Prof. V. Gadre. Date 5

LECTURE 36

THE POLYPHASE APPROACH
THE MODULATION APPROACH

$$x[n] \xrightarrow[\text{Z-trans}]{} X(Z)$$

appropr R_x

$$n \begin{cases} = 2m \\ = 2m+1 \end{cases}$$

Polyphase decomposition order 2.

To generalize:
Suppose order 3

$$n = \begin{cases} 3m & \text{over} \\ 3m+1 & \text{all integer} \\ 3m+2 & m \end{cases}$$

In general order M.

$$n = \begin{cases} Mm \\ Mm+1 \\ \vdots \\ Mm+(M-1) \end{cases}$$ over all integer m

To decompose $X(z)$ using polyphase decomposition of order M:

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n}$$

Now split $\sum_{n}$ into:

$$\sum_{n=-\infty}^{+\infty} \cdots (n) \cdots =$$

$$\sum_{l=0}^{M-1} \sum_{m=-\infty}^{+\infty} \cdots (Mm+l) \cdots$$

$$X(z) = \sum_{l=0}^{M-1} \sum_{m=-\infty}^{+\infty} x[Mm+l] \cdot z^{-(Mm+l)}$$

$$X(z) = \sum_{l=0}^{M-1} z^{-l} \Big\{ \underbrace{\sum_{m=-\infty}^{+\infty} x[Mm+l]\, z^{-Mm}}_{X_{l,M}(z^M)}$$

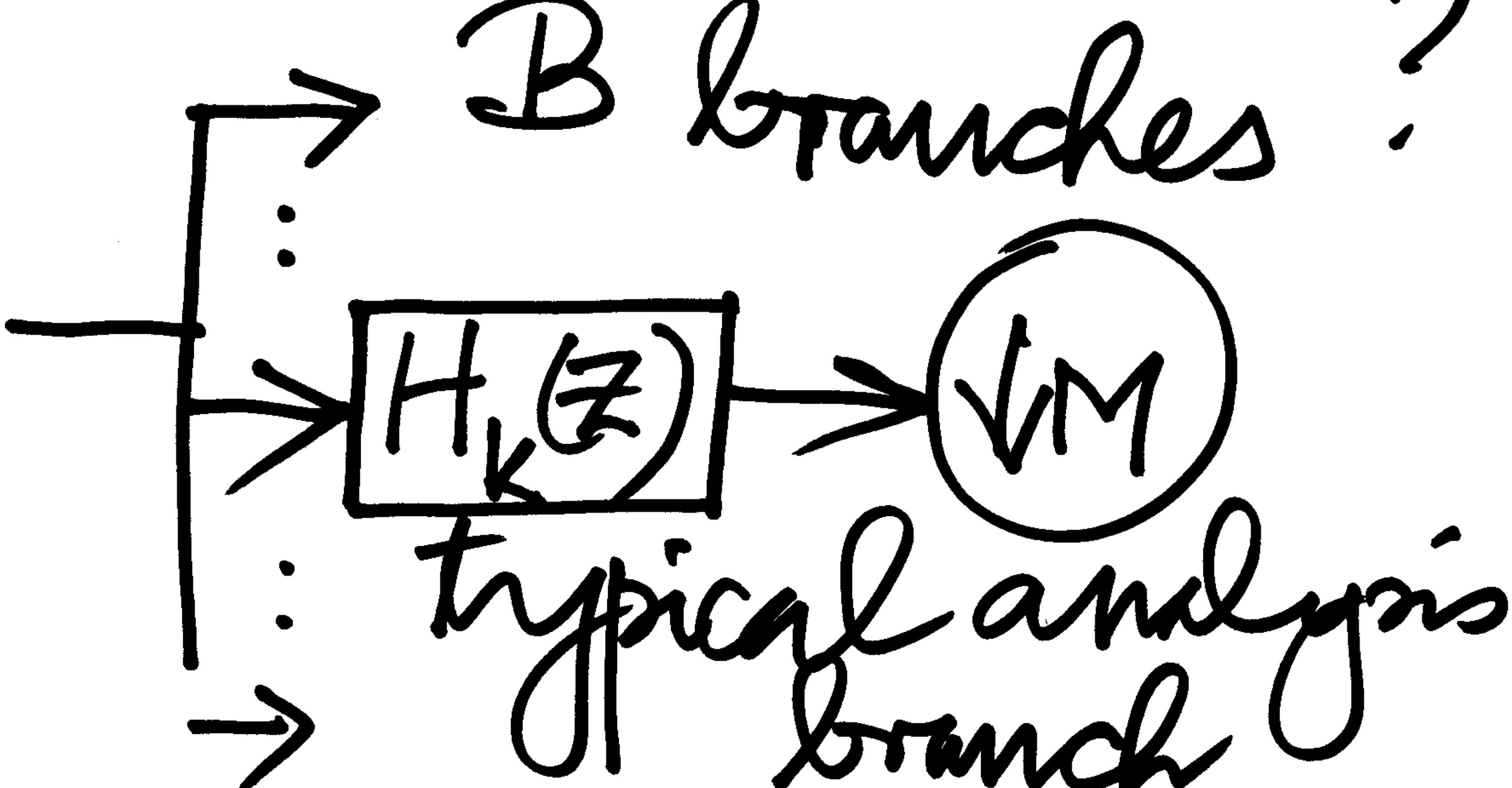
$$X_{l,M}(z) = \sum_{m=-\infty}^{+\infty} x[Mm+l]\, z^{-m}$$

$X_{l,M}(z)$ is essentially the z-transform of l^{th} polyphase component of $x[\cdot]$, order M.

$$X(z) = \sum_{l=0}^{M-1} z^{-l} X_{l,M}(z^M)$$

Analysis side:
General analysis branch in an M-band filter bank

M- band filter bank

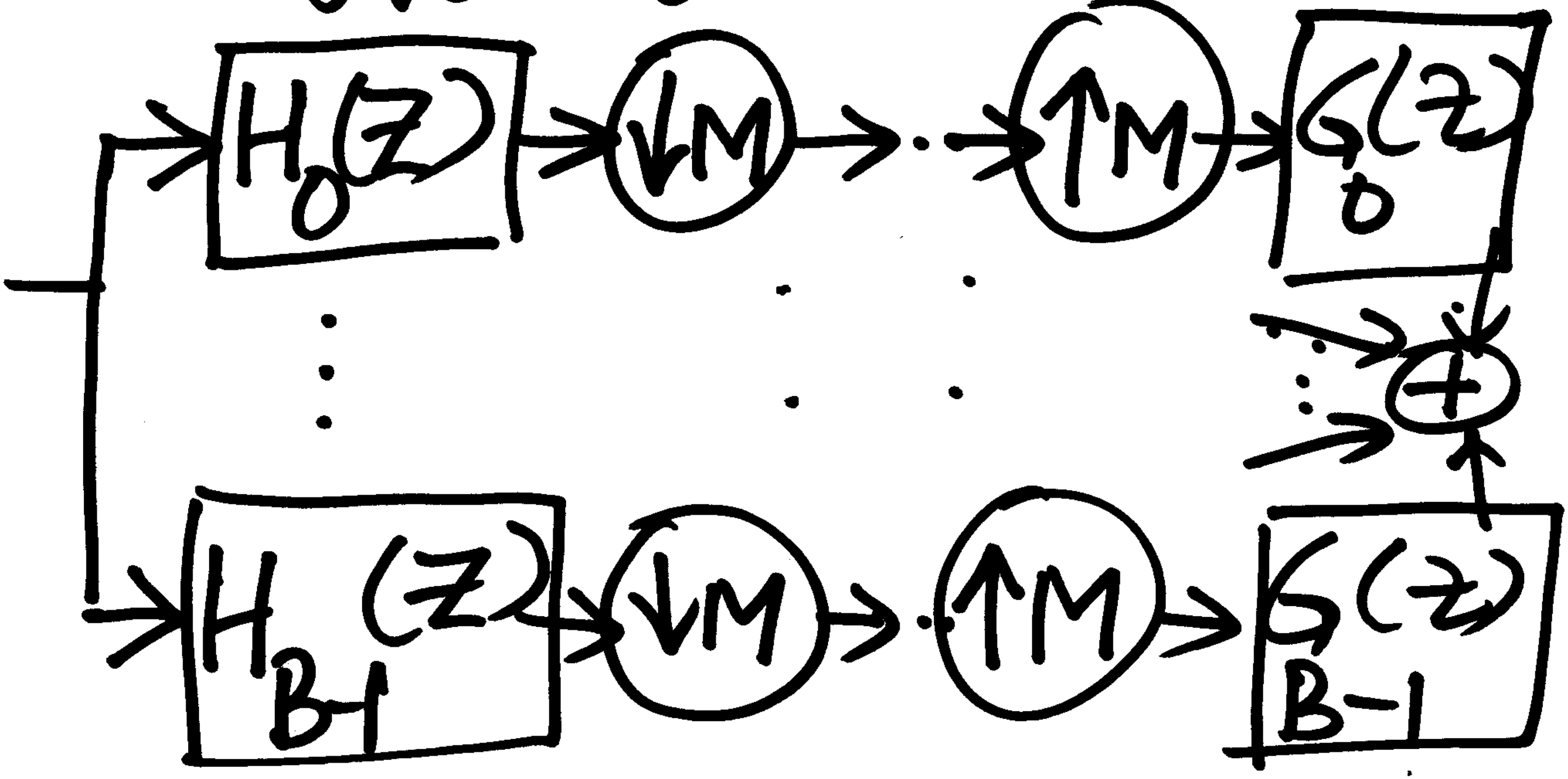


Typical synthesis branch:

$$\longrightarrow \uparrow M \longrightarrow \boxed{G_k(z)} \longrightarrow$$

B such branches

Overall :



$H_0(z)$
$\downarrow M$
$\uparrow M$
$G_0(z)$
$H_{B-1}(z)$
$\downarrow M$
$\uparrow M$
$G_{B-1}(z)$
+

$B = M$: Critically sampled M-band filter bank

$B < M$:

undersampled
M- band filter
bank

$B > M$:
oversampled
M-band filter
bank

Polyphase approach:
Decompose input, output, analysis filters, synthesis filters
— - —

-- into polyphase components of order M.

Consider the k^{th} branch.

$$
\begin{tikzcd}
\arrow[r, "X(z)"] & \boxed{H_k(z)} \arrow[r] & \downarrow M \arrow[r] & \uparrow M \arrow[d] \\
& & \arrow[l] \boxed{G_k(z)} &
\end{tikzcd}
$$

Z-domain analysis:

$$X(z) = \sum_{l=0}^{M-1} \left\{ x_{l,M}(z^M) \right\} z^{-l}$$

We could similarly write :

$$H_k(z) = \sum_{l=0}^{M-1} z^{-l} H_{k,l,M}(z^M)$$

$$X(z)H_k(z) \longrightarrow \downarrow M \longrightarrow$$

At this point,
0^{th} ~~order~~ polyphase
components of order
M.

$$X(z)H_k(z) = \sum_{l_1=0}^{M-1}\sum_{l_2=0}^{M-1} z^{-l_1} \cdot z^{-l_2} \cdot X_{l_1,M}(z^M) \cdot H_{k,l_2,M}(z^M)$$

The J^{th} polyphase component results when $Z^{-l_1} Z^{-l_2}$

__ __ __ ___

$$= \bar{z}^{-(l_1+l_2)}$$

contributes $\left(\bar{z}^{M}\right)^{l_0}$

l_0 some integer

ℓ_1	ℓ_2
0	0
1	$M-1$
$\vdots$	$\vdots$
$M-1$	1

Essential

$\ell_2 =$

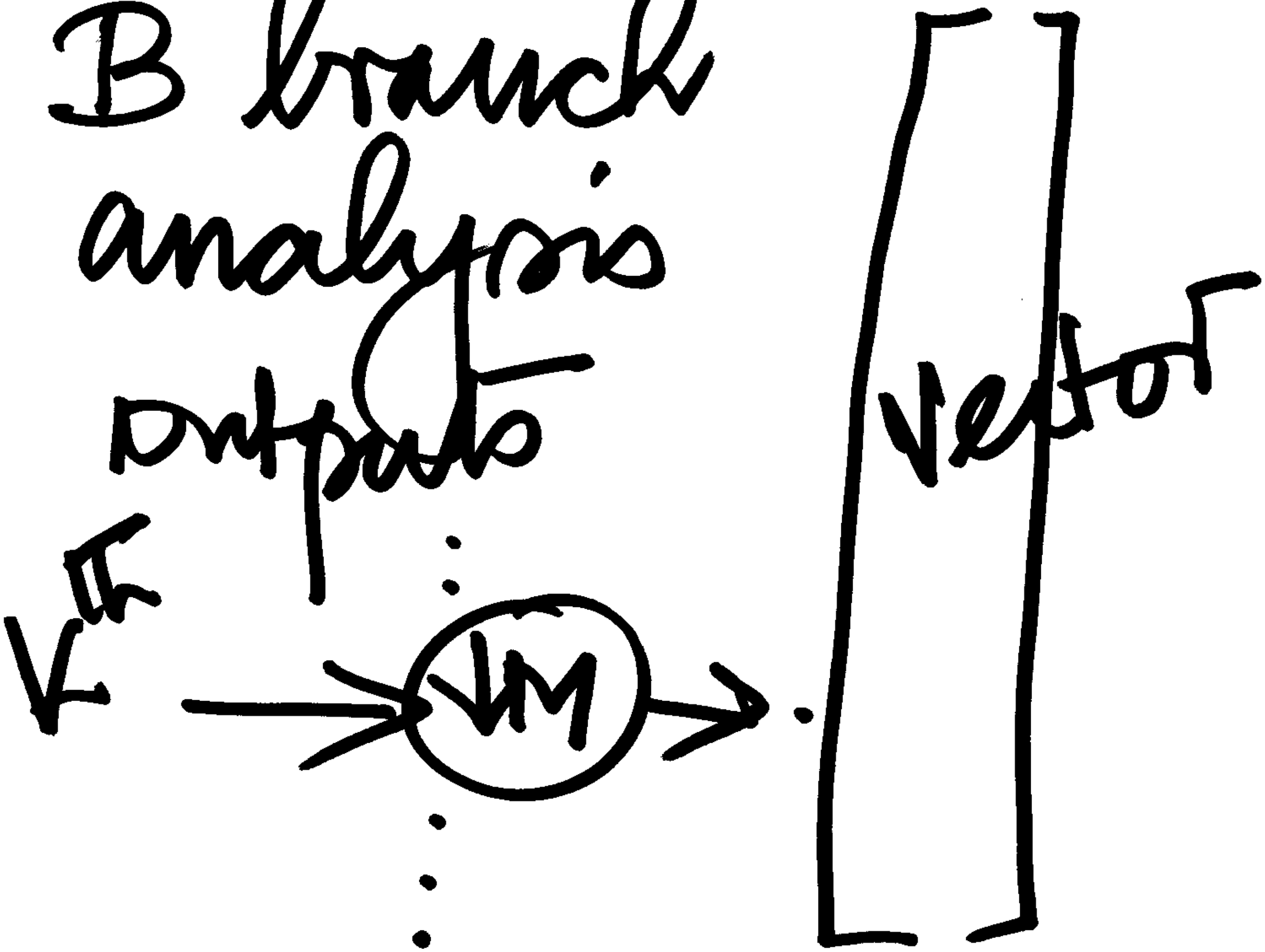
$(M-\ell_1)_M$

'modulo M'

$$X(z)H_k(z) \longrightarrow \boxed{\downarrow M}$$

$$X_{0,M}(z)H_{k,0,M}(z) + z^{-1}\sum_{l=1}^{M-1} X_{l,M}(z)H_{k,M-l,M}(z)$$

B branch analysis outputs



Analysis vector

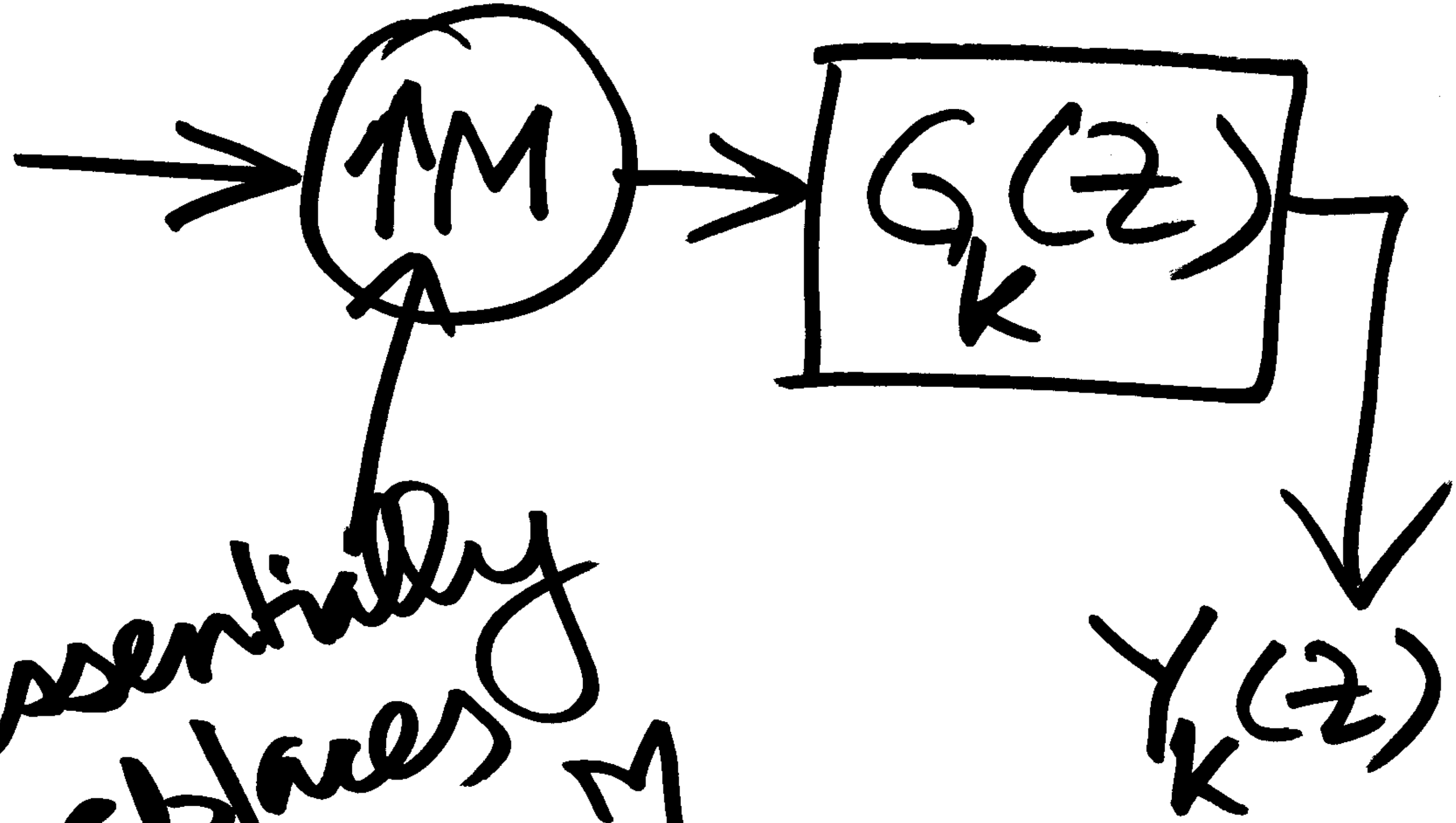
$$\begin{bmatrix} \\ - \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \underline{\quad k^{th} \text{ row} \quad} \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} x_{0,M} \\ x_{1,M} \\ \vdots \\ x_{M-1,M} \end{bmatrix}$$

k^{th} row :

$$H_{k,0,M}(\cdot) \cdot \bar{z}^{-1} H_{k,M-1,M}(\cdot) \cdots\cdots \bar{z}^{-1} H_{k,1,M}(z)$$

Analysis polyphase matrix with K^{th} row as described

$B \times M$ size.



essentially replaces

$z \leftarrow z^M$

$$Y_k(z) = \sum_{l=0}^{M-1} z^{-l} Y_{k,l,M}(z^M)$$

$$= \sum_{l=0}^{M-1} z^{-l} Y_{k,l,M}(z^M)$$

$$G_k(z) = \sum_{l=0}^{M-1} \left\{ G_{k,l,M}(z^M) \right\} z^{-l}$$

$$Y_{k,l,M}\left(z^M\right)$$

$$= G_{k,l,M}\left(z^M\right)\left(\begin{array}{l}\because \text{Output of } k^{th} \\ \text{upsampler as is}\end{array}\right)$$

Output of k^{th} upsampler as is:

$$\left[\, k^{th} \text{ row of analysis polyphase matrix} \,\right] \begin{bmatrix} X_{0,M} \\ X_{1,M} \\ \vdots \\ X_{M-1,M} \end{bmatrix}$$

$$z \rightarrow z^{M}$$

Output $Y(z)$

$$= \sum_{k=1}^{B} Y_k(z)$$

$$Y_{\ell,M}(Z^M) = \sum_{k=1}^{B} G_{k,\ell,M}(Z^M) \begin{pmatrix} \text{output of} \\ k^{\text{th}} \text{ upsampler} \\ \text{as is} \end{pmatrix}$$

$$\begin{bmatrix} Y_{0,M} \\ Y_{1,M} \\ \vdots \\ Y_{M-1,M} \end{bmatrix} = \begin{bmatrix} \text{SYNTHESIS POLYPHASE MATRIX} \\ M \times B \\ \text{Polyphase components of } g_k \text{'s.} \end{bmatrix}$$

Output polyphase components

l^{th} row of synthesis polyphase matrix =

$$\left[G_{0,l,M}(z^M) \; \cdots\cdots \; G_{B-1,l,M}(z^M) \right]$$

$$\underbrace{\begin{bmatrix}\text{Output} \\ \text{polyphase} \\ \text{vector}\end{bmatrix}(z^M)}_{M \times 1} = \underbrace{\begin{bmatrix}\text{Polyph} \\ \text{Synthesis} \\ \text{matrix}\end{bmatrix}(z^M)}_{M \times B} \underbrace{\begin{bmatrix}\text{Polyp} \\ \text{Analys} \\ \text{Matrix}\end{bmatrix}(z^M)}_{B \times M} \underbrace{\begin{bmatrix}\text{Inp} \\ \text{Poly} \\ \text{vector}\end{bmatrix}(z^M)}_{M \times 1}$$

Polyphase approach seen in detail

Modulation approach:

Treat downsampling as a sum of modulations example $M = 2$

$$\rightarrow \downarrow 2 \rightarrow \uparrow 2 \rightarrow$$

101010101010

multiplication by

multiplying sequence

$$1 \; \underbrace{0 \quad\quad 0}_{(M-1)} \; 1 \; \underbrace{0 \ldots 0}_{(M-1)} \; 1 \ldots$$

$\uparrow$

multiples of M