

Prof. V. P. Gadge

Date-6-1-11

LECTURE NO

40

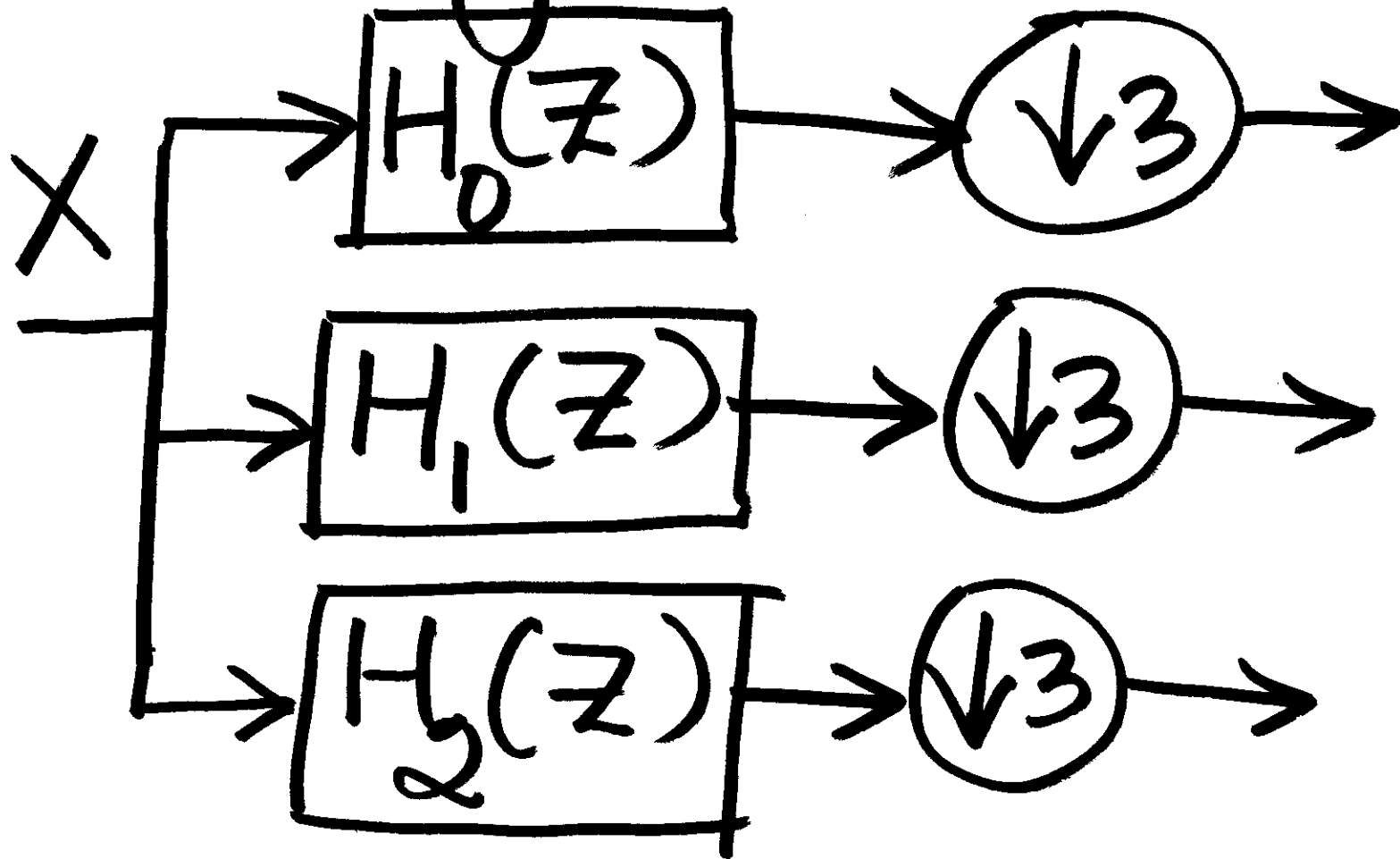
M-BAND FILTER

BANKS AND

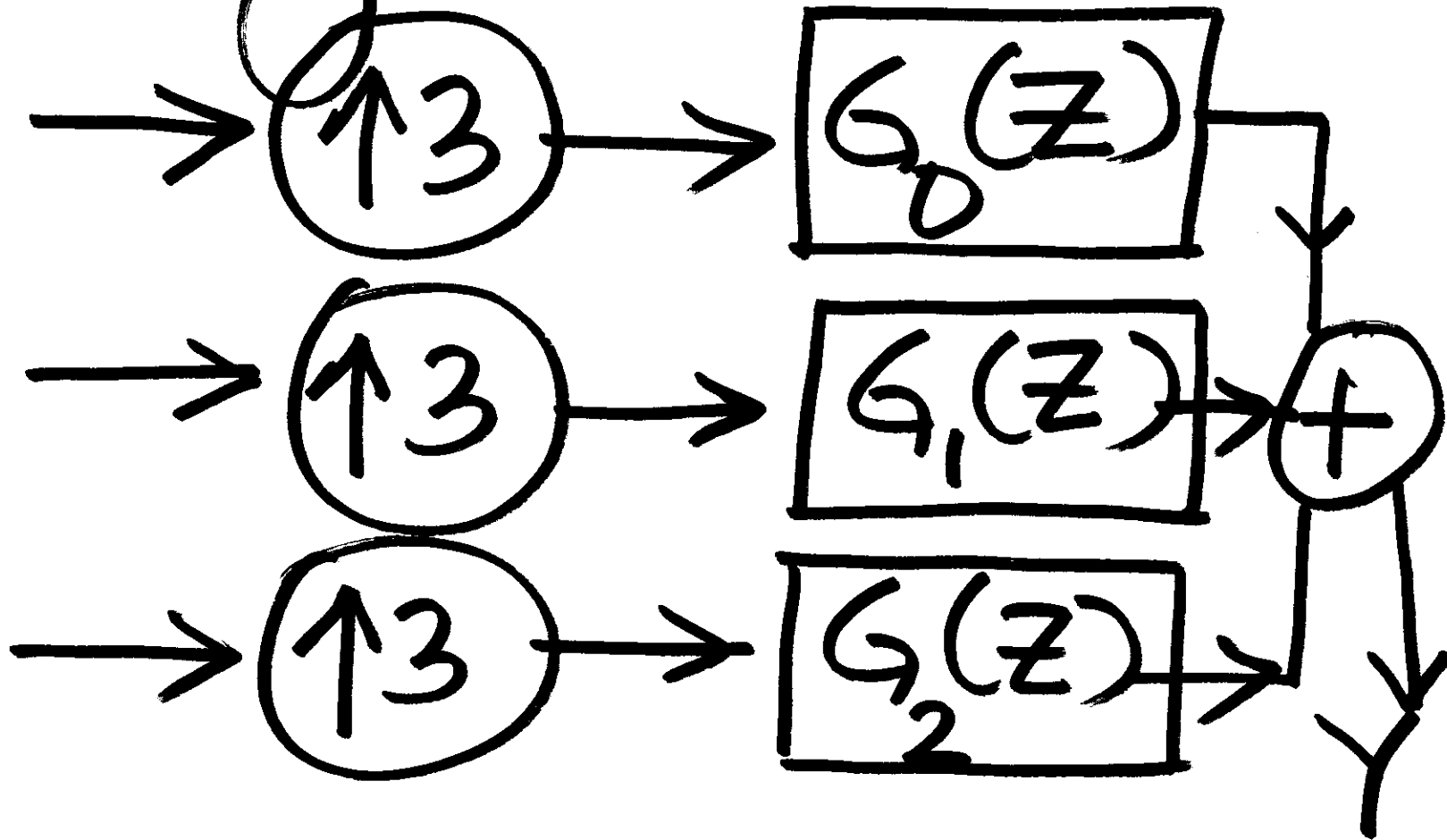
LOOKING AHEAD

3-band
filter bank
(Ideal)

Analysis side



Synthesis side



If 3 band system
has perfect
reconstruction,

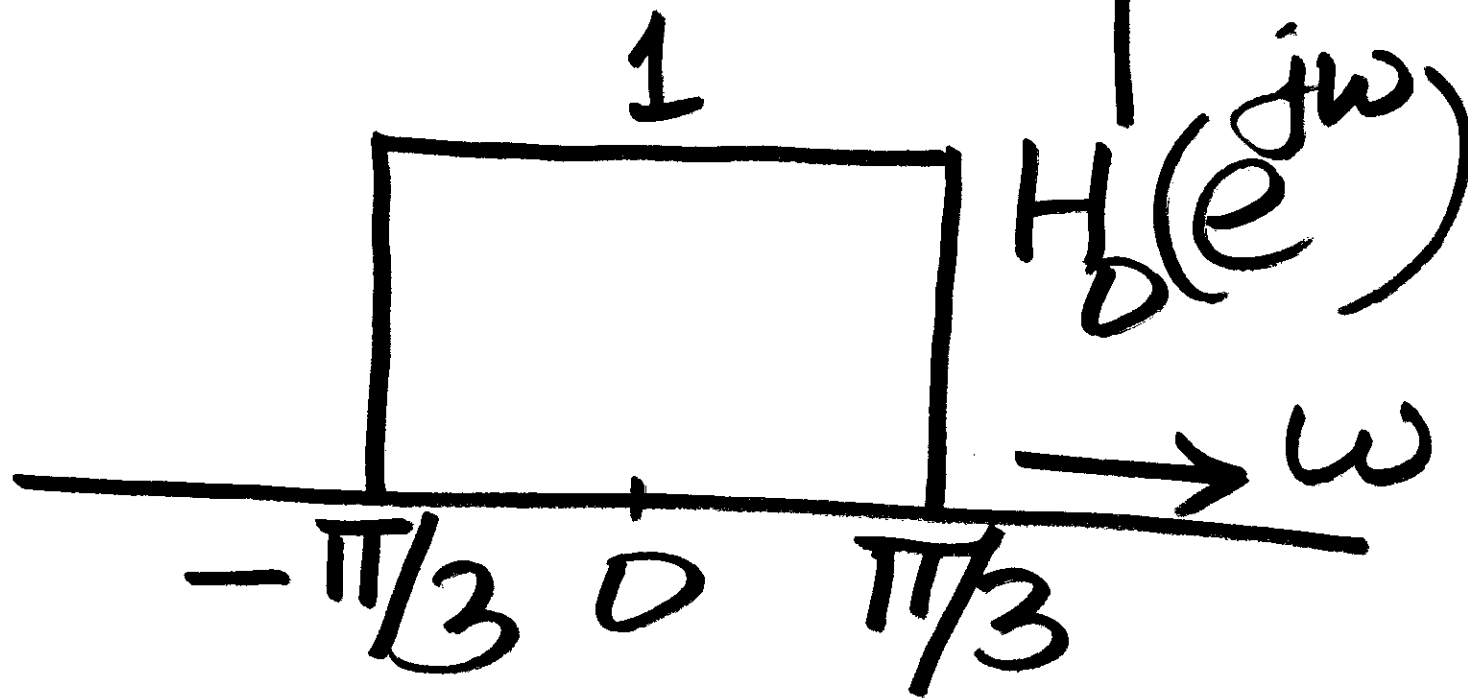
$$Y = X$$

Ideal 3-band
filter bank

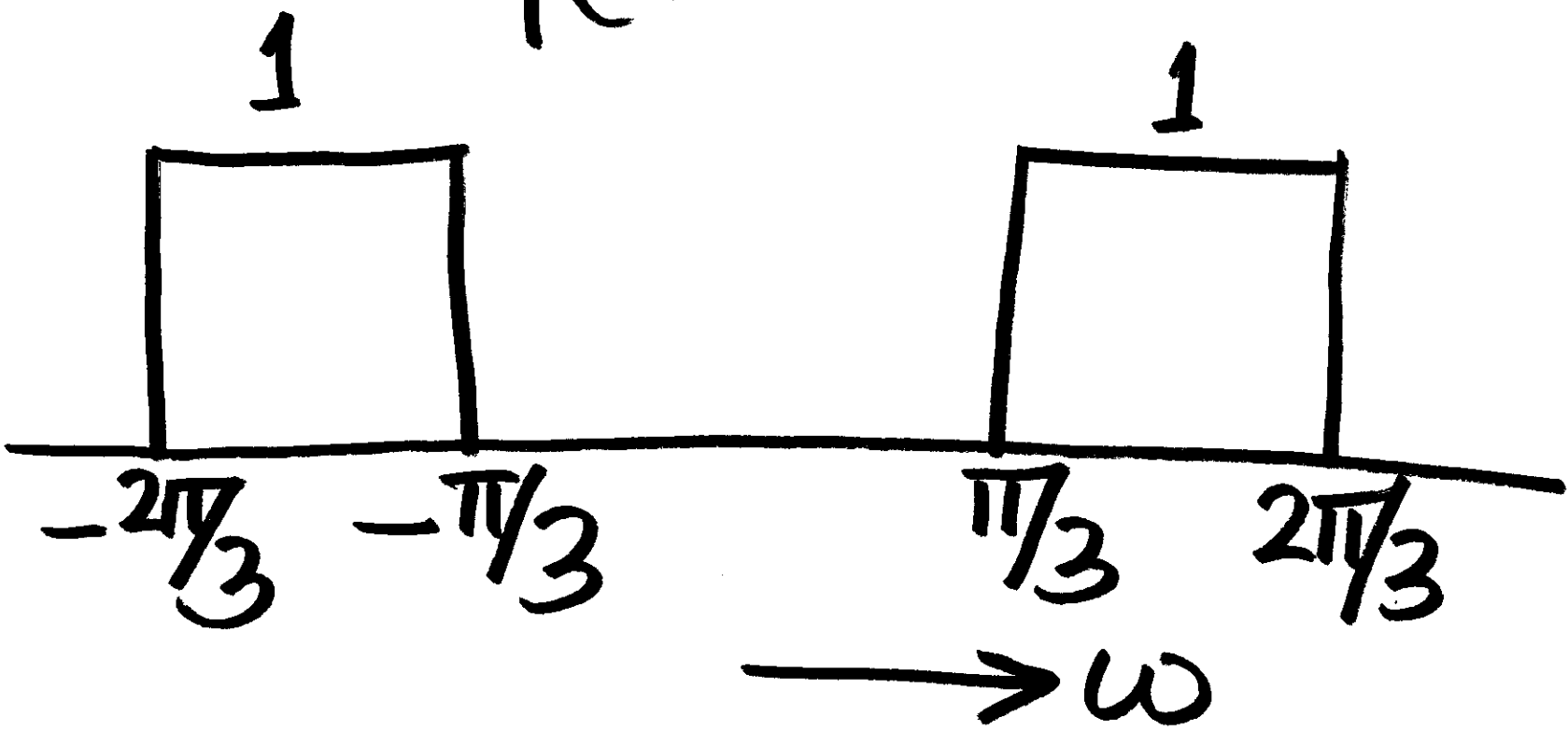
$$H_0 = G_0$$

$$H_1 = G_1 ; H_2 = G_2$$

Ideal frequency responses



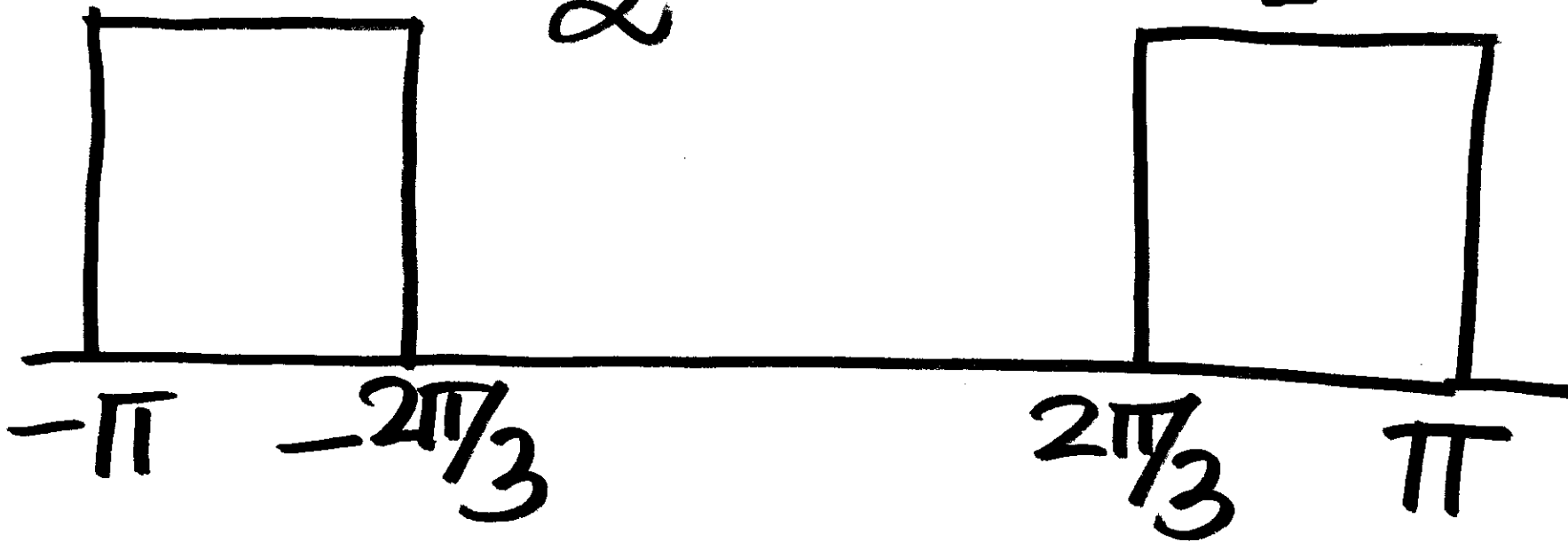
$$H_1(e^{j\omega})$$



$$H_2(e^{j\omega})$$

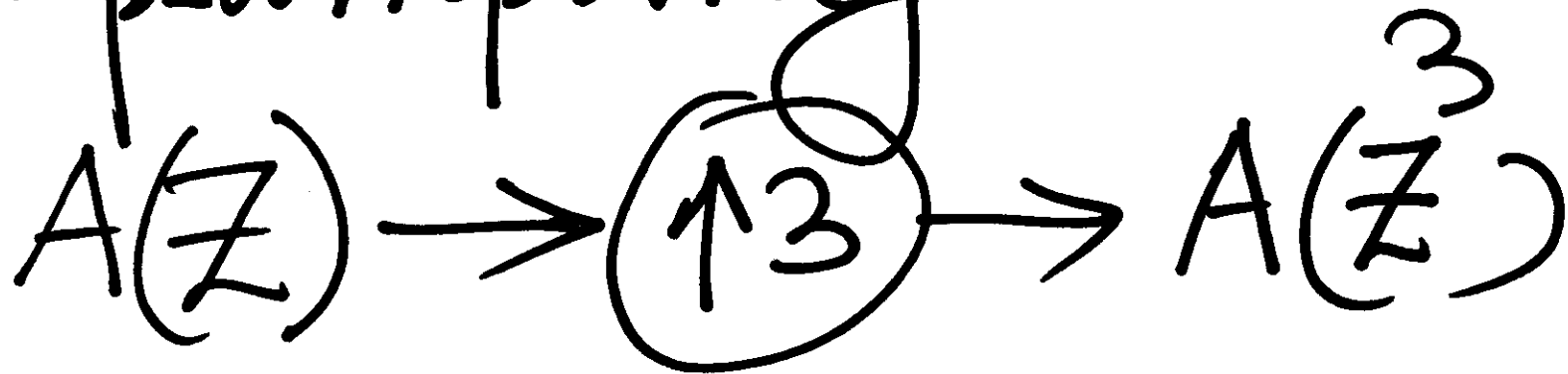
1

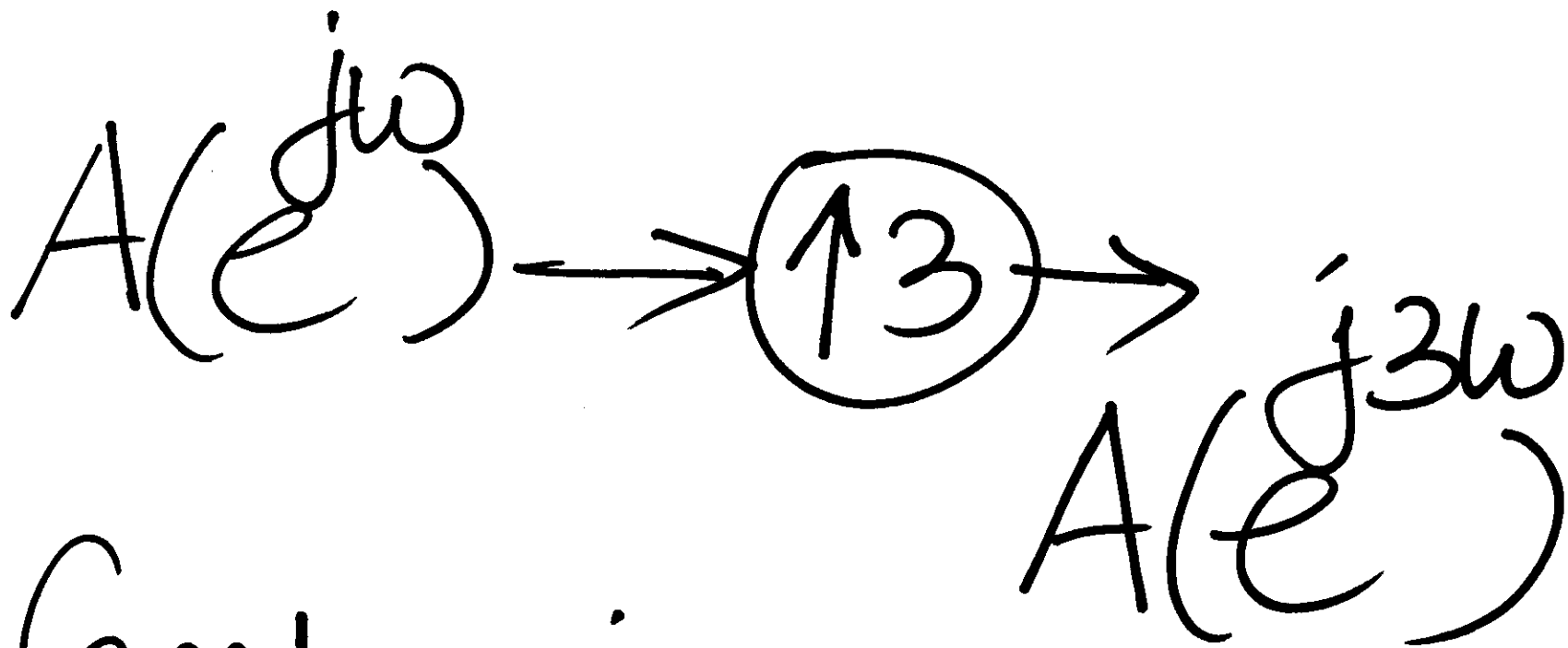
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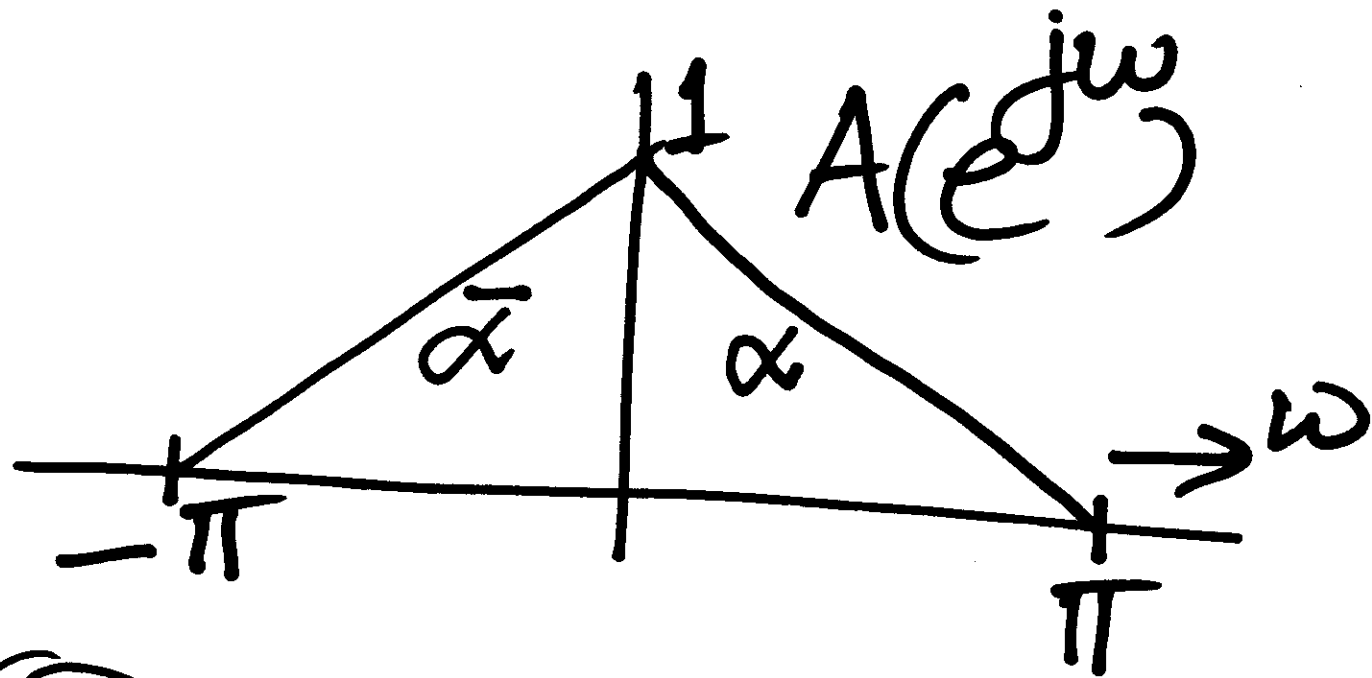
Frequency effect

of
upsampling

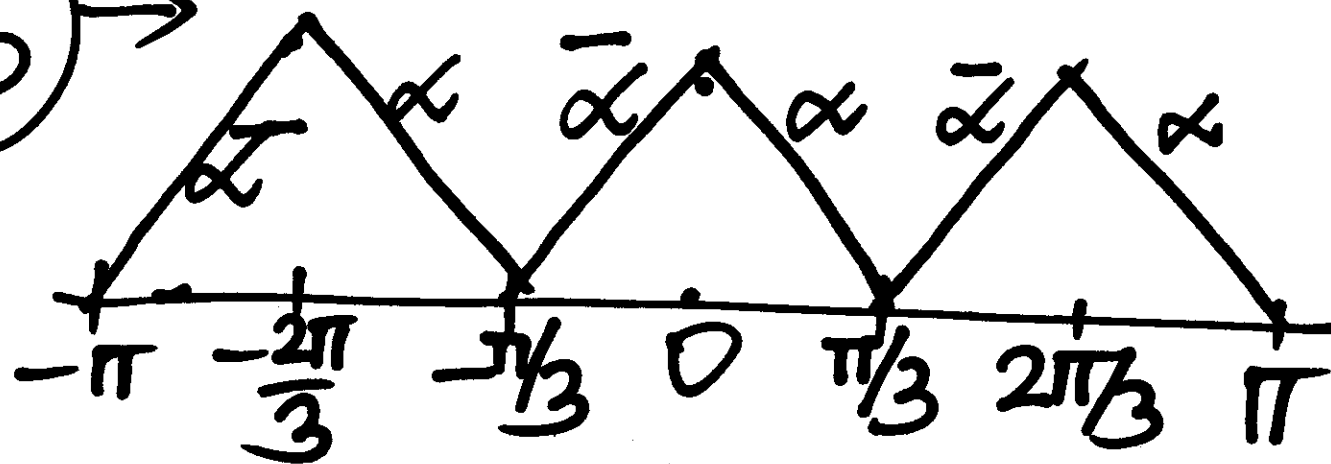


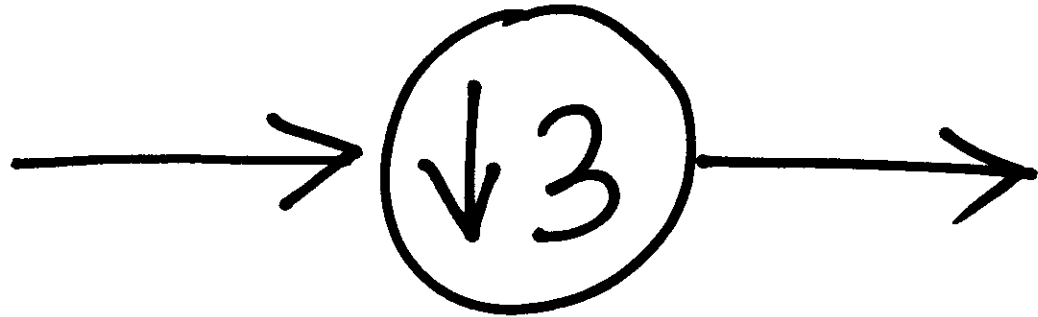


Compression
of frequency axis
by 3.



3





⇓ multiply by 3

∴ 1 0 0 1 0 0 1 0 0 1 0 0 1

-3 0 3 6 9

Then 'throw away' zeros.

This periodic sequence
can be written:

$$\frac{1}{3} \sum_{k=0}^2 e^{-j\frac{2\pi}{3}kn}$$

$$a[n] \rightarrow \textcircled{\downarrow 3} \rightarrow$$

\Downarrow equivalent to:

$$\left(a[n] \cdot \frac{1}{3} \cdot \sum_{k=0}^2 e^{-j \frac{2\pi}{3} kn} \right)$$

... followed by:

replace \mathbb{Z} by

$\mathbb{Z}^{\frac{1}{3}}$

(in \mathbb{Z} -
domain)

$$a[n] \rightarrow A(z)$$
$$A(e^{j\omega})$$

$$a[n] = \frac{1}{3} \sum_{k=0}^2 e^{-j\frac{2\pi}{3}kn}$$

Z transform \downarrow

$$\sum_{k=0}^2 \frac{1}{3} A(z e^{j\frac{2\pi}{3}k})$$

Now $Z \leftarrow Z^{1/3}$

$$A(Z) \rightarrow \sqrt[3]{Z} \rightarrow \sum_{k=0}^2 \frac{1}{3} Z^{1/3} e^{j \frac{2\pi}{3} k}$$

In the sinusoidal frequency domain

$$A(e^{j\omega}) \xrightarrow{\text{DFT}} \sqrt{3} \downarrow$$
$$\frac{1}{3} \sum_{k=0}^2 A(e^{j\frac{\omega}{3}} e^{j\frac{2\pi}{3}k})$$

Consider; $\frac{k=0,1,2}{\quad}$

$$A \left(e^{j\omega} \cdot e^{j\frac{2\pi}{3}k} \right)$$

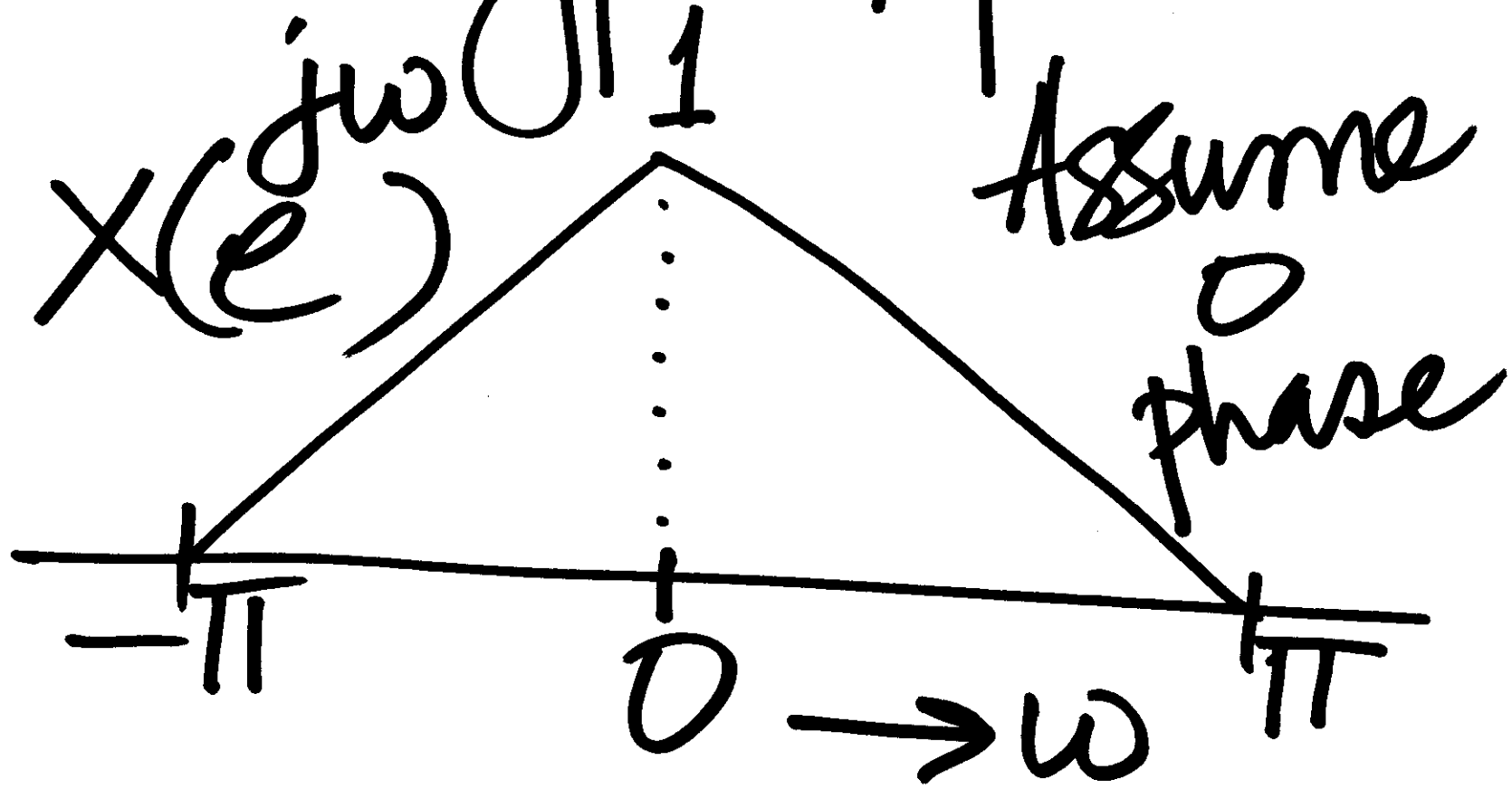
$$= A \left(e^{j\left(\omega + \frac{2\pi}{3}k\right)} \right)$$

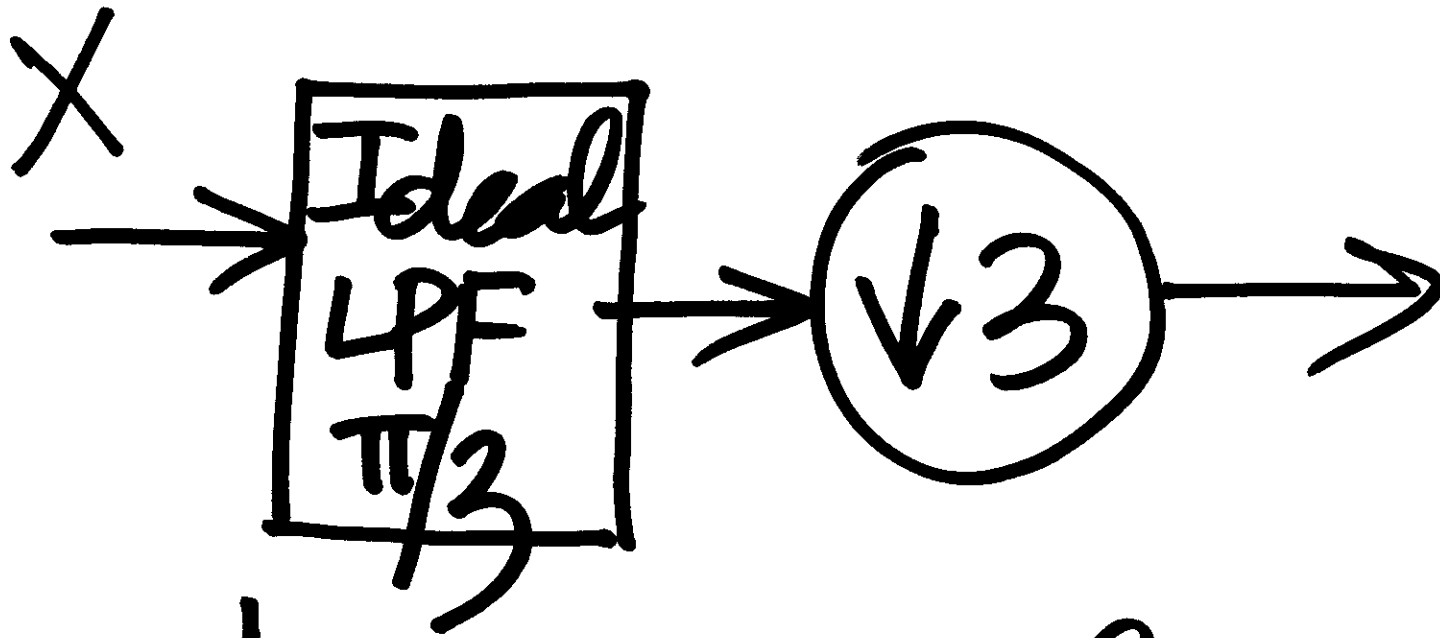
$$\sum_{k=0}^2 A(e^{j\omega} e^{j\frac{2\pi}{3}k})$$

Shift original DTFT
by $0, \frac{2\pi}{3}, \frac{4\pi}{3}$ and
add these Three!

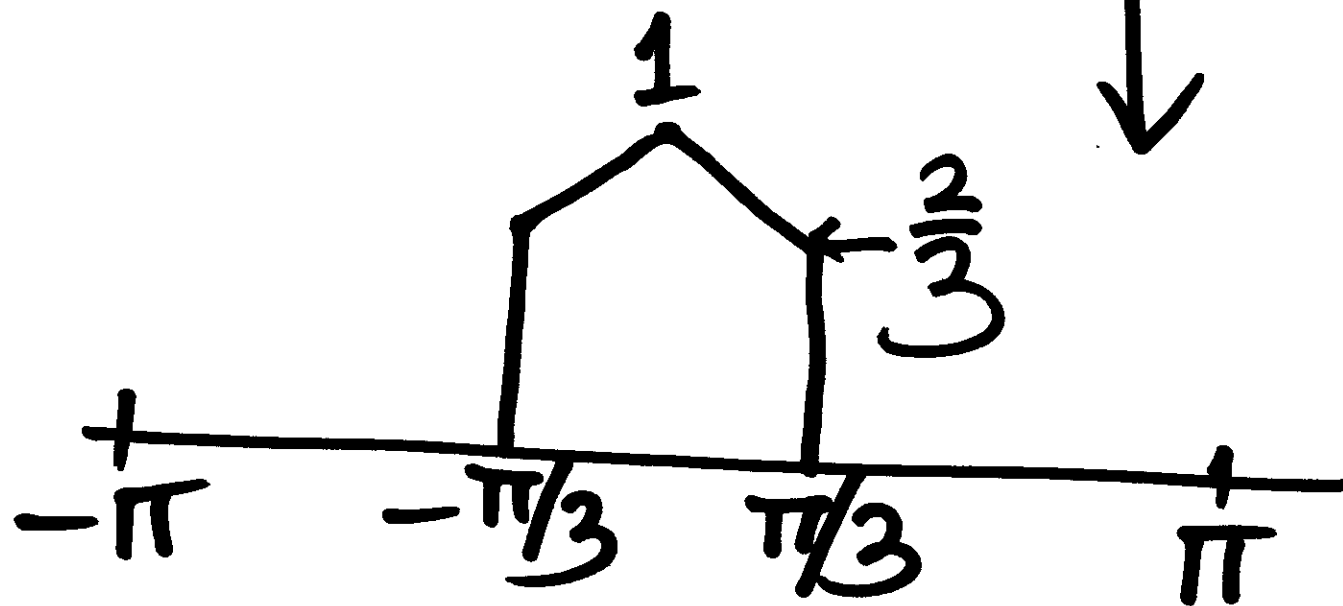
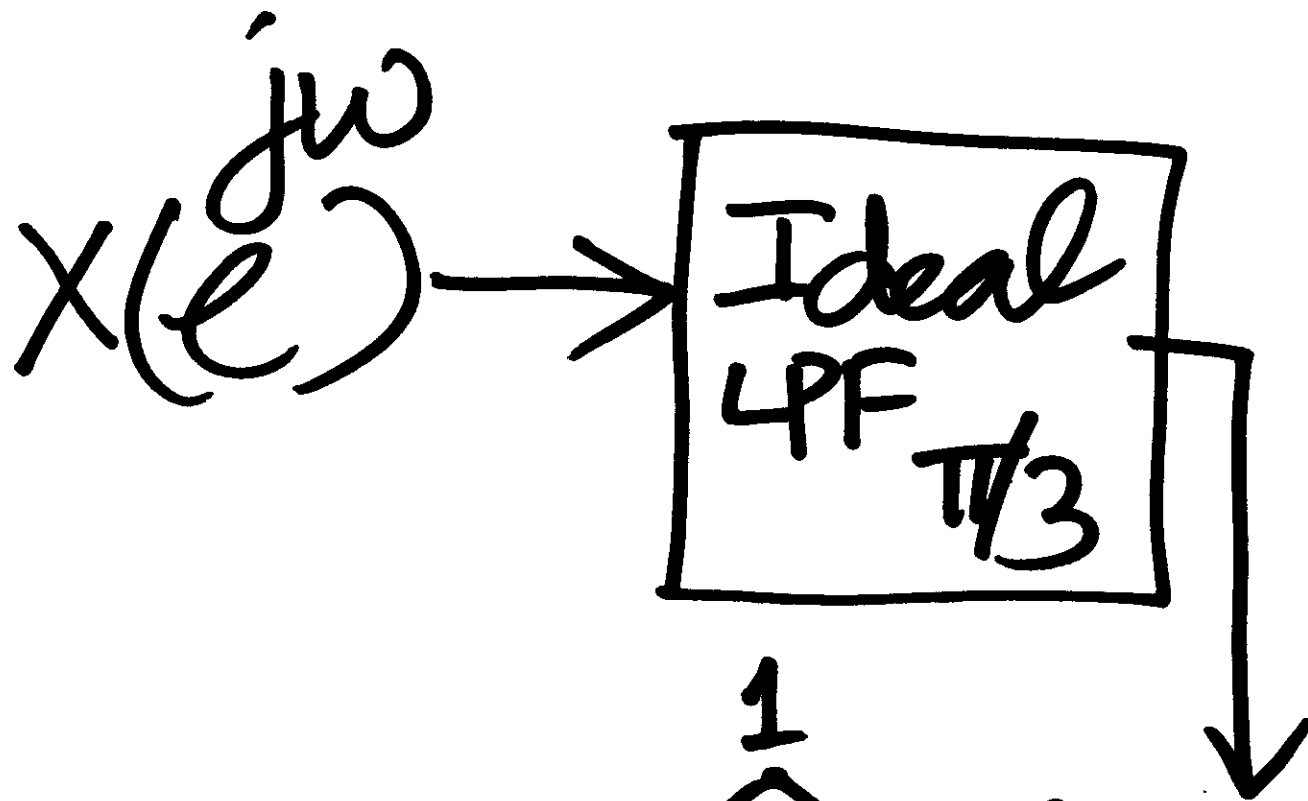
After thus shifting
and adding,
scale vertically by $\frac{1}{3}$
and horizontally,
stretch by $\frac{1}{3}$.

"Prototype" spectrum





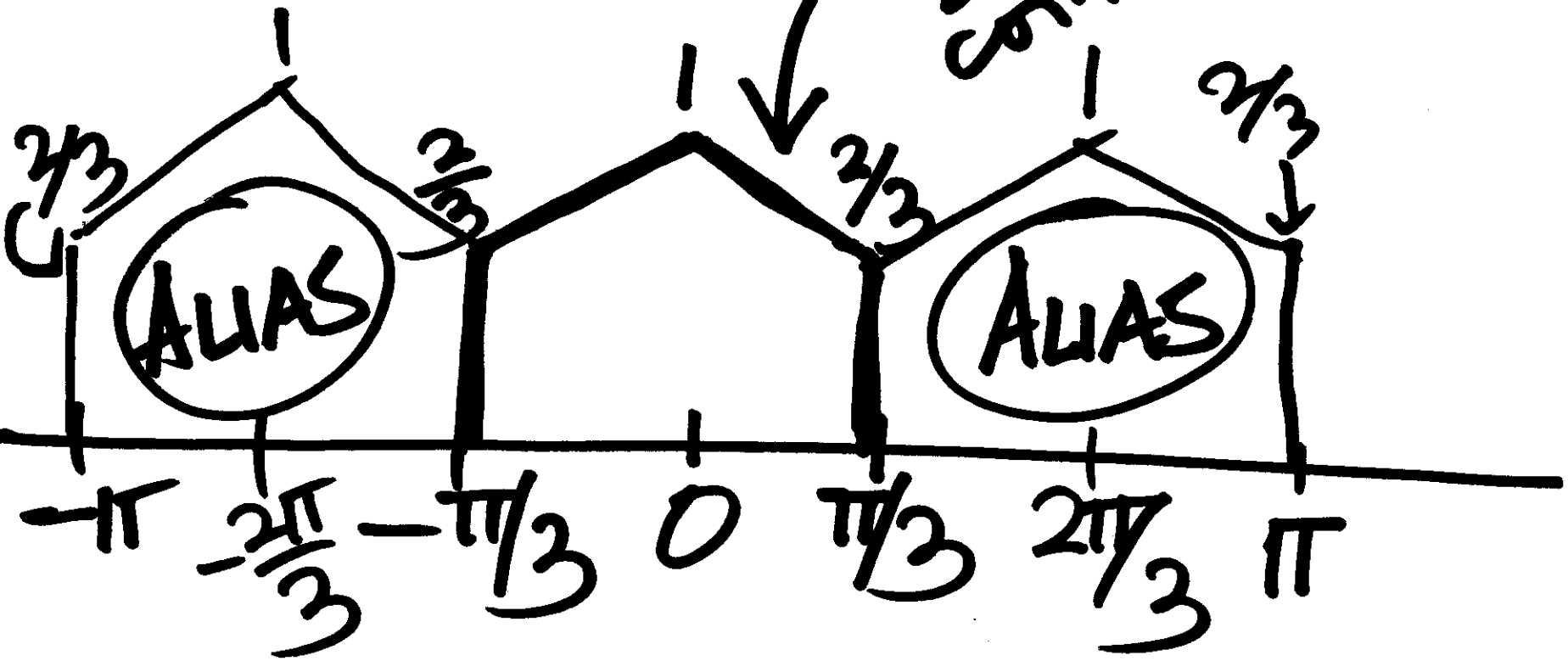
Analysis lowpass
branch

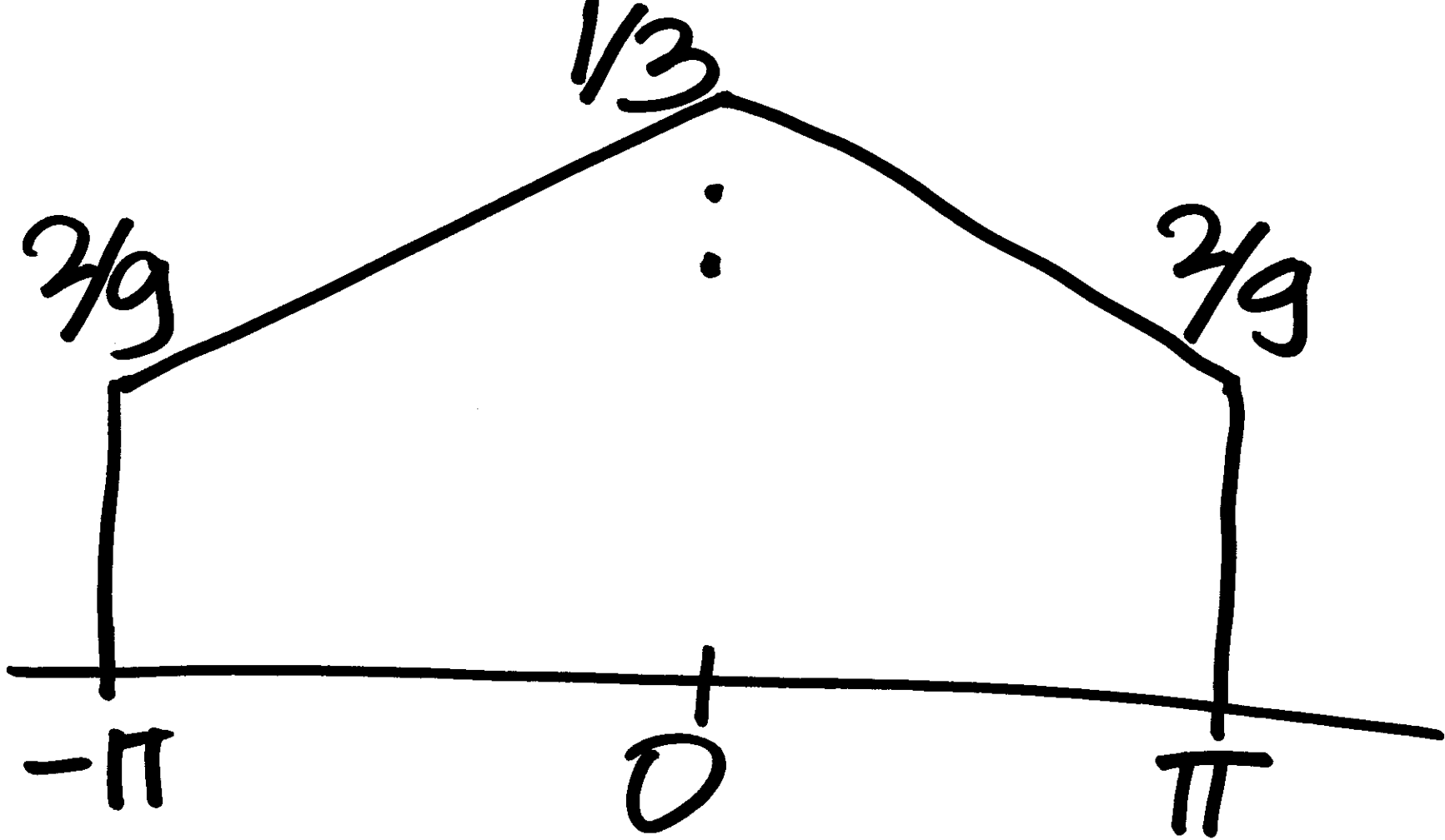


When

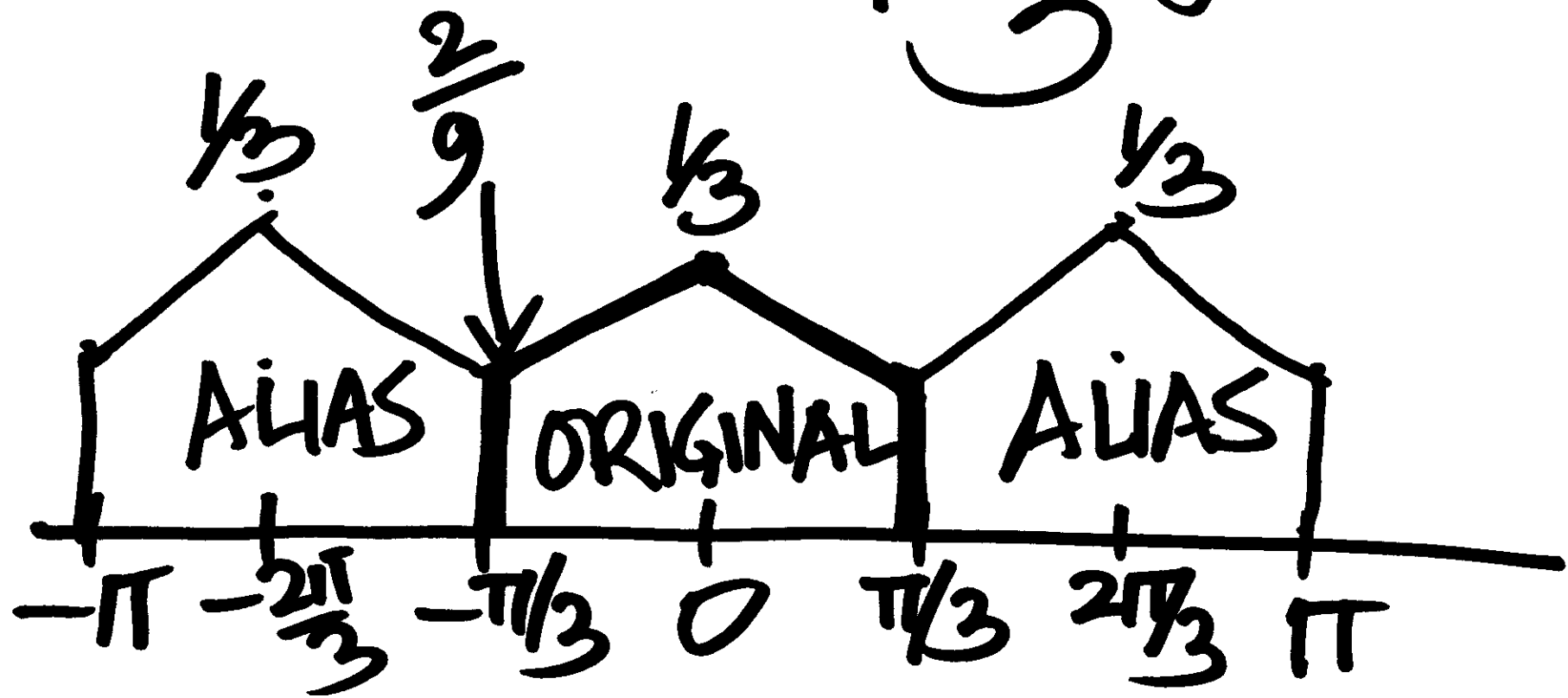
$\downarrow 3$

ORIGINAL SPECTRAL COMPONENT



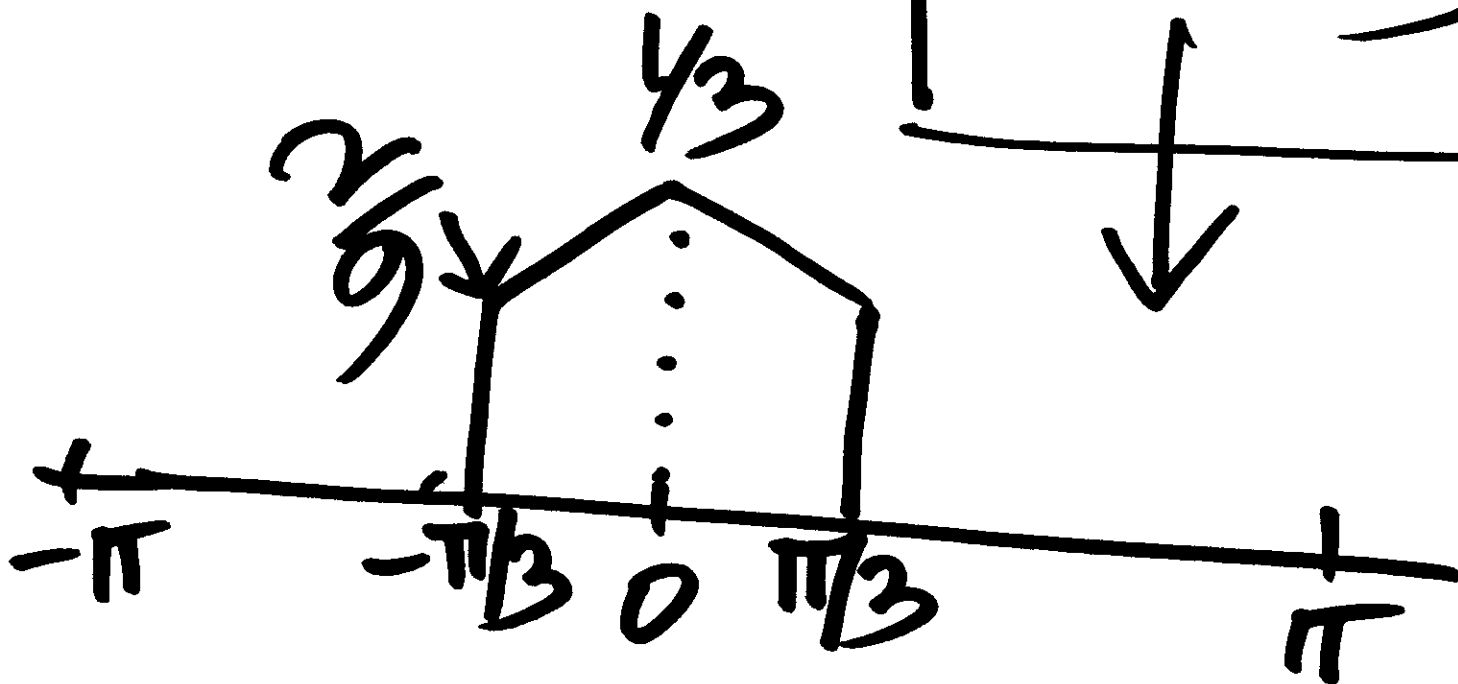
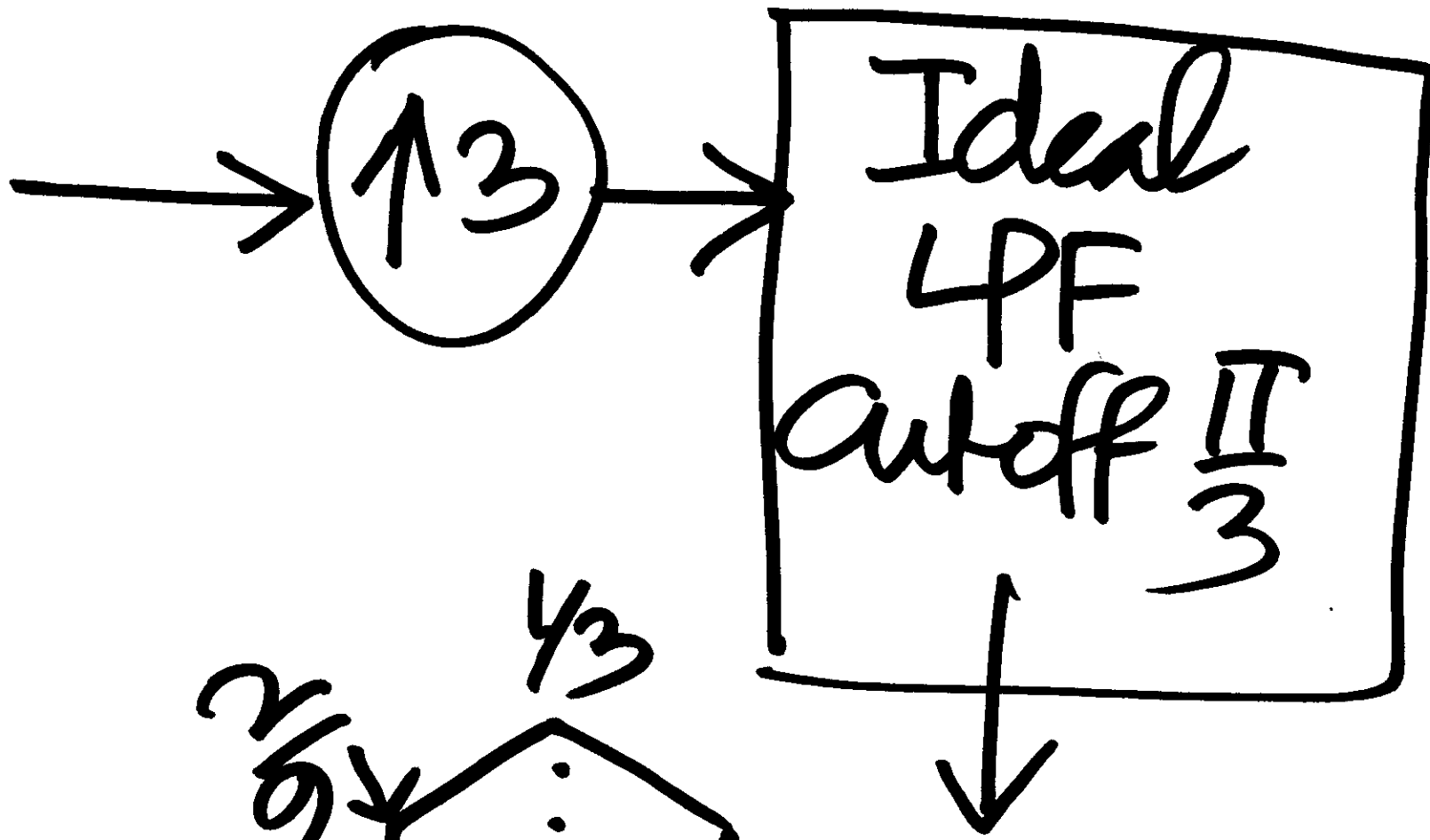


On upsampling by 3



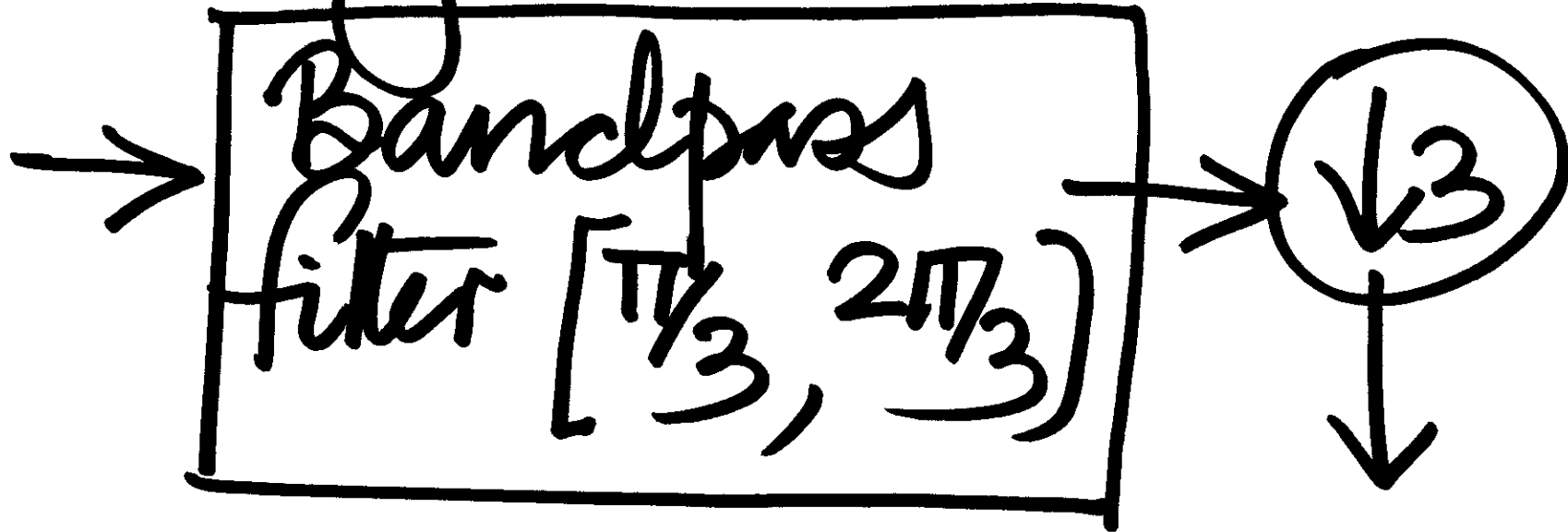
The synthesis filter
is an ideal
lowpass filter,
cutoff $\pi/3$

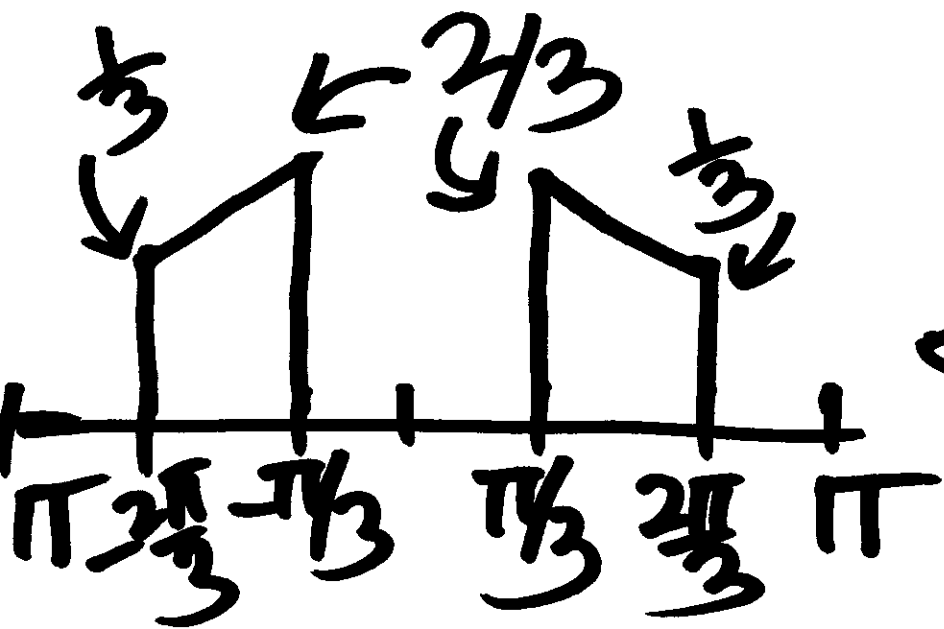
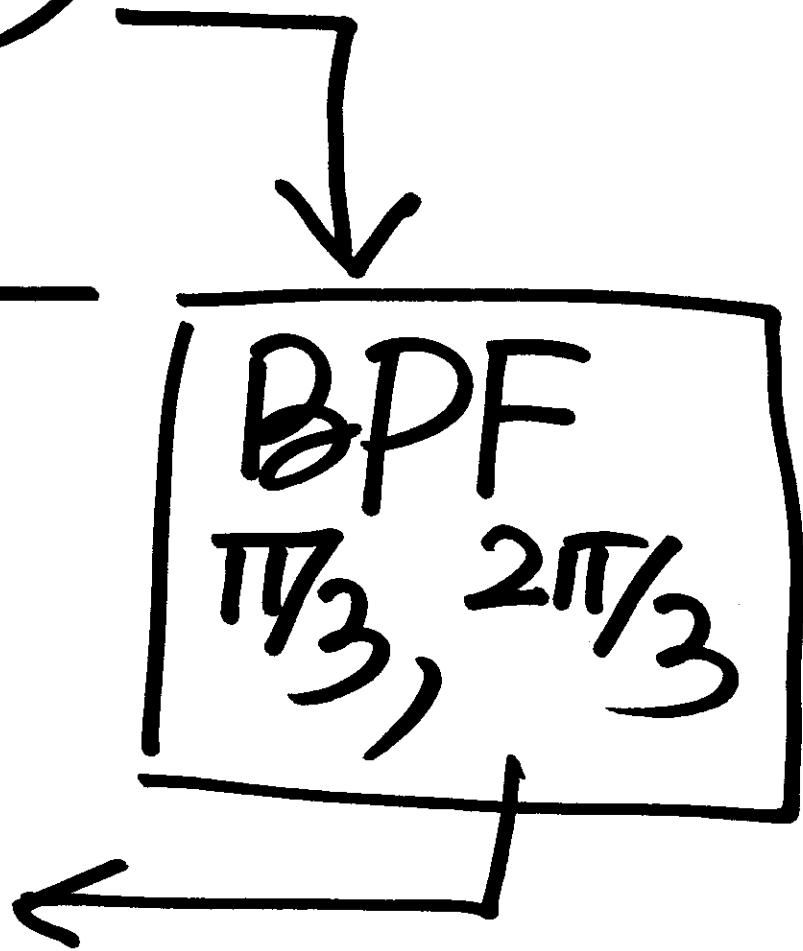
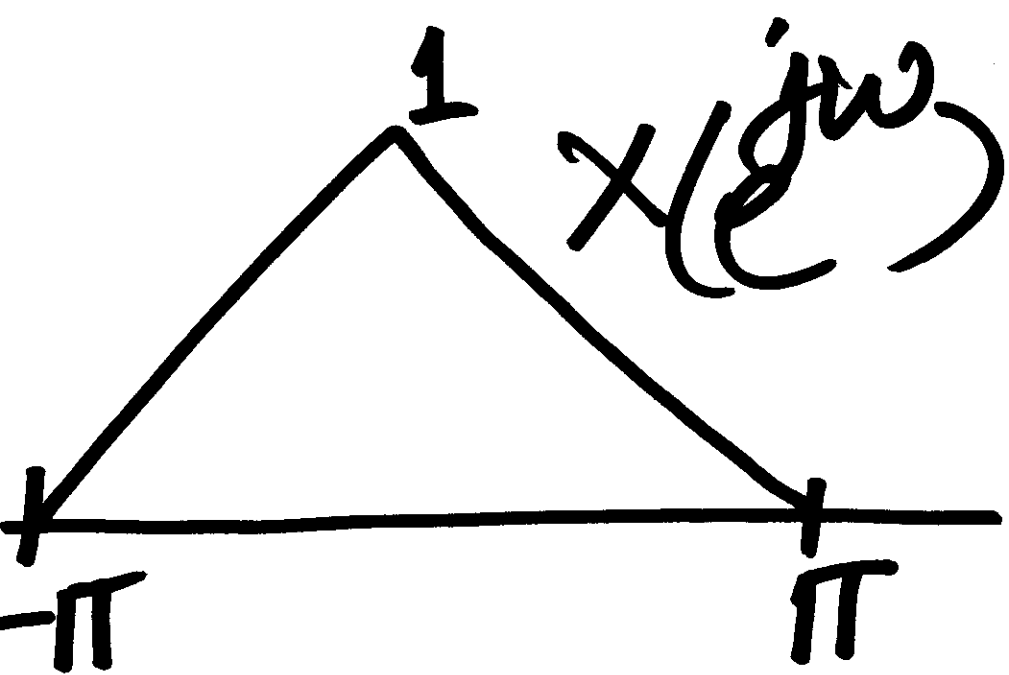
Retains the
original and
destroys the
aliases



"Middle branch"

Analysis side:





On

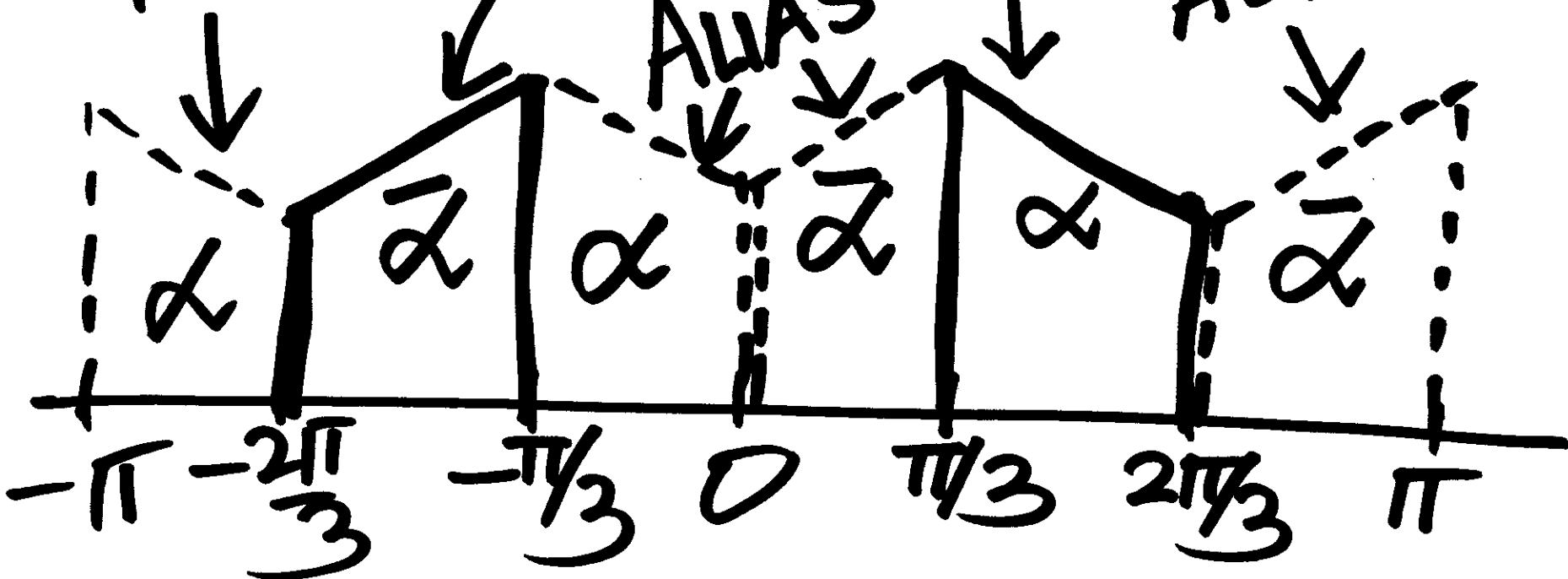
$\sqrt{3}$

ORIGINAL

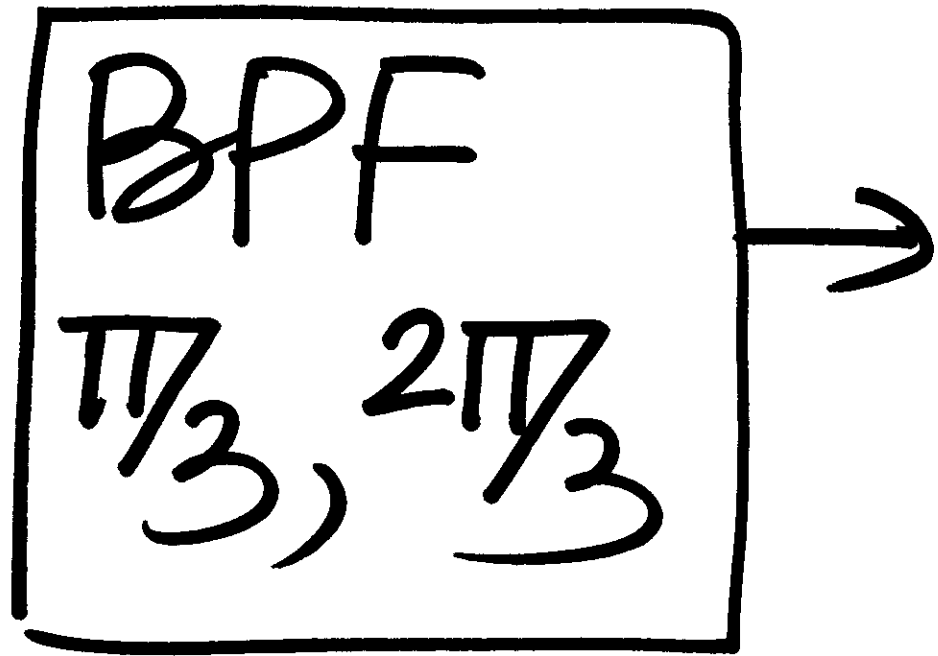
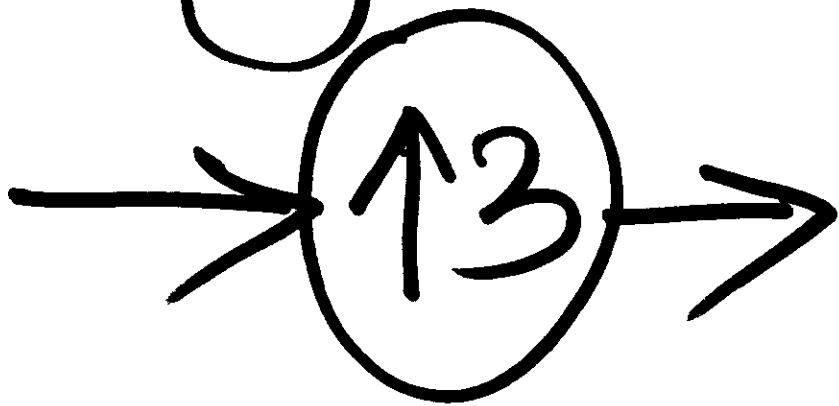
ALIAS

ALIAS

ALIAS



Synthesis filter



After the synthesis
filter,
original retained,
aliases removed.