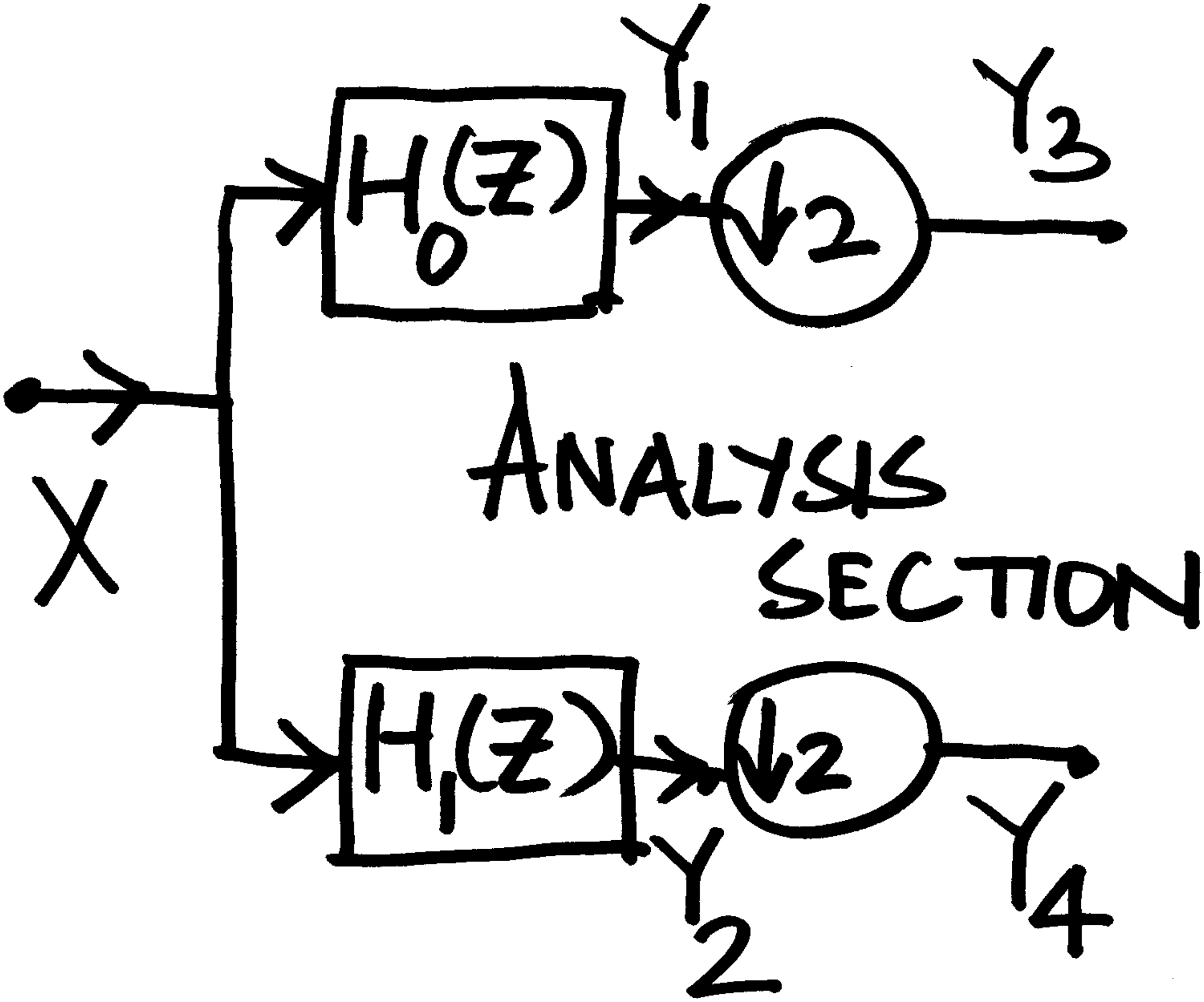
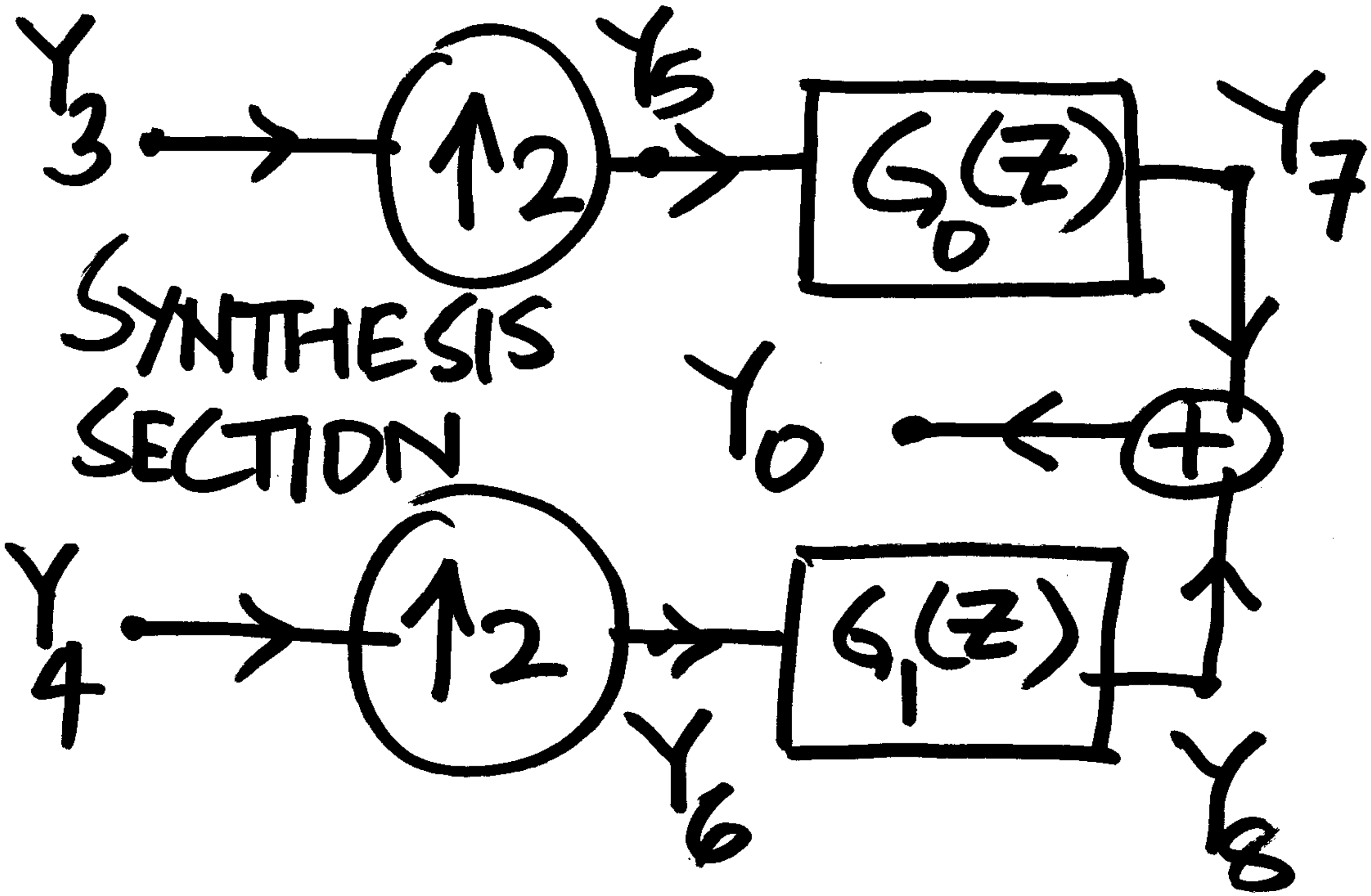
Prof. Gadre
Lec. No. 44
Date : 31/1/11

SESSION 44

TUTORIAL ON TWO BAND FILTER BANK


X
H0(Z)
H1(Z)
↓2
↓2
Y1
Y2
Y3
Y4
ANALYSIS
SECTION



Y3
↑2
Y5
G0(Z)
Y7
SYNTHESIS
SECTION
Y0
+
Y4
↑2
G1(Z)
Y6
Y8

On analysis side

$x[n]$, x } denotes time domain

$X(Z)$, X } denotes complex frequency domain

Haar Two Band Filter Bank

$$H_0(z) = (1 + z^{-1})$$

$$H_1(z) = (-1 + z^{-1})$$

$$G_0(z) = (1 + z^{-1})/2$$

$$G_1(z) = (1 - z^{-1})/2$$

$1/2$ factor can be either on analysis or synthesis.

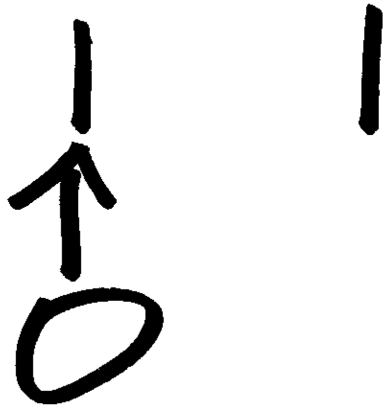
$x[n] =$

$$\underset{\substack{\uparrow \\ 0}}{7} \quad 5 \quad -4 \quad 6 \quad 3 \quad 8$$

as input

$$H_0(z) = 1 + z^{-1}$$

Corresponding impulse response



$= h_0[n]$

7 12 1 2 9 11 8

$\uparrow$

$O = x * h_0$

Output at Y_1

As expected, of length 7

$y_1 = x * h_0$

$$\begin{array}{cccccc} 7 & 5 & -4 & 6 & 3 & 8 \\ \uparrow \\ 0 \end{array}$$

$$* \quad \begin{array}{cc} 1 & 1 \\ \uparrow \\ 0 \end{array}$$

After downsampling by 2:

$$Y_1 \rightarrow \downarrow 2 \rightarrow Y_3$$

$$Y_3: \quad \underset{\substack{\uparrow \\ 0}}{7} \quad 1 \quad 9 \quad 8$$

$x * h_1$

$= -7 \quad 2 \quad 9 \quad -10 \quad 3 \quad -5 \quad 8$

$\uparrow$

0

Output y_2

As expected, of length 7

$$y_2 : x * h_1$$

$$H_1(z) = -1 + z^{-1}$$

$$h_1[n] = \underset{\substack{\uparrow \\ 0}}{-1} \quad 1$$

$$Y_2 \longrightarrow \downarrow 2 \longrightarrow Y_4$$

$$Y_4: \quad -7 \quad 9 \quad 3 \quad 8$$

$$\uparrow$$

$$0$$

Now the synthesis section

To upsample first!

$$Y_3 \rightarrow \uparrow 2 \rightarrow Y_5$$

$$Y_5 : \; \underset{\underset{0}{\uparrow}}{7} \; 0 \; 1 \; 0 \; 9 \; 0 \; 8$$

$$Y_4 \longrightarrow \uparrow 2 \longrightarrow Y_6$$

$$Y_6 = \underset{\substack{\uparrow \\ 0}}{-7} \quad 0 \quad 9 \quad 0 \quad 3 \quad 0 \quad 8$$

$$Y_6 \rightarrow \boxed{G_1(z)} \rightarrow Y_8$$

$$(1-z^{-1})/2$$

$$y_8[n] = \frac{y_6[n] - y_6[n-1]}{2}$$

$$Y_5 \rightarrow \boxed{G_0(z)} \rightarrow Y_7$$

$$(1+z^{-1})/2$$

$$Y_7: \ \frac{7}{2} \ \frac{7}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{9}{2} \ \frac{9}{2} \ \frac{8}{2} \ \frac{8}{2}$$

$$y_8[n] =$$

$$\underset{\substack{\uparrow \\ 0}}{-\frac{7}{2}} \quad \frac{7}{2} \quad \frac{9}{2} \quad -\frac{9}{2} \quad \frac{3}{2} \quad -\frac{3}{2} \quad \frac{8}{2} \quad -\frac{8}{2}$$

$$
\begin{array}{llllllllll}
Y_7: & \frac{7}{2} & \frac{7}{2} & \frac{1}{2} & \frac{1}{2} & \frac{9}{2} & \frac{9}{2} & \frac{8}{2} & \frac{8}{2} \\[6pt]
Y_8: & -\frac{7}{2} & \frac{7}{2} & \frac{9}{2} & -\frac{9}{2} & \frac{3}{2} & -\frac{3}{2} & \frac{8}{2} & -\frac{8}{2} \\
\hline
+: & 0 & 7 & 5 & -4 & 6 & 3 & 8 & 0 \\
 & \uparrow & & & & & & & \\
 & 0 & & & & (Y_0) & & &
\end{array}
$$

We notice: that

$$y_D[n] = x[n-1]$$

Delay has occurred on account of CAUSALITY need!

Now we <u>periodize</u> the input

New periodic input

$$\tilde{x}[n] = \sum_{k=-\infty}^{+\infty} x[n+kN]$$

<u>$N \geq 6$</u>

For simplicity take $\underline{N=6}$

$$\cdots 3\ 8\ \underbrace{\underset{0}{7}\ 5\ {-4}\ 6\ 3\ 8}_{\text{focus on}}\ 7\ 5 \cdots$$

$$Y_1: \underbrace{15\ 12\ 1\ 2\ 9\ 11\ 15}$$

$$Y_2: \underbrace{1\ \ 2\ \ 9\ -10\ \ 3\ -5\ \ 1}$$

$$Y_3: \underbrace{15 \quad 1 \quad 9 \quad 15}$$

$$Y_4: \underbrace{1 \quad 9 \quad 3 \quad 1}$$

$$Y_5: \underbrace{15\ 0\ 1\ 0\ 9\ 0\ 15}$$

$$Y_6: \underbrace{1\ 0\ 9\ 0\ 3\ 0\ 1}$$

$$Y_7 : \underbrace{\tfrac{15}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{9}{2}\ \tfrac{9}{2}\ \tfrac{15}{2}}$$

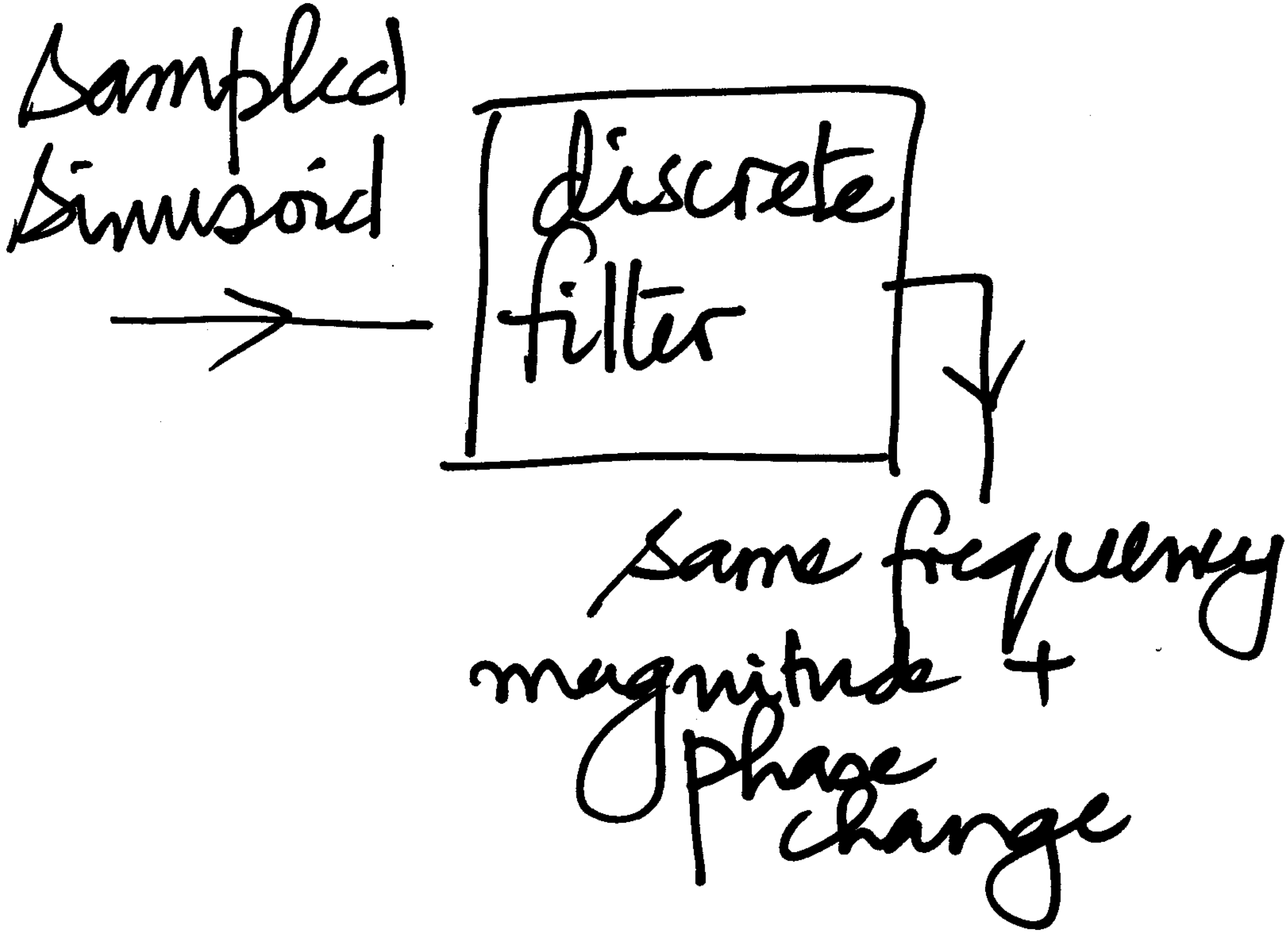
$$Y_8 : \underbrace{-\tfrac{1}{2}\ \tfrac{9}{2}\ -\tfrac{9}{2}\ \tfrac{3}{2}\ -\tfrac{3}{2}\ \tfrac{1}{2}}$$

$$+ : \underbrace{7\ 5\ -4\ 6\ 3\ 8}$$

Frequency domain tutorial study:

Input

$$x[n] = \cos\frac{\pi}{4}n + \cos\frac{3\pi}{4}n$$


Sampled
Sinusoid
discrete
filter
Same frequency
magnitude +
phase
change

$1+z^{-1} \quad 1+e^{-j\omega}$

$-1+z^{-1} \quad -1+e^{-j\omega}$.

$|\cdot|$ change $=$ $2\cos\omega$.

$e^{-j\frac{\omega}{2}}\left(e^{j\frac{\omega}{2}}+e^{-j\omega/2}\right)$

Phase change

$$= -\frac{\omega}{2}.$$

$$X(e^{j\omega}) \longrightarrow \downarrow 2 \longrightarrow \frac{1}{2}\left\{X\left(e^{j\frac{\omega}{2}}\right) + X\left(e^{j\left(\frac{\omega}{2} \pm \pi\right)}\right)\right\}$$

$$X(e^{j\omega}) \longrightarrow \uparrow 2 \longrightarrow X(e^{j2\omega})$$