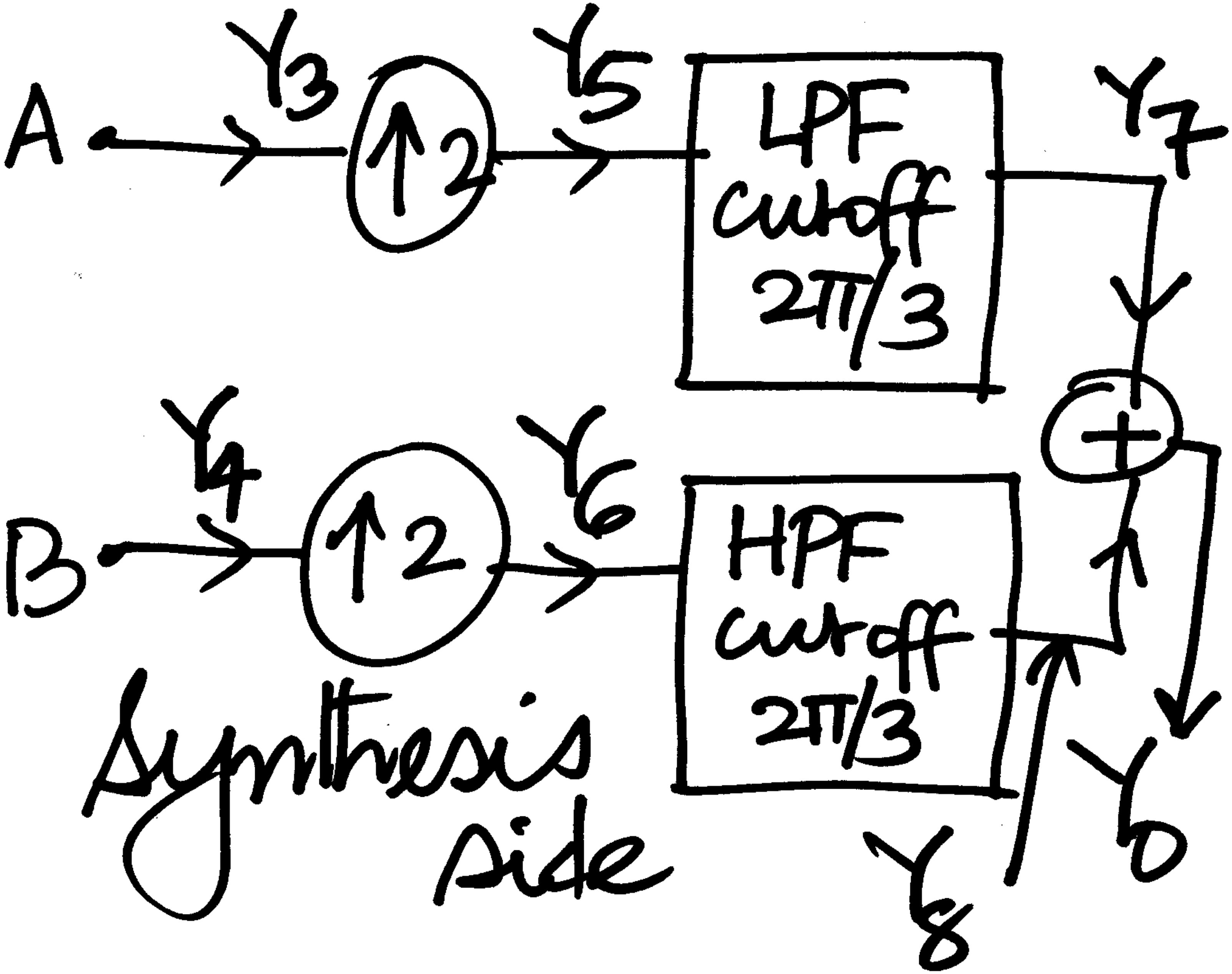
Prof. V.M. Gadre
Lec. 45
Date - 10-2-11

SESSION 45

TUTORIAL - FREQUENCY DOMAIN ANALYSIS OF TWO BAND FILTER BANK



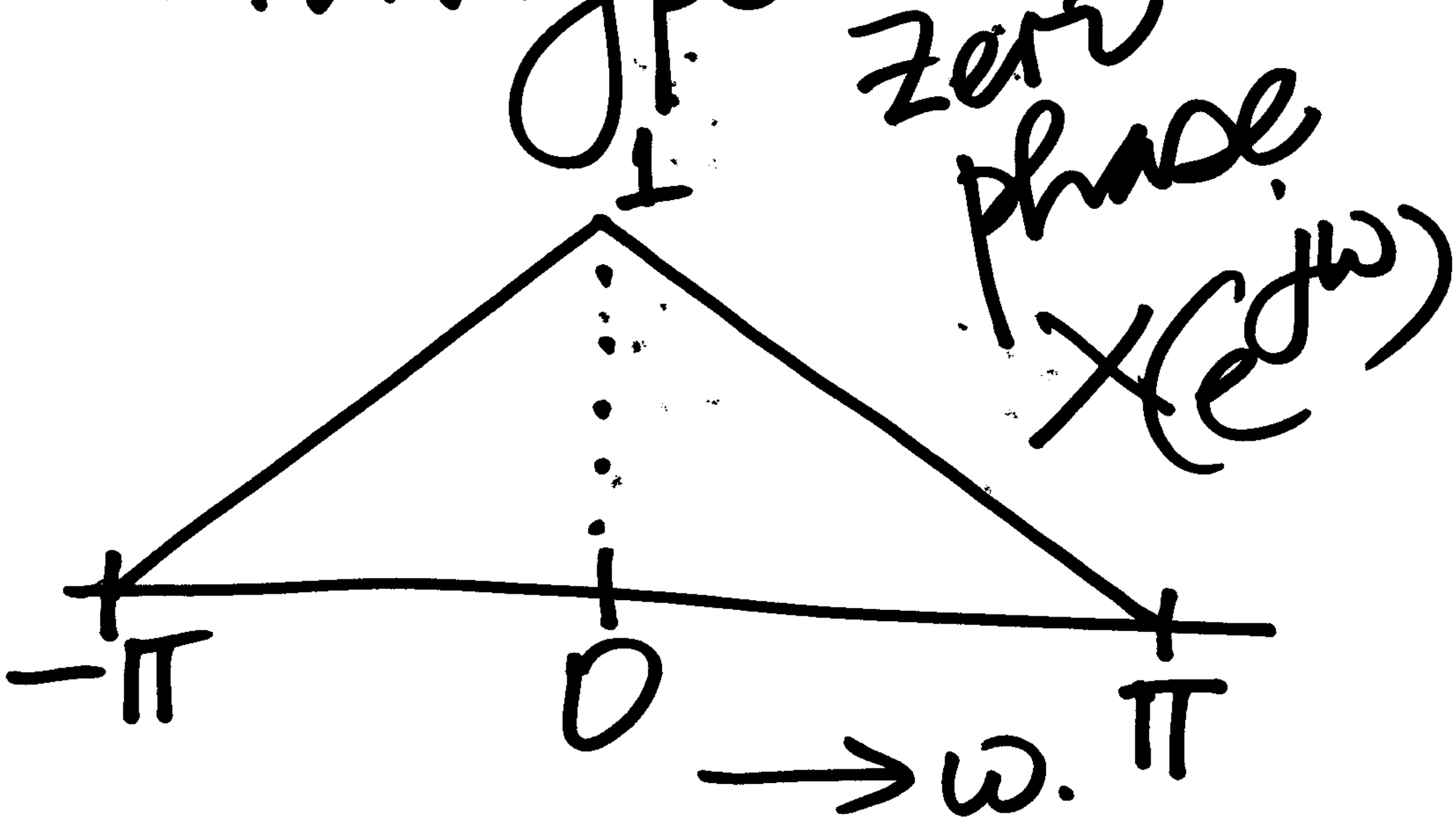
Analysis Side


A
B
Y3
Y4
↑2
↑2
Y5
Y6
LPF cutoff 2π/3
HPF cutoff 2π/3
Y7
Y8
+
Y0
Synthesis side

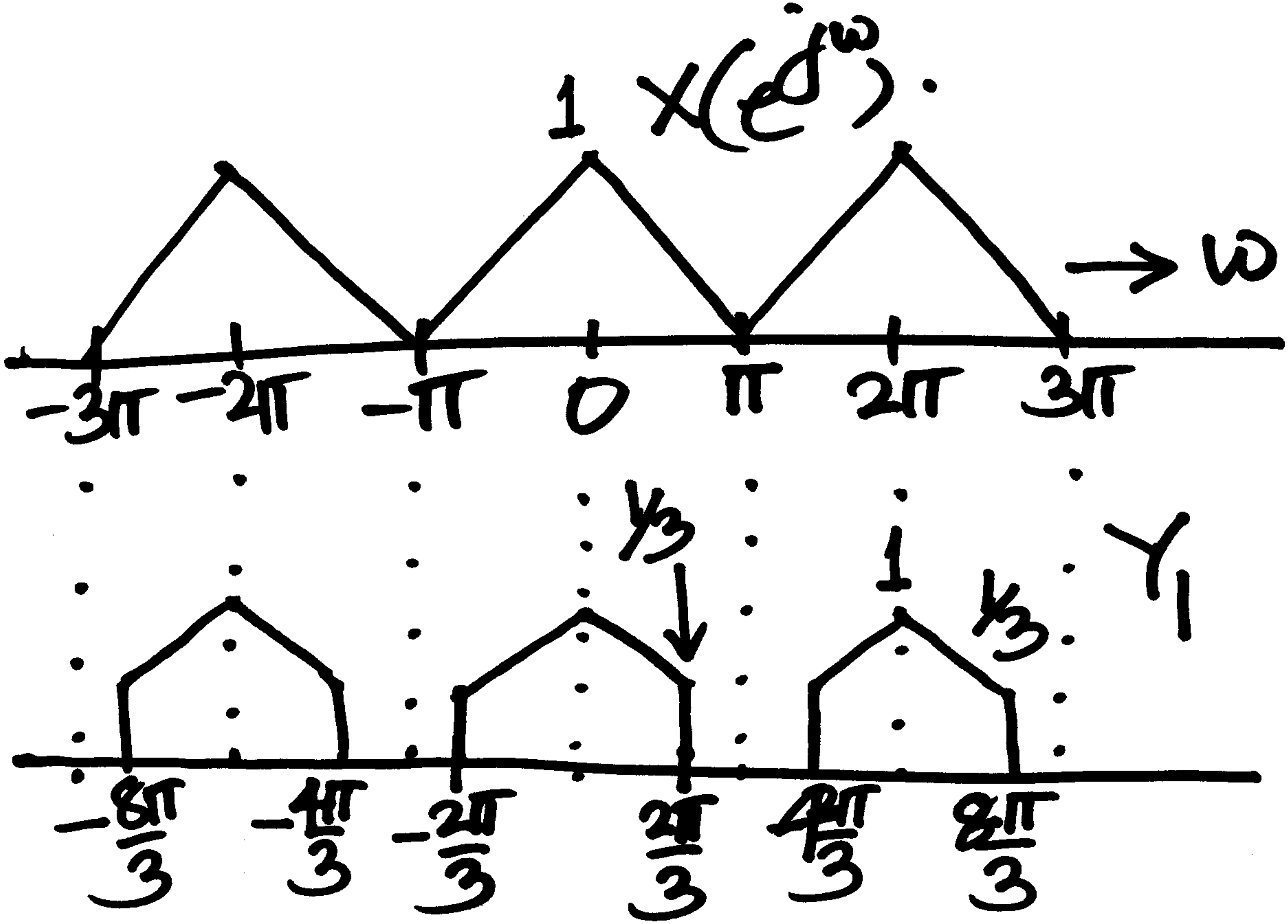
Sinusoidal frequency domain

Discrete Time Fourier Transforms (DTFTs)

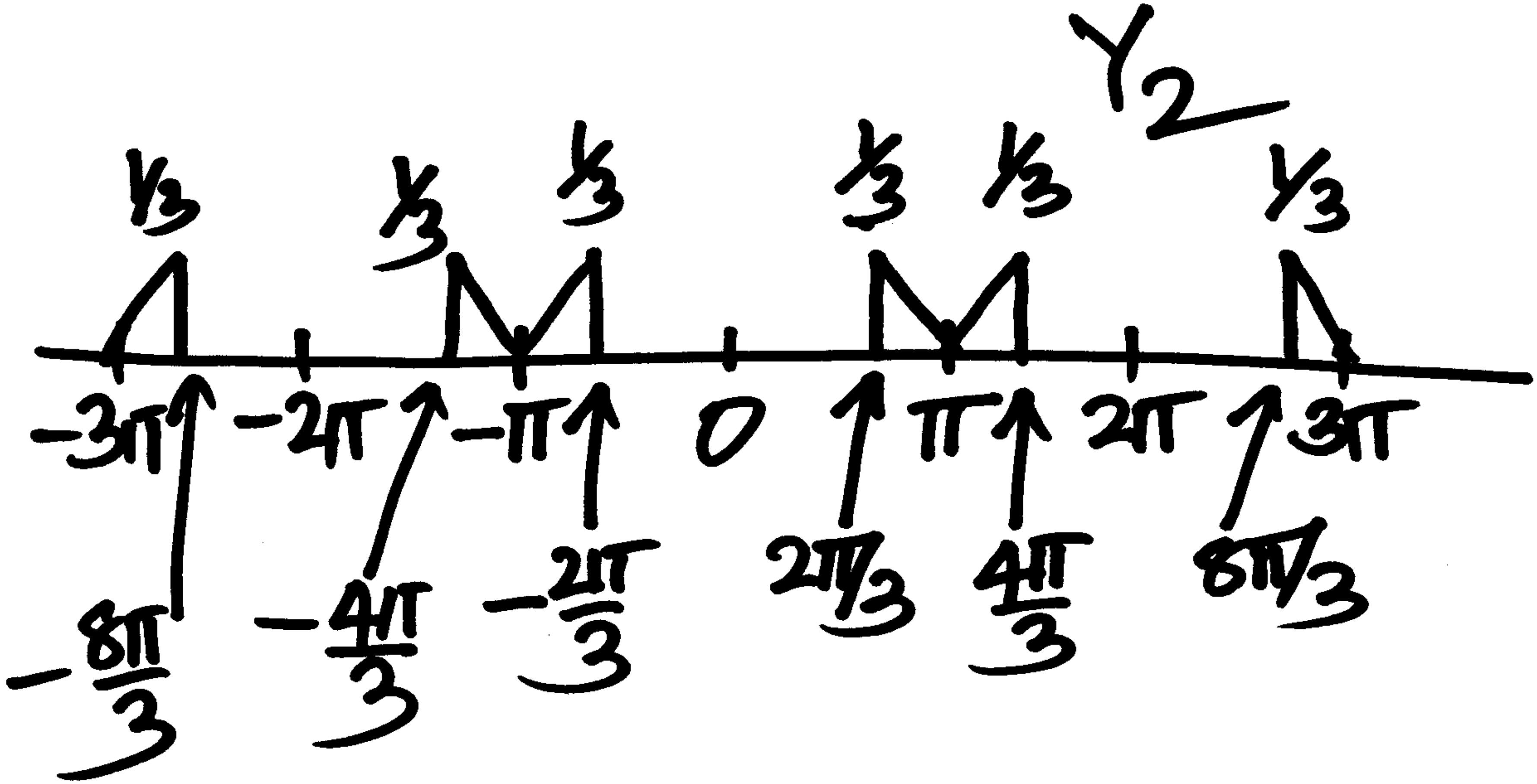
"Prototype"



In general, it is always a good idea to consider 3 successive periods of 2π: $[-3\pi, -\pi]$, $[-\pi, +\pi]$, $[+\pi, +3\pi]$



1
$X(e^{j\omega})$
$\rightarrow \omega$
-3π -2π $-\pi$ 0 π 2π 3π
$1/3$
1
$1/3$
Y_1
$-\frac{8\pi}{3}$ $-\frac{4\pi}{3}$ $-\frac{2\pi}{3}$ $\frac{2\pi}{3}$ $\frac{4\pi}{3}$ $\frac{8\pi}{3}$


Y2
1/3 1/3 1/3 1/3 1/3 1/3
−3π −2π −π 0 π 2π 3π
−8π/3 −4π/3 −2π/3 2π/3 4π/3 8π/3

Z domain

$$g_{in}[n] \rightarrow \downarrow 2 \rightarrow \uparrow 2 \rightarrow g_{out}[n]$$

$G_{in}(Z)$ $\qquad$ $G_{out}(Z)$

$$G_{out}(z) = \frac{1}{2}\left\{ G_{in}(z) + G_{in}(-z) \right\}$$

$$z \leftarrow e^{j\omega}$$

$$G_{out}(e^{j\omega}) = \frac{1}{2}\left\{ G_{in}(e^{j\omega}) + G_{in}(e^{j(\omega \pm \pi)}) \right\}$$

$G_{out}(z) =$

$\frac{1}{2}$ original DTFT

$+\frac{1}{2}$ aliased DTFT

$(z \leftarrow -z)$

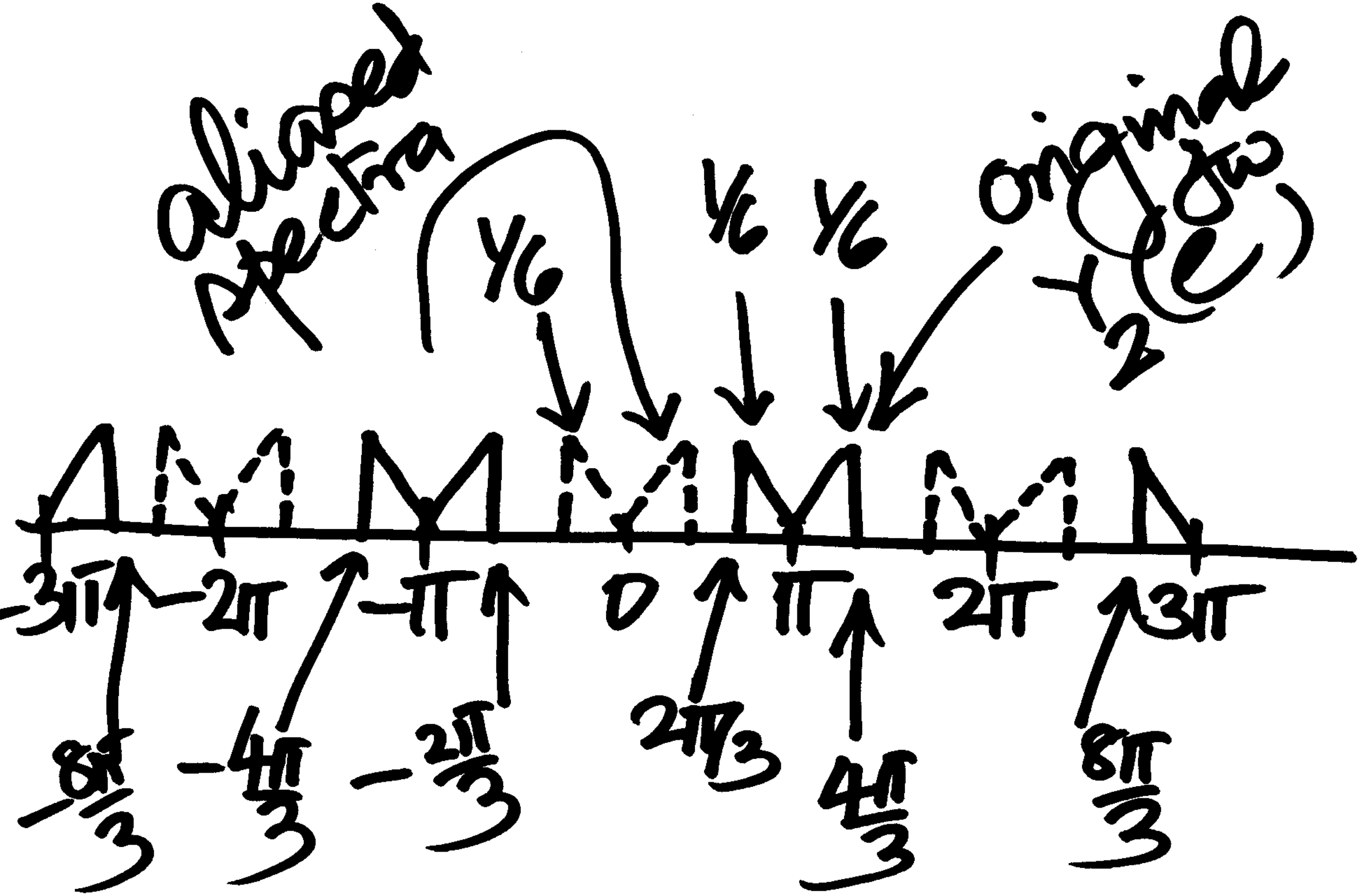
$$e^{j\omega} \longleftarrow - e^{j\omega}$$

$$= e^{\pm j\pi} e^{j\omega}$$

$$= e^{j(\omega \pm \pi)}$$

$$Y_2 \rightarrow \downarrow 2 \rightarrow \uparrow 2 \rightarrow Y_6$$

$$Y_6(e^{j\omega}) = \frac{1}{2}\left\{ Y_2(e^{j\omega}) + Y_2(e^{j(\omega \pm \pi)}) \right\}$$


aliased spectra
1/6
1/6
1/6
1/6
original $Y(e^{j\omega})$
1/2
-3π
-2π
$-\pi$
0
π
2π
3π
$-\frac{8\pi}{3}$
$-\frac{4\pi}{3}$
$-\frac{2\pi}{3}$
$2\pi/3$
$\frac{4\pi}{3}$
$\frac{8\pi}{3}$

HPF with cutoff $2\pi/3$

retains $\frac{1}{3} Y\left(e^{j\omega}\right)$

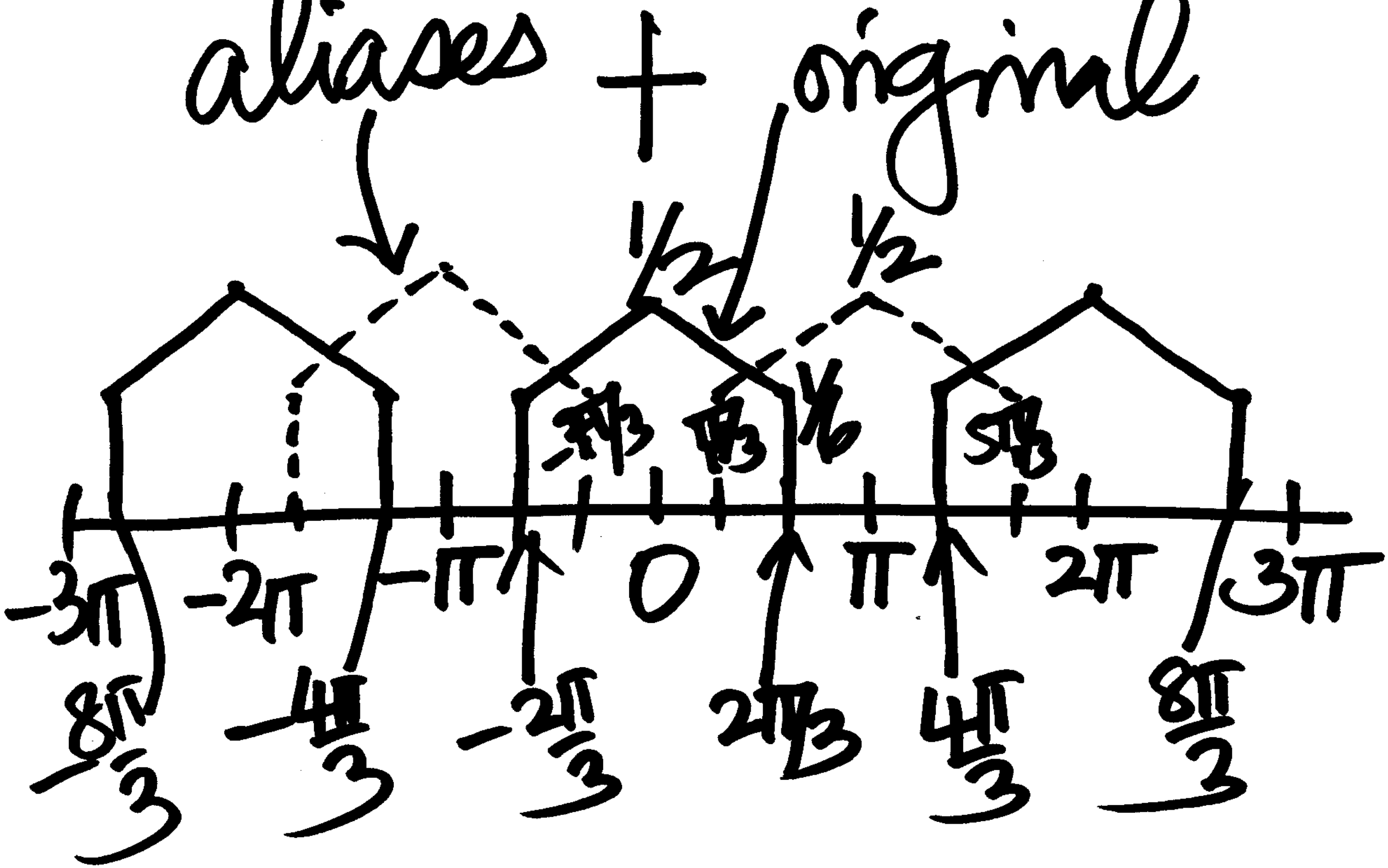
and destroys $\frac{1}{3} Y\left(e^{j(\omega \pm 2\pi)}\right)$.

(aliases).

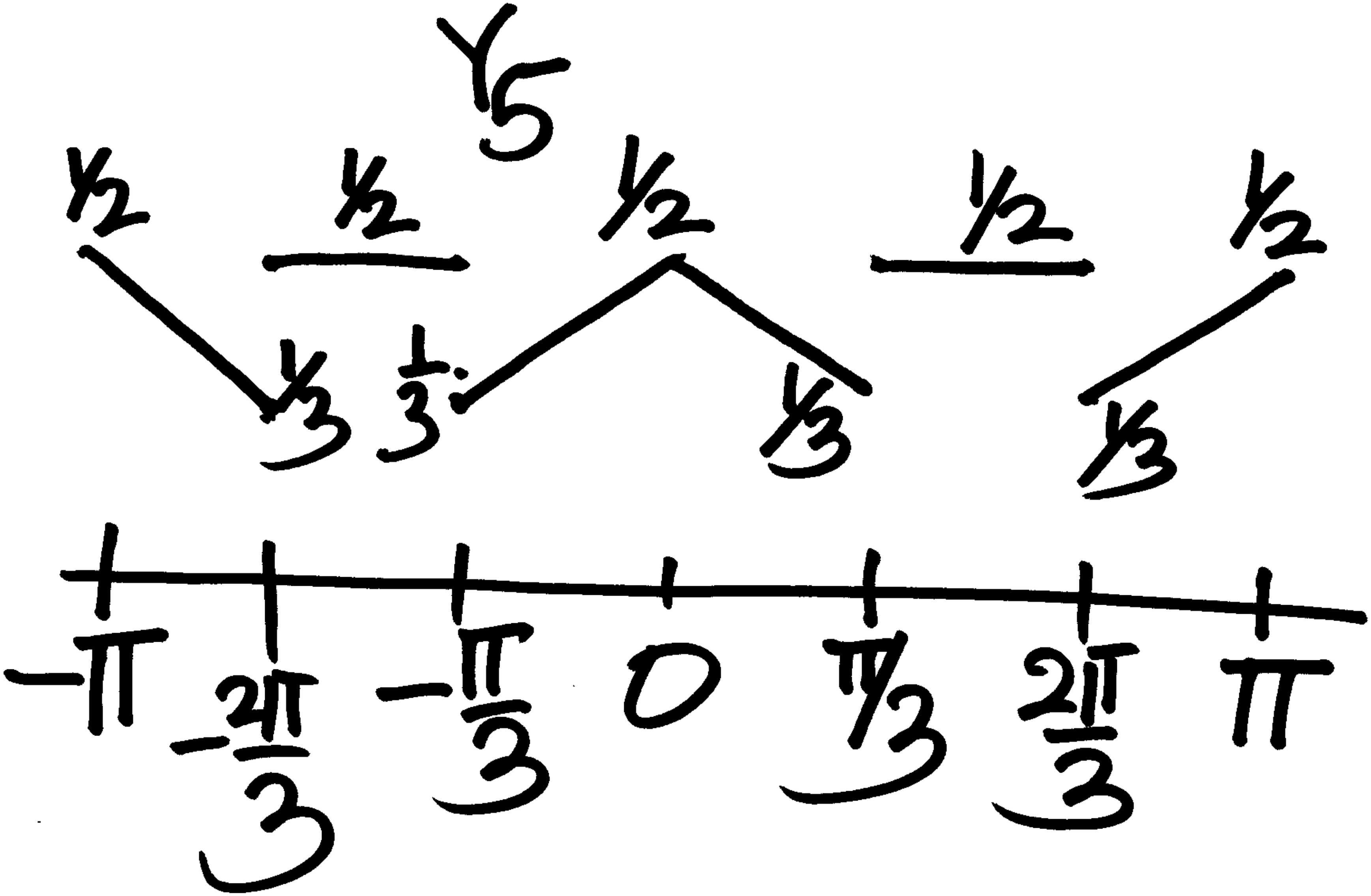
$$Y_1 \longrightarrow \boxed{\downarrow 2} \longrightarrow \boxed{\uparrow 2} \longrightarrow Y_5$$

$$Y_5(e^{j\omega}) =$$

$$\underset{\text{original}}{\tfrac{1}{2} Y_1(e^{j\omega})} + \underset{\text{alias.}}{\tfrac{1}{2} Y_1(e^{j(\omega \pm \pi)})}$$

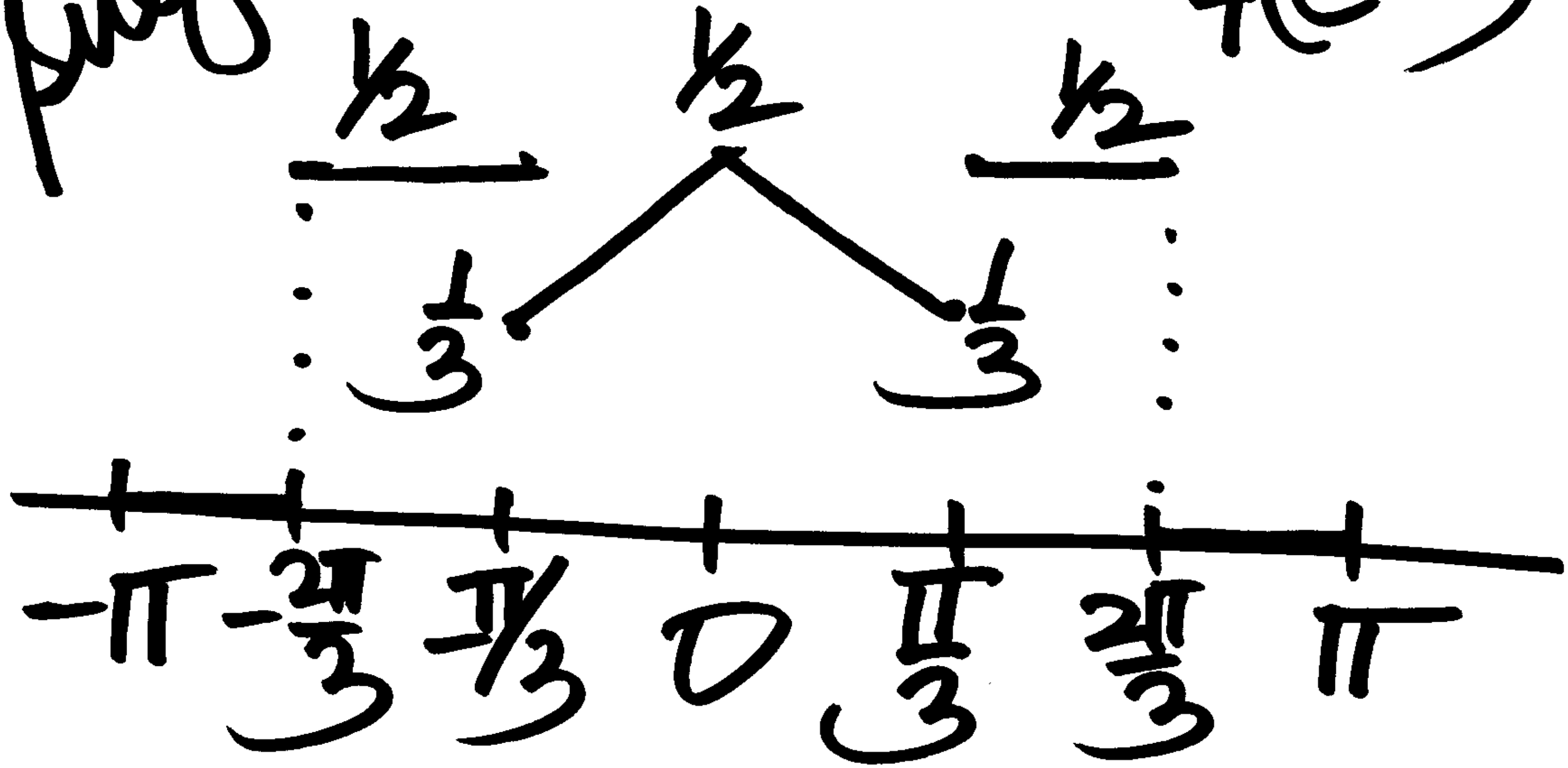


aliases + original
1/2
1/2
π/3
1/6
5π/3
-3π
-2π
-π
0
π
2π
3π
-8π/3
-4π/3
-2π/3
2π/3
4π/3
8π/3

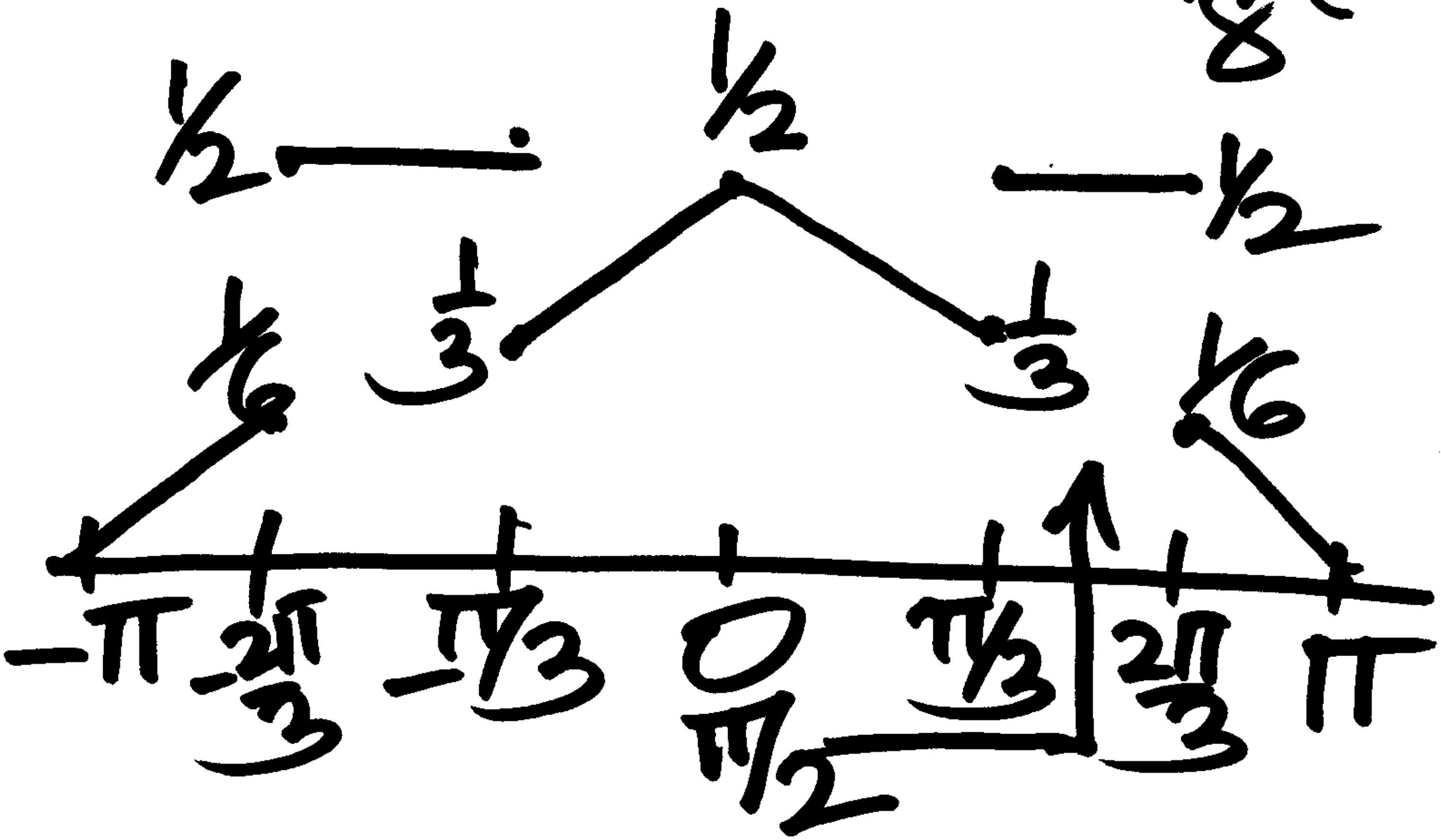

Y5
1/2
1/2
1/2
1/2
1/2
1/3
1/3
1/3
1/3
−π
−2π/3
−π/3
0
π/3
2π/3
π

Subjecting to LPF

$Y_7(e^{j\omega})$



$$Y_6(e^{j\omega}) = Y_7(e^{j\omega}) + Y_8(e^{j\omega})$$



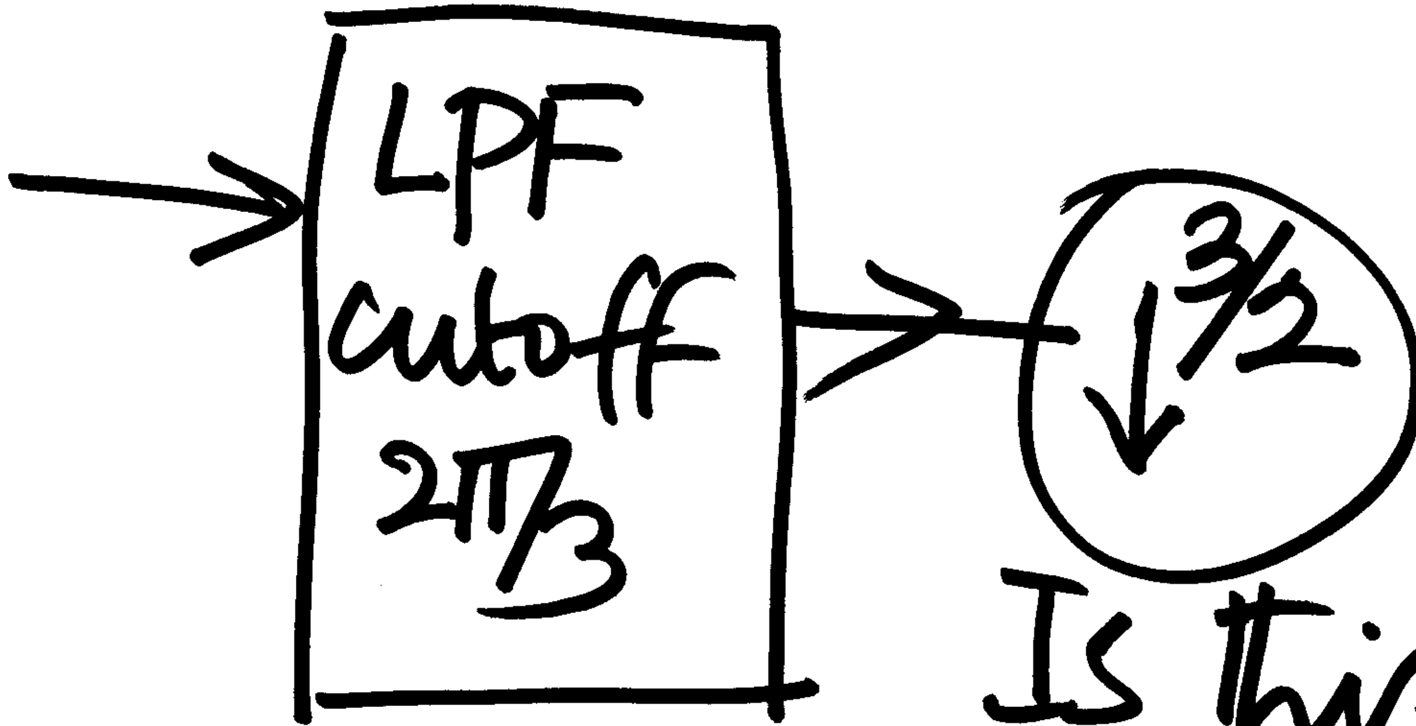
Aliasing has occured in a band of extent $\frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}$ on <u>either side</u> of $\frac{\pi}{2}$!

Perhaps we could interprete $\downarrow 3/2$

as:

$$\longrightarrow \downarrow 3 \longrightarrow \uparrow 2 \longrightarrow$$

?



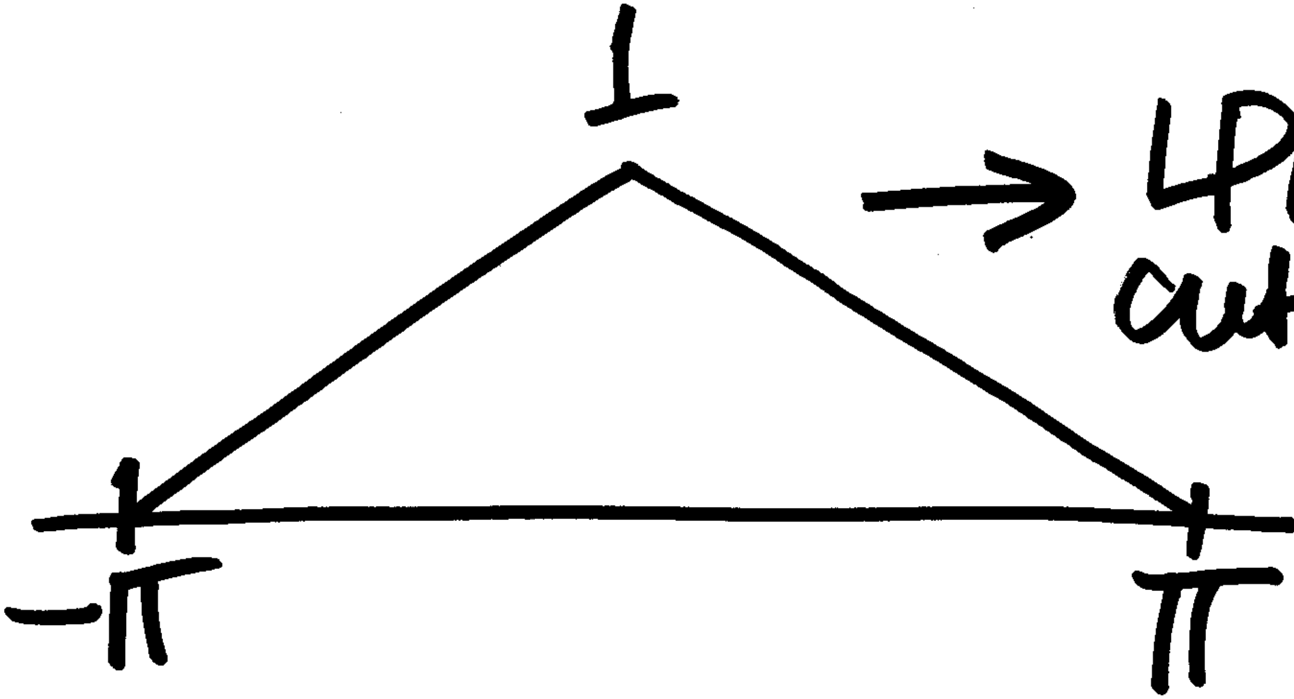
Is this possible?

Perhaps it may be better to do

$$\longrightarrow \uparrow 2 \longrightarrow \downarrow 3 \longrightarrow$$

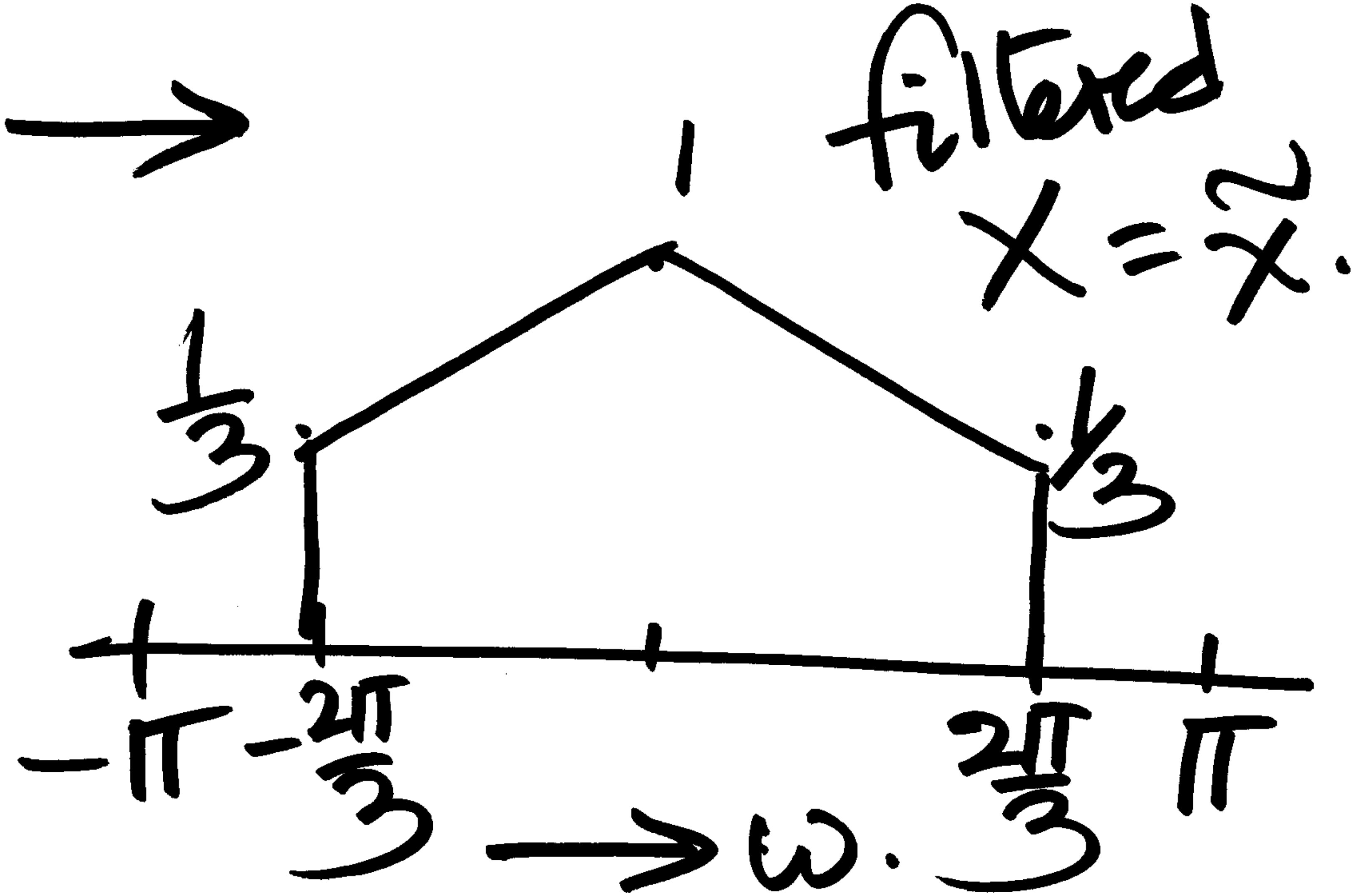
no 'loss' of information

incur 'loss' only in the end!

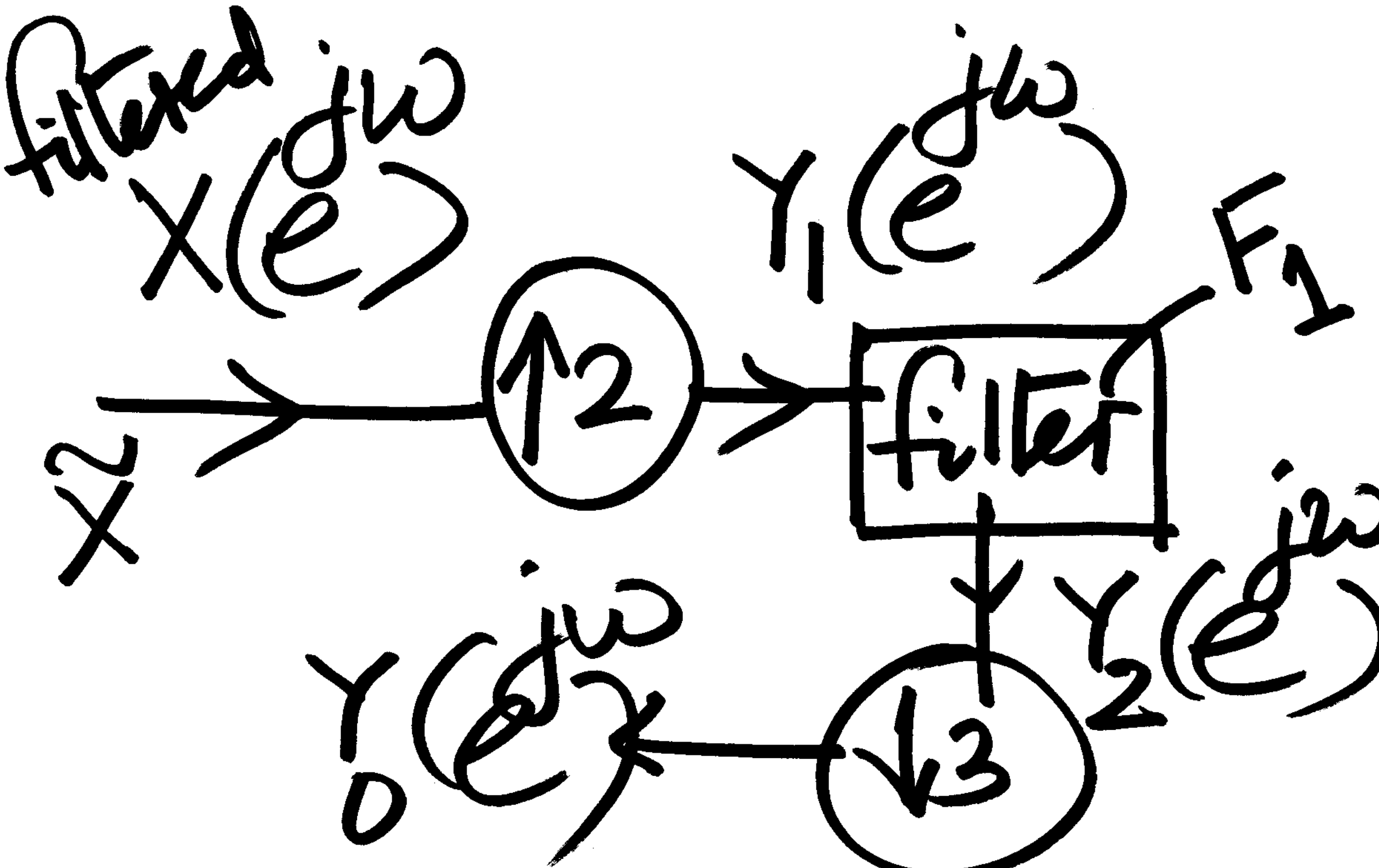


$\rightarrow$ LPF cutoff $\frac{2\pi}{3}$

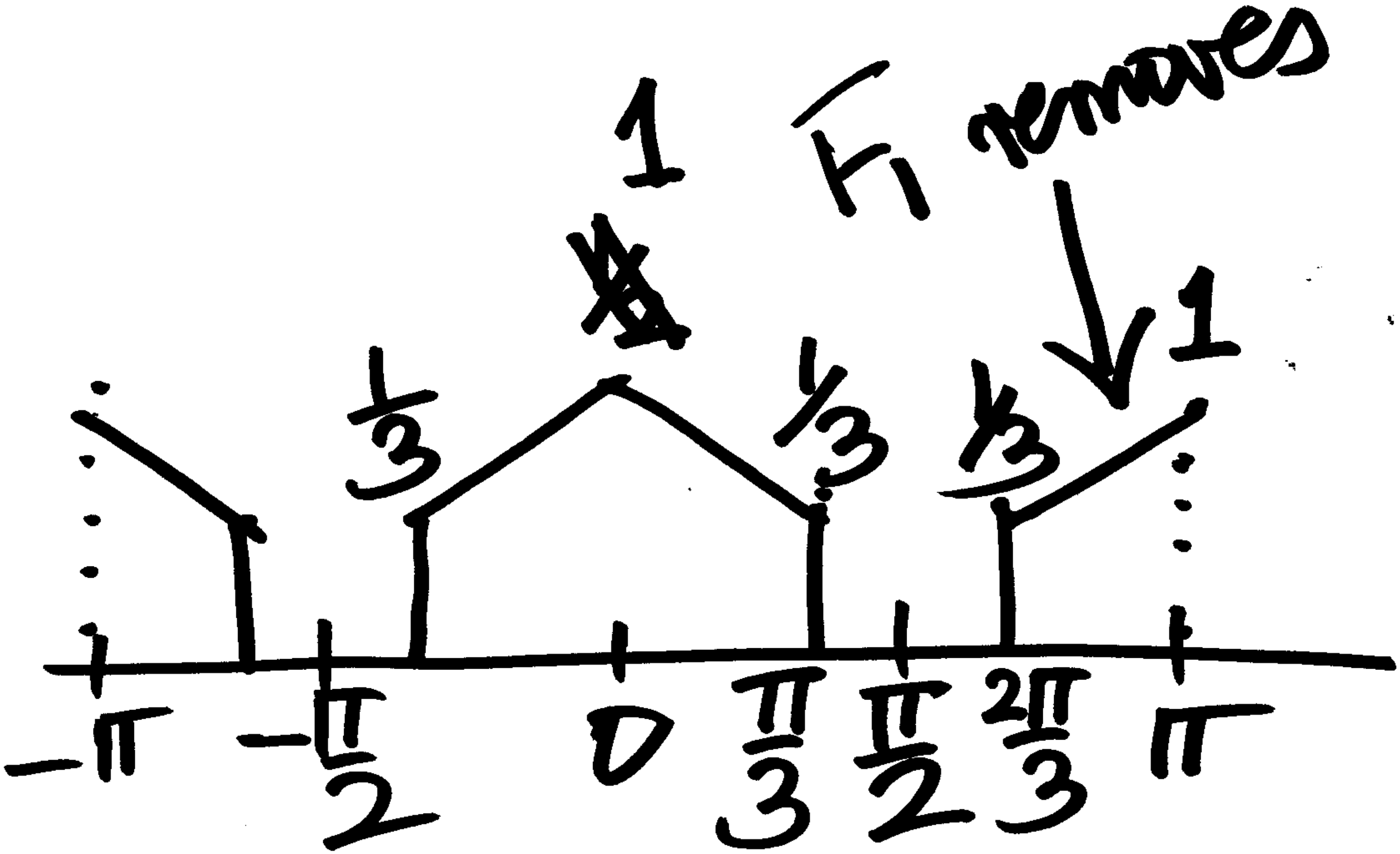
'Prototype' $X(e^{jw})$



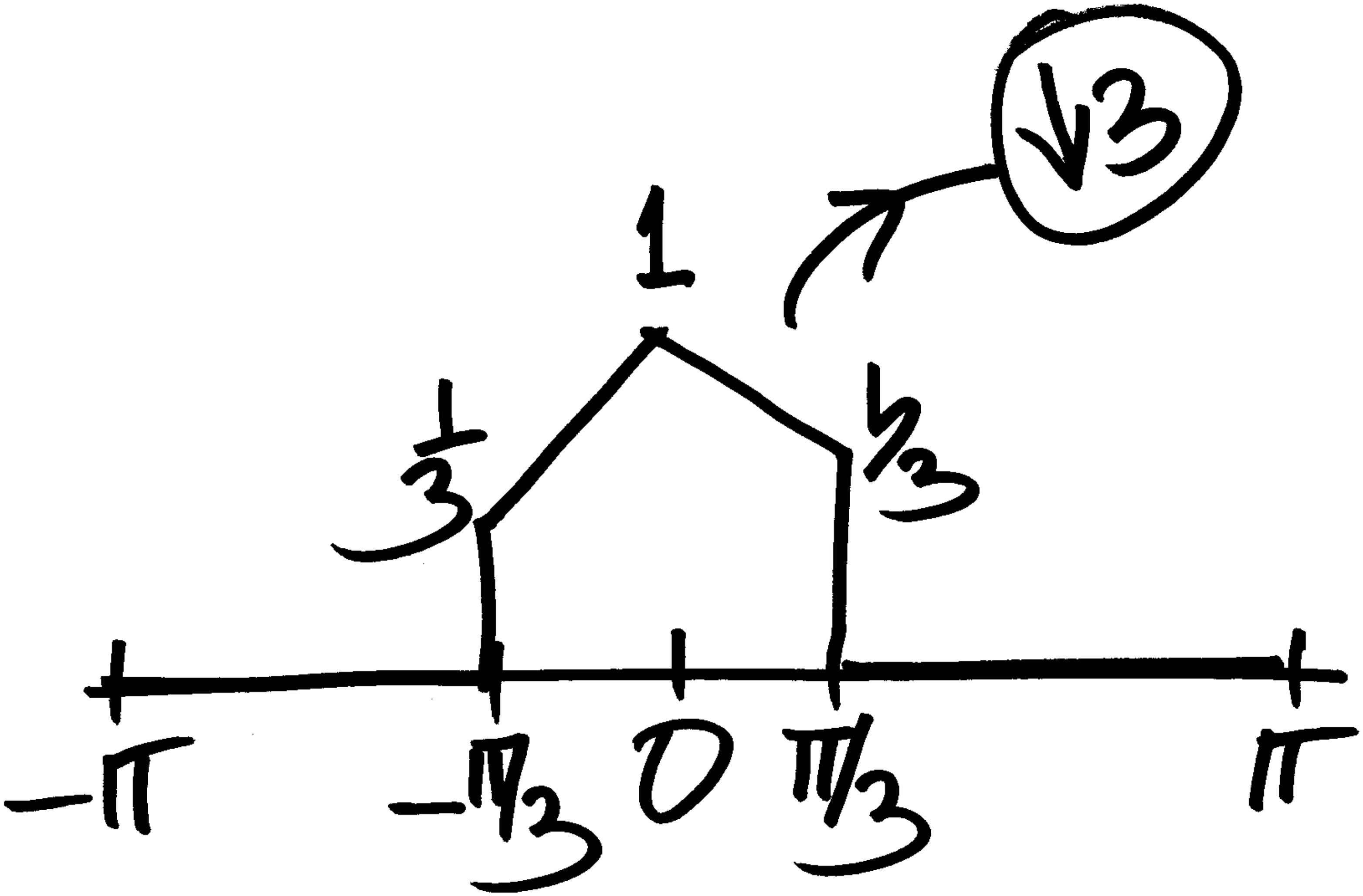
filtered
$X = \tilde{X}$.
1
$\frac{1}{3}$
$\frac{1}{3}$
$-\pi$
$-\frac{2\pi}{3}$
$\frac{2\pi}{3}$
π
$\longrightarrow \omega$.



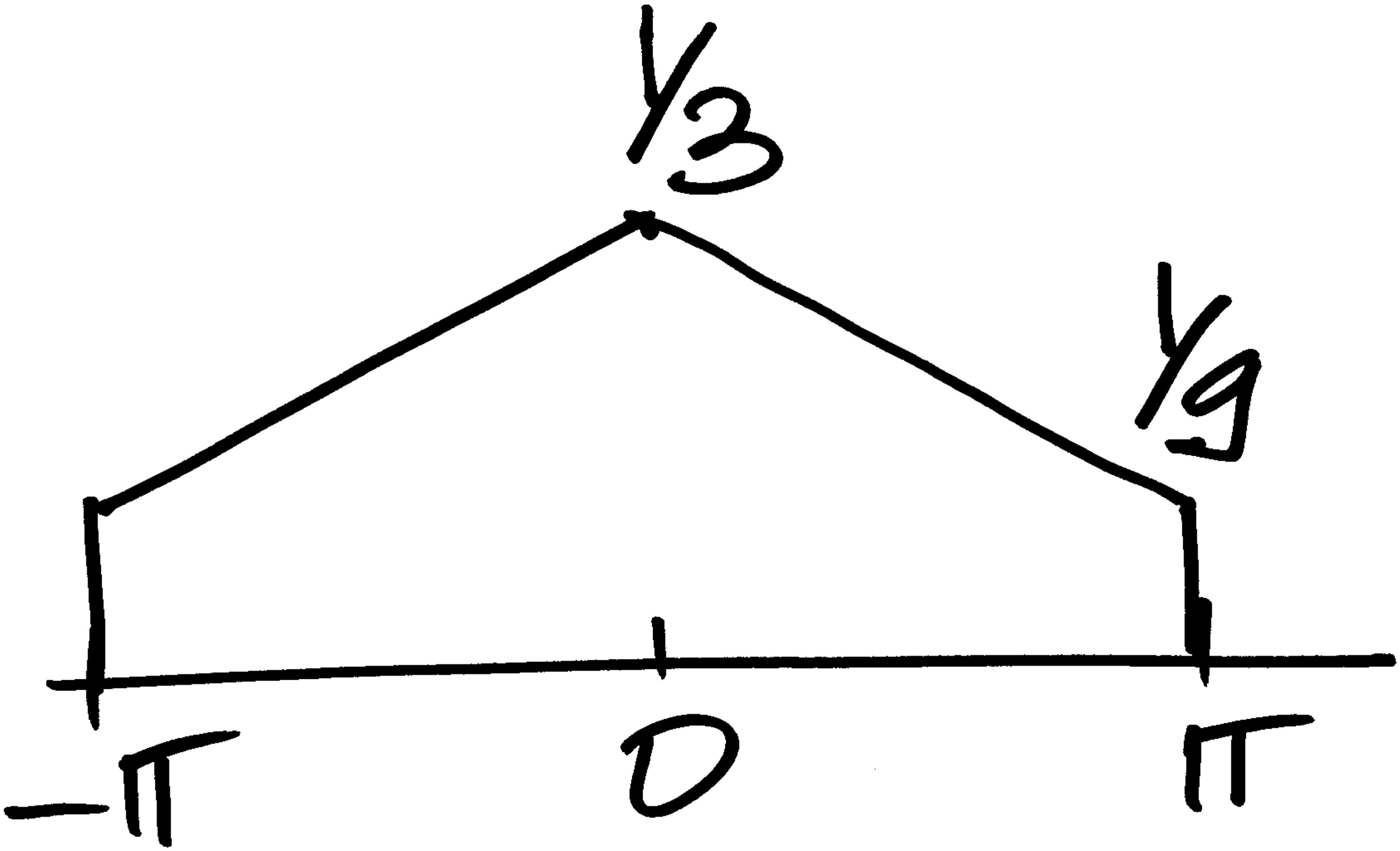
filtered
$X(e^{j\omega})$
$\tilde{x}$
$\uparrow 2$
$Y_1(e^{j\omega})$
filter
F_1
$Y_2(e^{j2\omega})$
$\downarrow 3$
$Y_0(e^{j\omega})$


F_1 removes
1
1/3
1/3
1/3
1
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{3}$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$
π

F_1 needs to be an ideal lowpass filter cutoff $\frac{\pi}{2}$



√3
1
1/3
1/3
−π
−π/3
0
π/3
π



1/3
1/9
-π
0
π