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Lect. 37
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LECTURE 37

- MODULATION ANALYSIS AND THE 3-BAND FILTER BANK
- APPLICATIONS

Final step in the polyphase approach:

$$\begin{bmatrix} \text{Output} \\ \text{polyphase} \\ \text{vector, order } M \end{bmatrix} = \ \ldots$$

Analysis polyphase matrix

Input polyphase vector

Synthesis polyphase matrix

ALL ORDER M

Product of the synthesis and analysis polyphase matrices of order M = ___

A square matrix

$M \times M$

What should this matrix be, for perfect reconst?

For perfect reconstruction we require:

$$Y(z) = c_0 z^{-D} X(z)$$

$$Y(z) = \sum_{k=0}^{M-1} z^{-k} Y_{k,M}(z^M)$$

$$X(z) = \sum_{k=0}^{M-1} z^{-k} X_{k,M}(z^M)$$

$$\sum_{K=0}^{M-1} Y_{K,M}\left(z^{M}\right) \cdot z^{-K} =$$

$$C_0 Z^{-D} \cdot \sum_{K=0}^{M-1} Z^{-K} X_{K,M}\left(z^{M}\right)$$

$$= c_0 \sum_{k=0}^{M-1} z^{-(D+k)} X_{\leq M}(z^M).$$

$D+k$:

D ↑ fixed for all k

Example:

$M = 3$

$D = 5$

k	$k+1$		
0	5	$\equiv$	2
1	6	$\equiv$	0
2	7	$\equiv$	1

$\mod 3.$

For perfect reconstruction

Syn Matrix x Analy Matri (polyphase)

= — - —

Each row and column has exactly one entry $\in \mathbb{Z}^{-L}$

L: depends on D

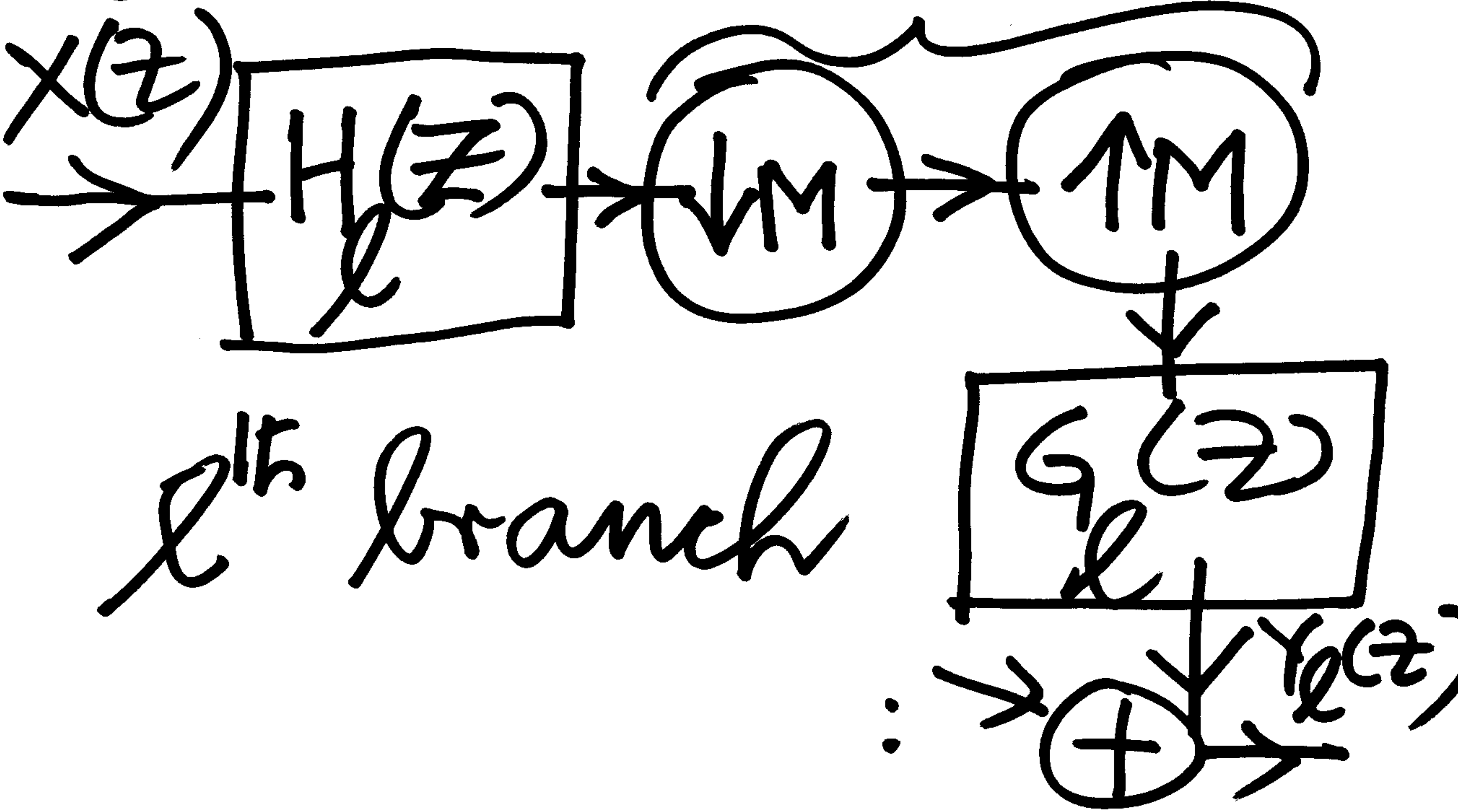
For example,
$D = 5$, $M = 3$
polyphase vectors
and matrices 3
written in z

then $L = 3$

If written in z,

$L = 1$

One branch:



ℓ^{th} branch

$$\longrightarrow \downarrow M \longrightarrow \uparrow M \longrightarrow$$

essentially multiplication by a periodic seq $p_M[n]$

$$P_M[n] = 1, \text{ n a multiple of M}$$

$$= 0 \text{ else}$$

Consider one period
$P_M(n)$ restricted
to $0 \ldots (M-1)$
Obtain its DFT

$$\tilde{P}_M[k] = \sum_{n=0}^{M-1} P_M[n] W_M^{-nk}$$

$$W_M = e^{j\frac{2\pi}{M}}$$

$$\tilde{P}_M[k] = 1$$

$$k = 0, \ldots, (M-1)$$

$$P_M[n] = \text{Inverse DFT} = \frac{1}{M}\sum_{k=0}^{M-1} 1 \cdot W_M^{nk}$$

for all n.

$$X(z) \longrightarrow \boxed{H_\ell(z)} \longrightarrow X(z)\,H_\ell(z)$$

— —

modulated by

$$\frac{1}{M} \sum_{k=0}^{M-1} W_M^{kn}$$

When we modulate a sequence by α^n, $z \leftarrow z\alpha^{-1}$ in the Z-Transform

$$X(z)H_l(z) \rightarrow \downarrow M \rightarrow \uparrow M \rightarrow$$

$$\frac{1}{M}\sum_{k=0}^{M-1} X(zW_M^{-k})\,H_l(zW_M^{-k})$$

$$Y_l(z) = G_l(z) \cdot \sum_{k=0}^{M-1} X\left(zW_M^{-k}\right) \cdot H_l\left(zW_M^{-k}\right)$$

l^{th} row of modulation matrix

$$= G_l(z)\left[H_l(zW_M^{-0})\ H_l(zW_M^{-1}) \cdots \cdots\cdots H_l(zW_M^{-(M-1)})\right]$$

For perfect reconstruction we first want alias cancellation

Alias cancellation means: no contribution from $X(zW_M^{-k}),\ \underline{\underline{k \neq 0}}$

$$\underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{\text{vector of } Y_\ell} = \begin{bmatrix} \text{Modulation} \\ \text{matrix} \\ \\ \end{bmatrix} \underbrace{\begin{bmatrix} \text{Input} \\ M \times 1 \\ \\ \end{bmatrix}}_{\text{modulation vector}}$$

Essentially we
ask for: first column of modulation matrix
is the only nonzero column

A more general condition:
Sums of columns in modal matrix $= 0 \quad \forall k \neq 0$.

Example
$M=3$ and
3 channels

Modulation matrix:

$$\begin{bmatrix} G_0(z) & 0 & 0 \\ 0 & G_1(z) & 0 \\ 0 & 0 & G_2(z) \end{bmatrix} \begin{bmatrix} H_0(z) & H_0(zW_3^{-1}) & H_0(z \end{bmatrix}$$

Similarly

H_1, H_2

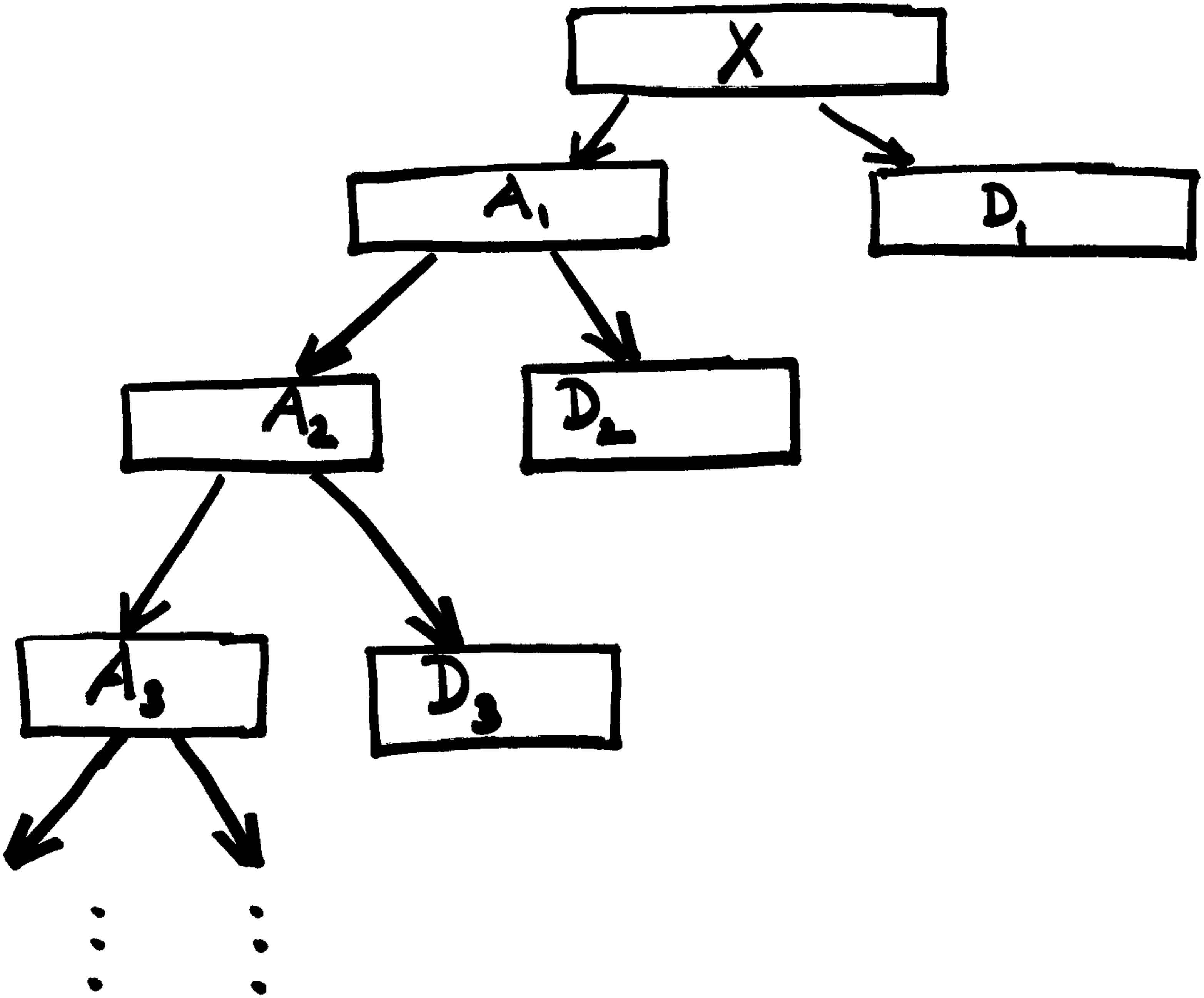
$$\sum_{l=0}^{2} G_l(z) H_l(zW_3^{-k}) = 0$$

$$k = 1, 2$$

APPLICATION OF WAVELE

IN

DATA MINING


X
A_1
D_1
A_2
D_2
A_3
D_3