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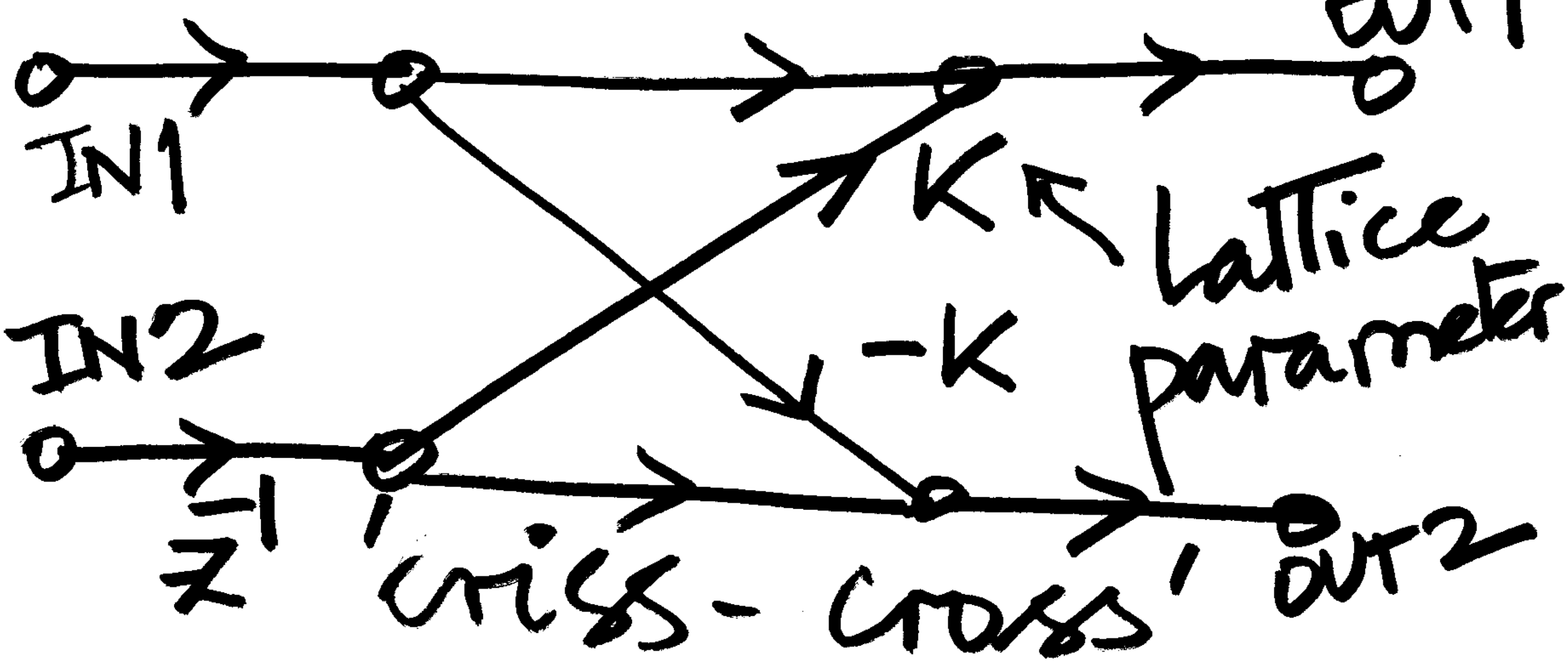
Date. 30-3-10

LECTURE 35

THE 'LIFTING' STRUCTURE AND POLYPHASE MATRICES

Module: 'a LATTICE STAGE'



We seek a relation between

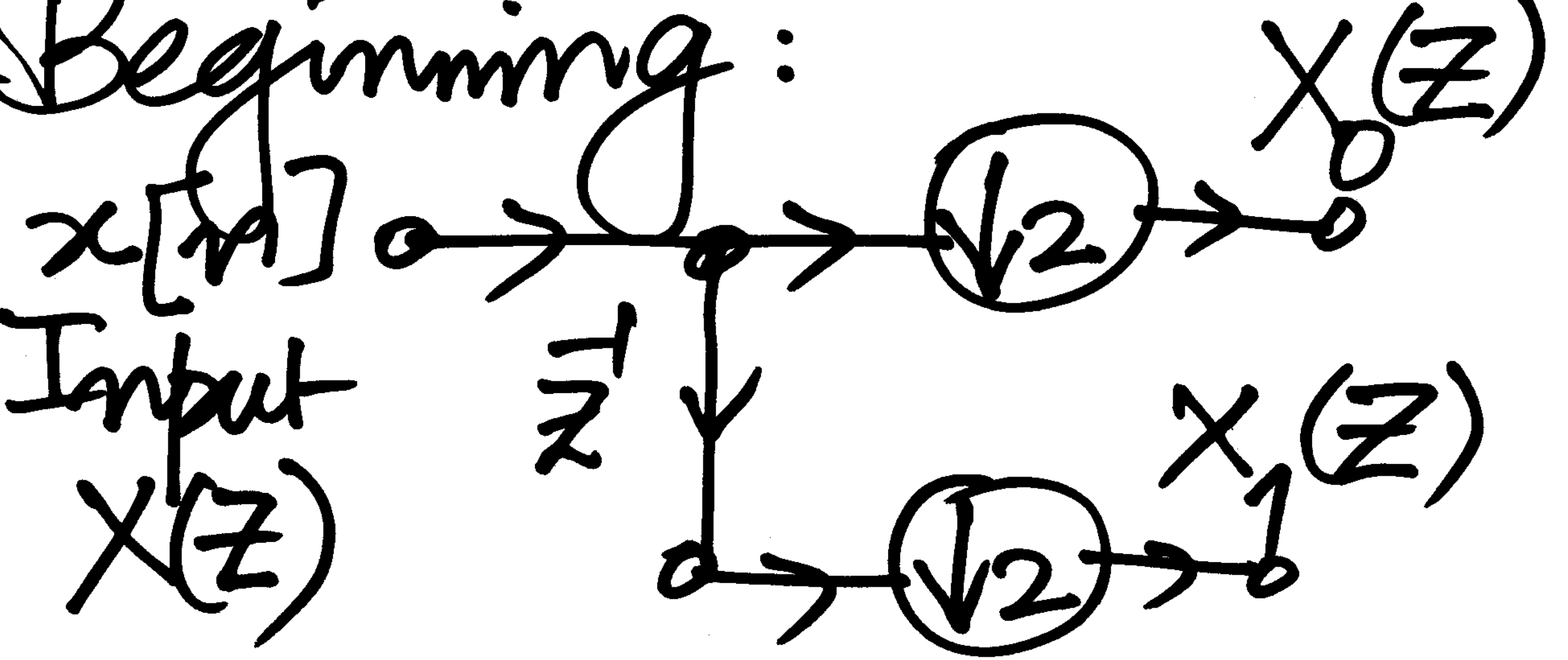
$X(z)$, $X_0(z)$,

$X_1(z)$

'POLYPHASE'

Beginning:

related to



Graphically:

$x[n] \rightarrow \qquad \cdots \quad x_{-1} \; x_0 \; x_1 \; x_2 \cdots$

$n \rightarrow \; -6 \; -5 \; -4 \; -3 \; -2 \; -1 \; 0 \; 1 \; 2 \; 3$

On the x_0 branch:

$$\ldots \; x_{-4} \; x_{-2} \; \underset{\substack{\uparrow \\ 0}}{x_0} \; x_2 \; x_4 \; x_6 \; \ldots$$

$x_0[n]$

On the x_1 branch:

$$\cdots\ x_{-3}\quad x_{-1}\quad \underset{\substack{\uparrow \\ 0}}{x_1}\quad x_3\quad x_5\quad x_7\ \cdots$$

$x_1[n]$

x is obtained by interleaving the sequence on X_0 branch and that on X_1 branch.

$$x_0[n] = x[2n]$$
$$\forall n \in \mathbb{Z}$$
$$x_1[n] = x[2n+1]$$
$$\forall n \in \mathbb{Z}$$

It is easy to see:

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n]\, z^{-n}$$

can be decomposed:

$$= \sum_{n=-\infty}^{+\infty} x[2n] z^{-2n}$$

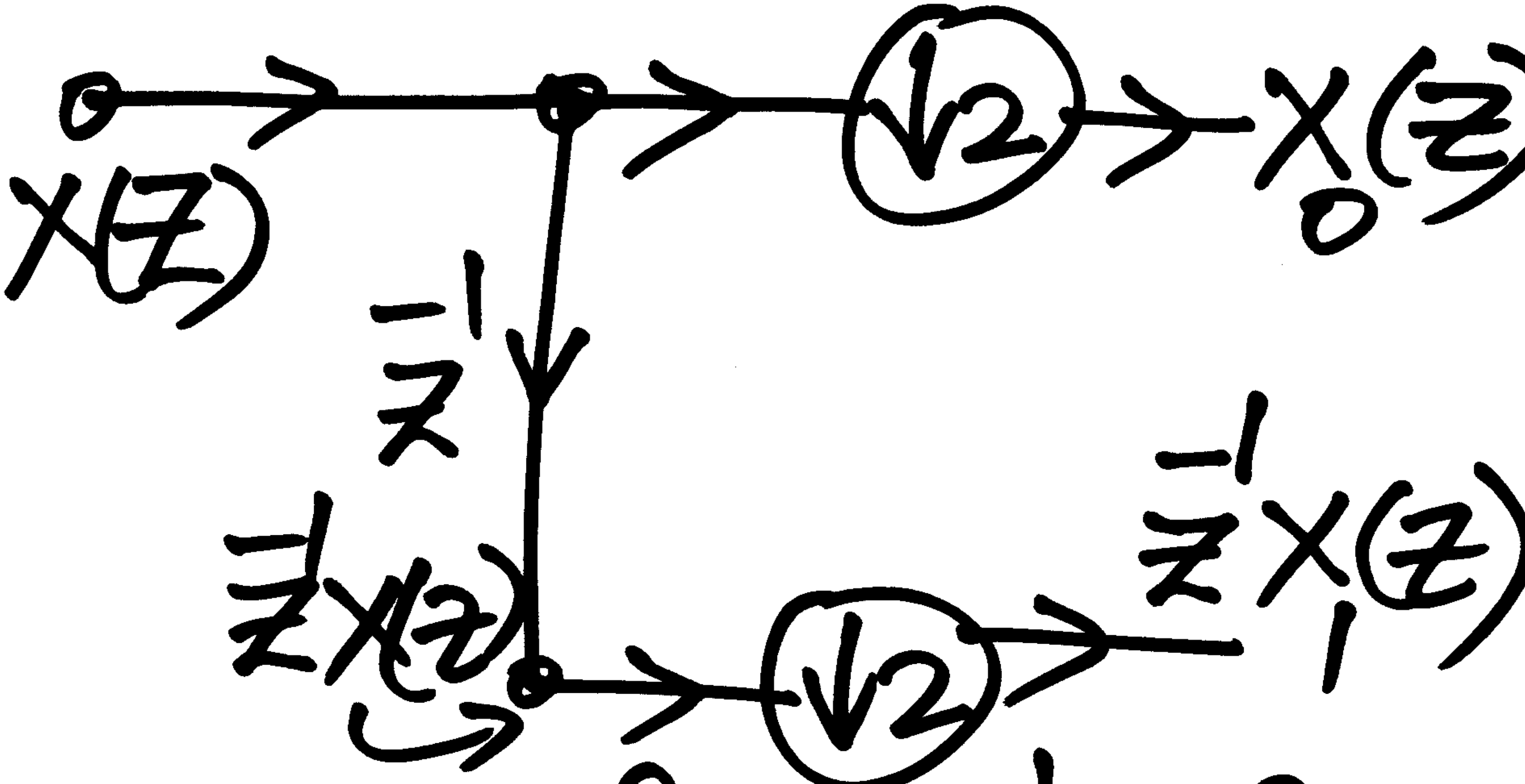
$$+ \sum_{n=-\infty}^{+\infty} x[2n+1] z^{-(2n+1)}$$

$$= \sum_{n=-\infty}^{+\infty} x_0[n] \left(z^2\right)^{-n}$$

$$+ \sum_{n=-\infty}^{+\infty} x_1[n] \cdot z^{-1} \cdot \left(z^2\right)^{-n}$$

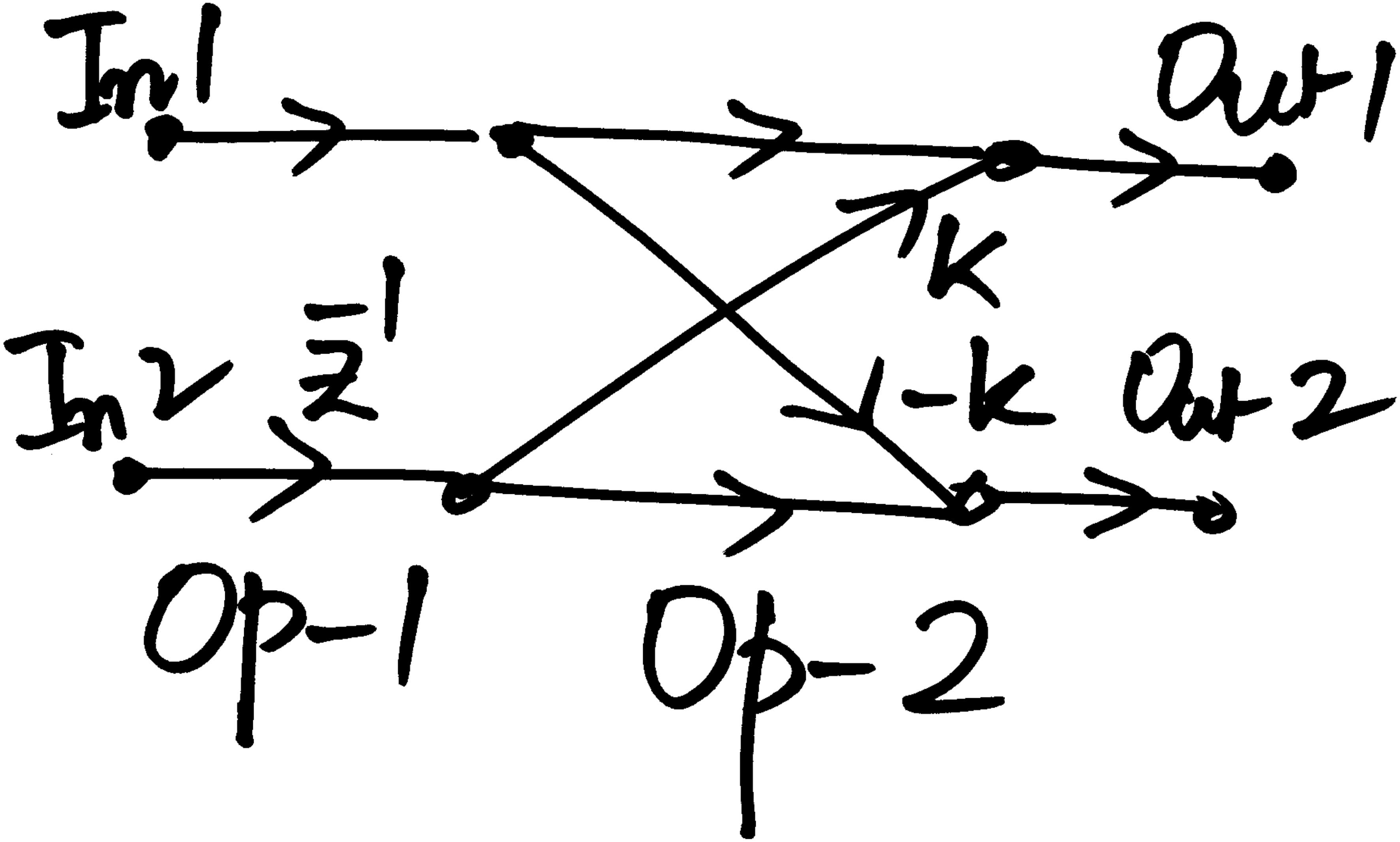
$x_0[n]$ and $x_1[n]$ are called the 'polyphase components' of $x[n]$.

$$X(z) = X_0(z^2) + z^{-1} X_1(z^2)$$



$$X(z) = X_0(z^2) + z^{-1} X_1(z^2)$$

Each stage of lattice: 2x2 matrix operation on polyphase components


In 1
In 2
z^{-1}
k
1-k
Out 1
Out 2
Op-1
Op-2

Matrix Corresponding to Op-1:

$$\begin{bmatrix} \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} In1 \\ In2 \end{bmatrix}$$

matrix corresponding to Op-2:

$$\begin{bmatrix} Out1 \\ Out2 \end{bmatrix} = \begin{bmatrix} 1 & K \\ -K & 1 \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

$$\begin{bmatrix} Out2 \\ Out1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & K \\ -K & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix}}_{\text{Polyphase matrix}} \begin{bmatrix} In1 \\ In2 \end{bmatrix}$$

We could consider decomposing

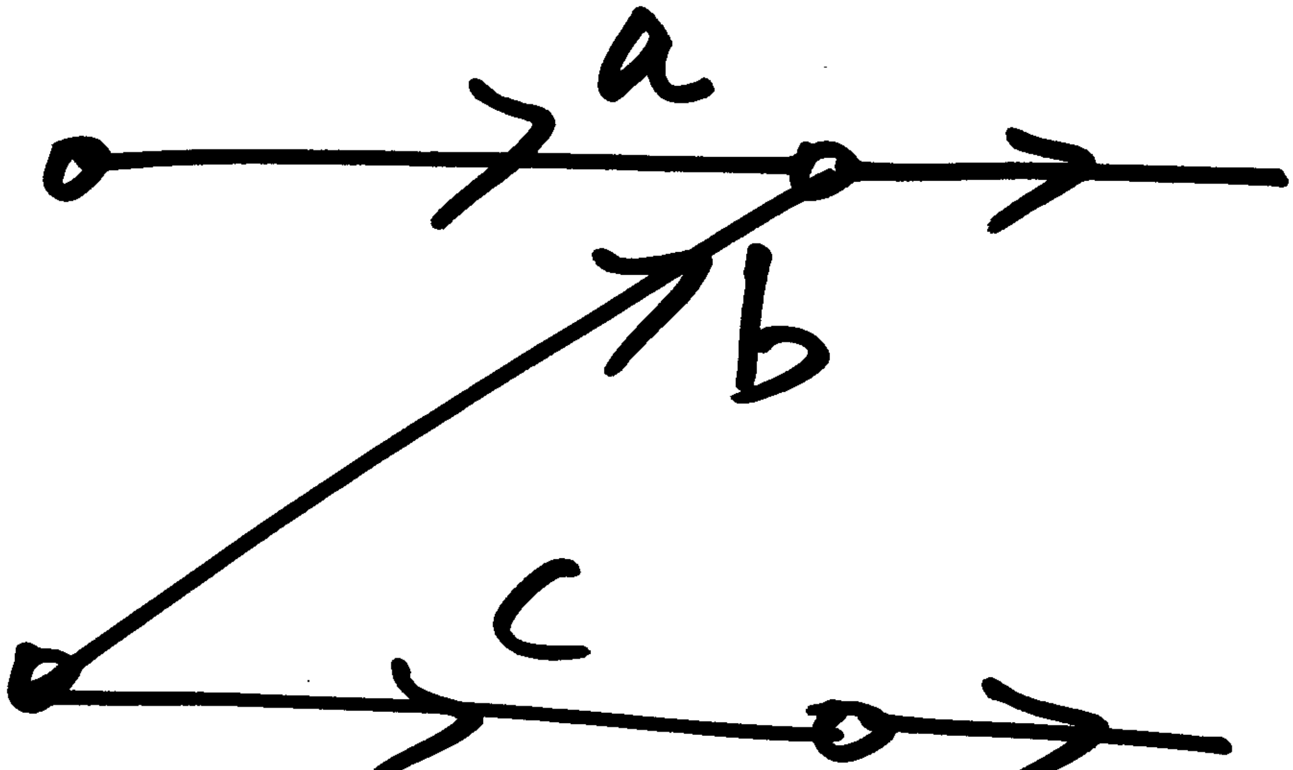
$$\begin{bmatrix} 1 & K \\ K & 1 \end{bmatrix} = \begin{bmatrix} \cdot & \cdot \\ 0 & \cdot \end{bmatrix} \begin{bmatrix} \cdot & 0 \\ \cdot & \cdot \end{bmatrix}$$

upper triangular

lower triangular

Consider upper triangular matrix

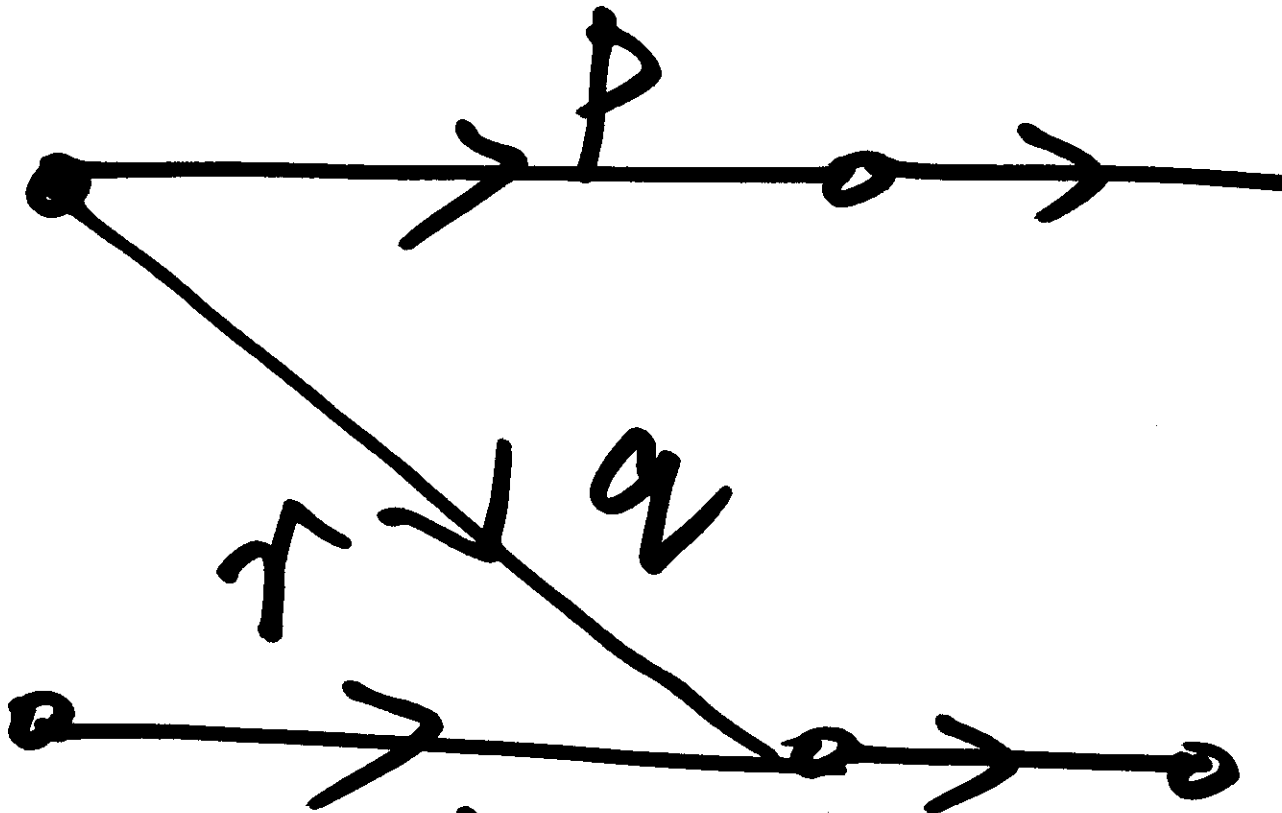
$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$



'upper triangular'
computation

'lower triangular' matrix

$$\begin{bmatrix} p & 0 \\ q & r \end{bmatrix}$$



'lower triangular computation'

Let if possible

$$\begin{bmatrix} 1 & k \\ -k & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \begin{bmatrix} p & 0 \\ q & r \end{bmatrix}$$

4 equations, 6 variables

$$\left.\begin{aligned} ap+bq &= 1 \\ cq &= -k \\ br &= k \\ cr &= 1 \end{aligned}\right\}$$

We exploit the degrees of freedom by choosing very simple values for some entries

Consider choosing

$p = 1$

$r = 1$

whereupon --

$$a + bq = 1$$
$$cq = -k$$
$$b = k$$
$$c = \underline{1}$$

We have:

$$a + k(-k) = 1$$

$$\Rightarrow a = 1 + k^2$$

$b = k,$

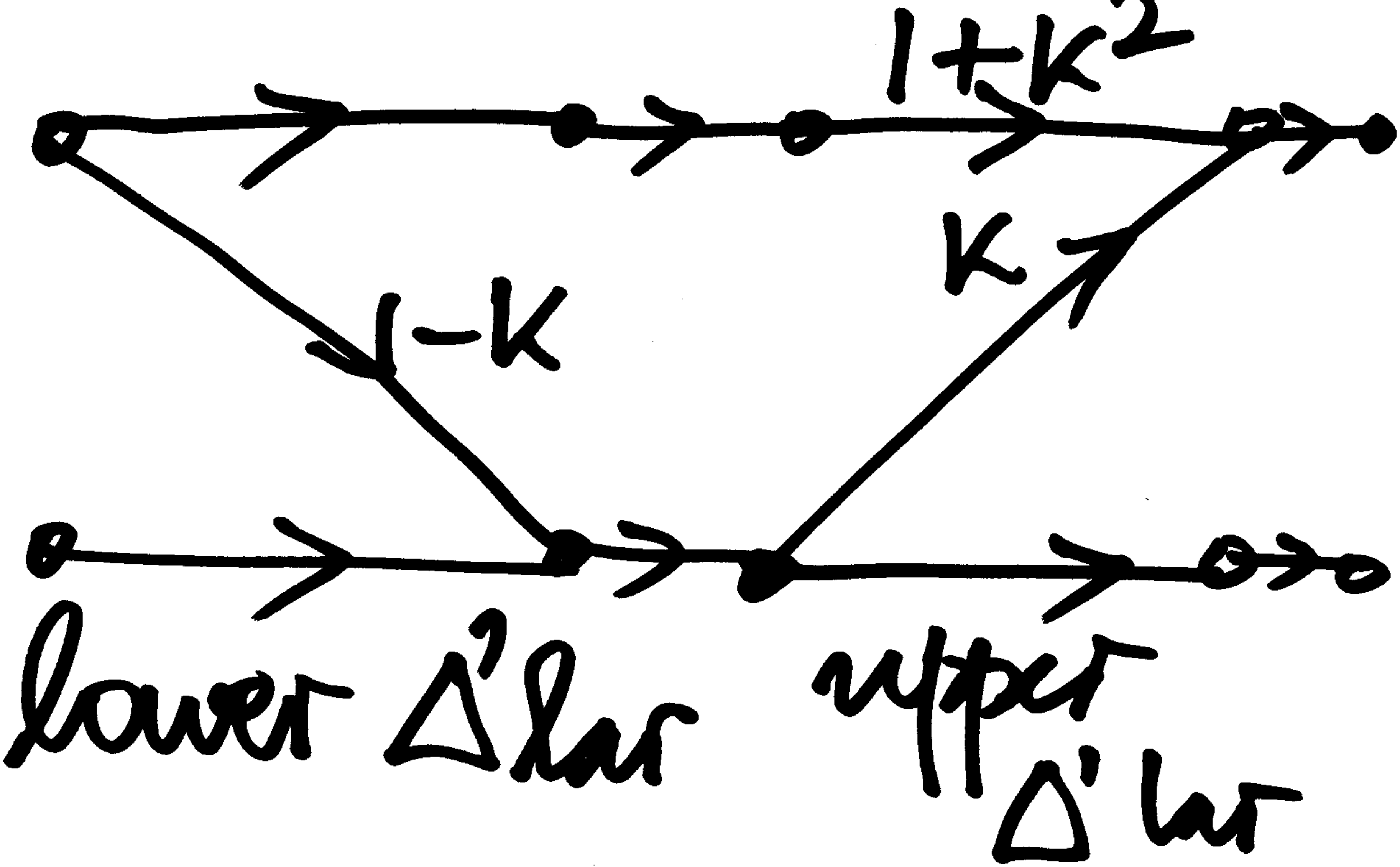
$c = 1, \quad q = -k$

$$\begin{bmatrix} 1 & k \\ -k & 1 \end{bmatrix} = \text{Verified}$$

$$\begin{bmatrix} 1+k^2 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -k & 1 \end{bmatrix}$$

Computationally we have broken down

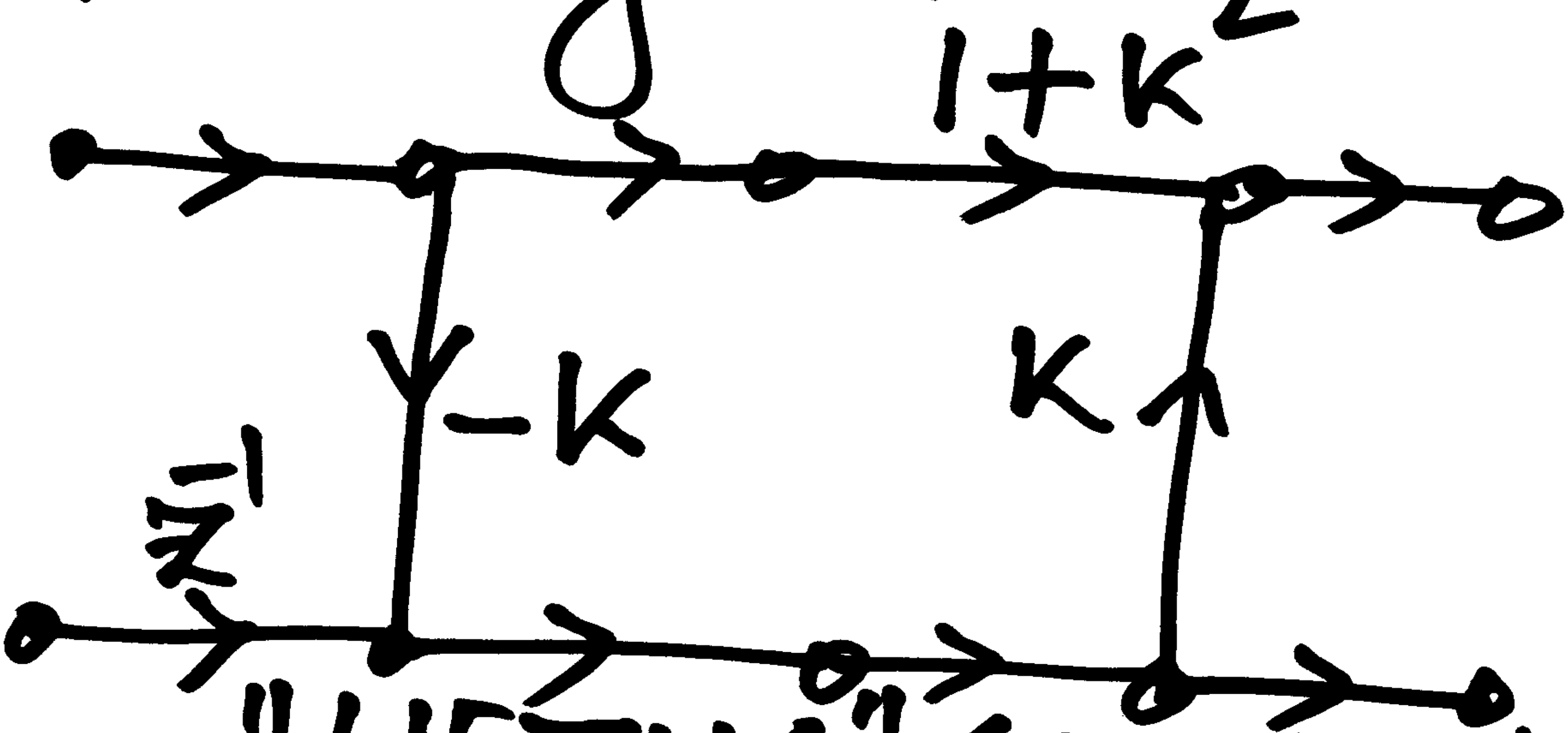
into ....



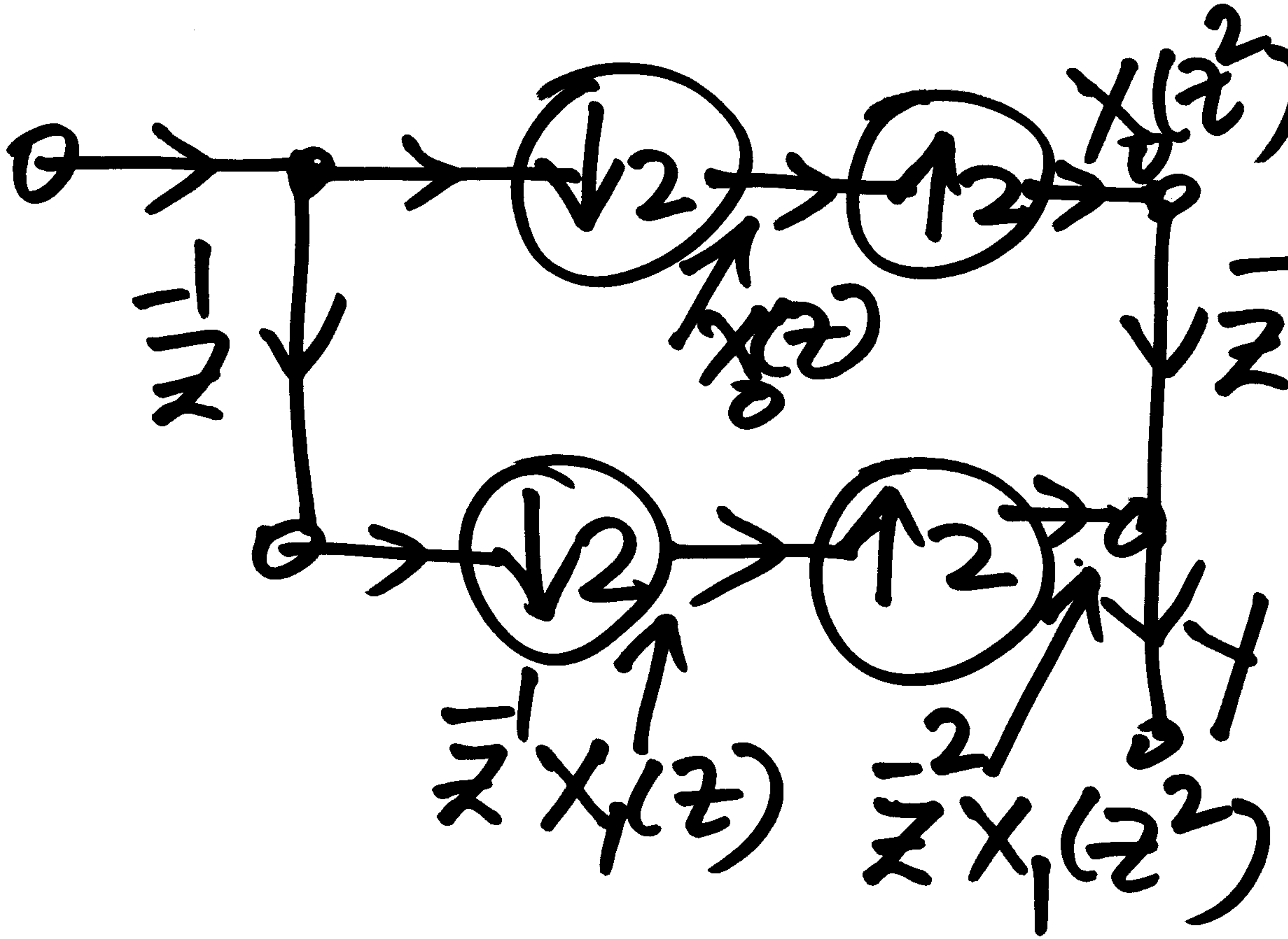
$1+K^2$
K
$1-K$
lower Δ'lar
upper Δ'lar

Redrawing, and including delay operator

All together:



"LIFTING" STAGE



↓2
↑2
$X_0(z^2)$
$X_0(z)$
z^{-1}
z^{-1}
↓2
↑2
$z^{-1}X_1(z)$
$z^{-2}X_1(z^2)$
Y

$$Y(z) = z^{-1} X_0(z^2) + z^{-2} X_1(z^2)$$

$$= z^{-1} X(z)$$