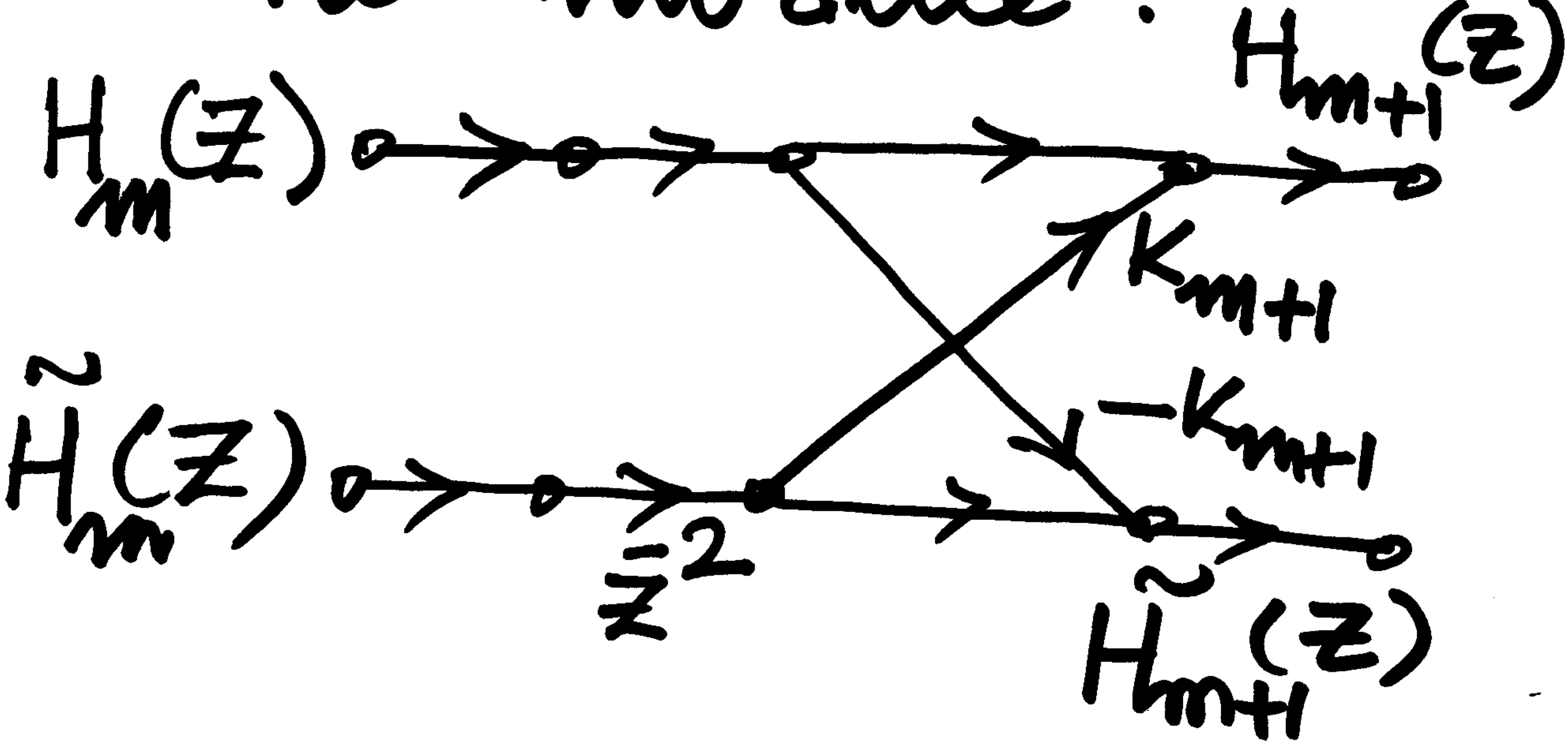
Prof V. Gadre - Lec - 34
Date 23-3-10

LECTURE 34

CONSTRUCTING THE LATTICE AND ITS VARIANTS.

One 'module':



Given:

$$\tilde{H}_m(z) = z^{-(2m-1)} H_m(-z^{-1})$$

we have ....

$$\tilde{H}_{m+1}(z) = z^{-(2(m+1)-1)} H_{m+1}(-z^{-1})$$

Conjugate Quadrature relation is carried

Construction of lattice structure:

Go the other way:

$$H_{m+1} \longrightarrow H_m$$

$$\tilde{H}_{m+1} \qquad \tilde{H}_m$$

Inductive (recursive) lattice relations:

$$\begin{matrix} H_n \\ \tilde{H}_n \end{matrix} \longrightarrow \begin{matrix} H_{n+1} \\ \tilde{H}_{n+1} \end{matrix}$$

$$H_{m+1}(z) = H_m(z) + K_{m+1} z^{-2} \tilde{H}_m(z)$$

$$\tilde{H}_{m+1}(z) = z^{-2} \tilde{H}_m(z) - K_{m+1} H_m(z).$$

Consider:

$$H_{m+1}(z) - K_{m+1}\tilde{H}_{m+1}(z)$$

$$- - - -$$

Equal to:

$$H_m(z) + K_{m+1}^2 H_m(z) + K_{m+1} z^{-2} \tilde{H}_m(z) - K_{m+1} z^{-2} \tilde{H}_m(z)$$

$$= \left(1 + K_{m+1}^2\right) H_m(z)$$

(eliminated $\tilde{H}_m(z)$)

$$H_m(z) = \frac{H_{m+1}(z) - K_{m+1}\tilde{H}_{m+1}(z)}{1 + K_{m+1}^2}$$

Obtain K_{m+1}?

look at forward recursion!

$$H_{m+1}(z) \triangleq \text{length } 2(m+1)$$

$$= \underbrace{H_m(z)}_{\text{length } 2m} + K_{m+1}\, z^{-2}\, \underbrace{\tilde{H}_m(z)}_{\text{length } 2m}$$

We shall show inductively:

Coeff of z^0 in $H_m(z) = 1$. $\forall m$.

Basis step: true:

System functions:

$$1 + K_1 z^{-1} \leftrightarrow H_1(z)$$

$$K_1 + z^{-1} \quad (\text{true})$$

Let it be true for $H_m(z)$, $m \geq 1$.

$$H_{m+1}(z) = H_m(z) + K_{m+1} \bar{z}^{-2} \tilde{H}_m(z)$$

This recursive step "carries" the z^{0} term from $H_m(z)$ to $H_{m+1}(z)$!

$$\tilde{H}_m(z) = z^{-(2m-1)} H_m(-z^{-1})$$

$$H_m(z) = 1 + h_1^{(m)} z^{-1} + h_2^{(m)} z^{-2} + \cdots + h_{2m-1}^{(m)} z^{-(2m-1)}$$

$$\tilde{H}_m(z) = z^{-(2m-1)} \left\{ 1 - h_1^{(m)} z + h_2^{(m)} z^2 \cdots - h_{2m-1}^{(m)} z^{(2m-1)} \right\}$$

$$\tilde{H}_m(z) =$$

$$-h_{2m-1}^{(m)} + \cdots + h_2^{(m)} z^{-(2m-1)+2}$$

$$\cdots + z^{-(2m-1)}$$

$\underline{\underline{\text{important}}}$

As a consequence of

$$H_{m+1}(z) = H_m(z) + \underbrace{K_{m+1} z^{-2} \tilde{H}_m(z)}_{\text{Contributes } z^{-(2m+1)} \text{ Term}}$$

Coeff of $z^{-(2m+1)}$

(highest neg power of z)

$= K_{m+1}$

Once we know K_{m+1}
we can 'peel'
off one module:

H_{m+1} : known

$\tilde{H}_{m+1}$: can be constructed

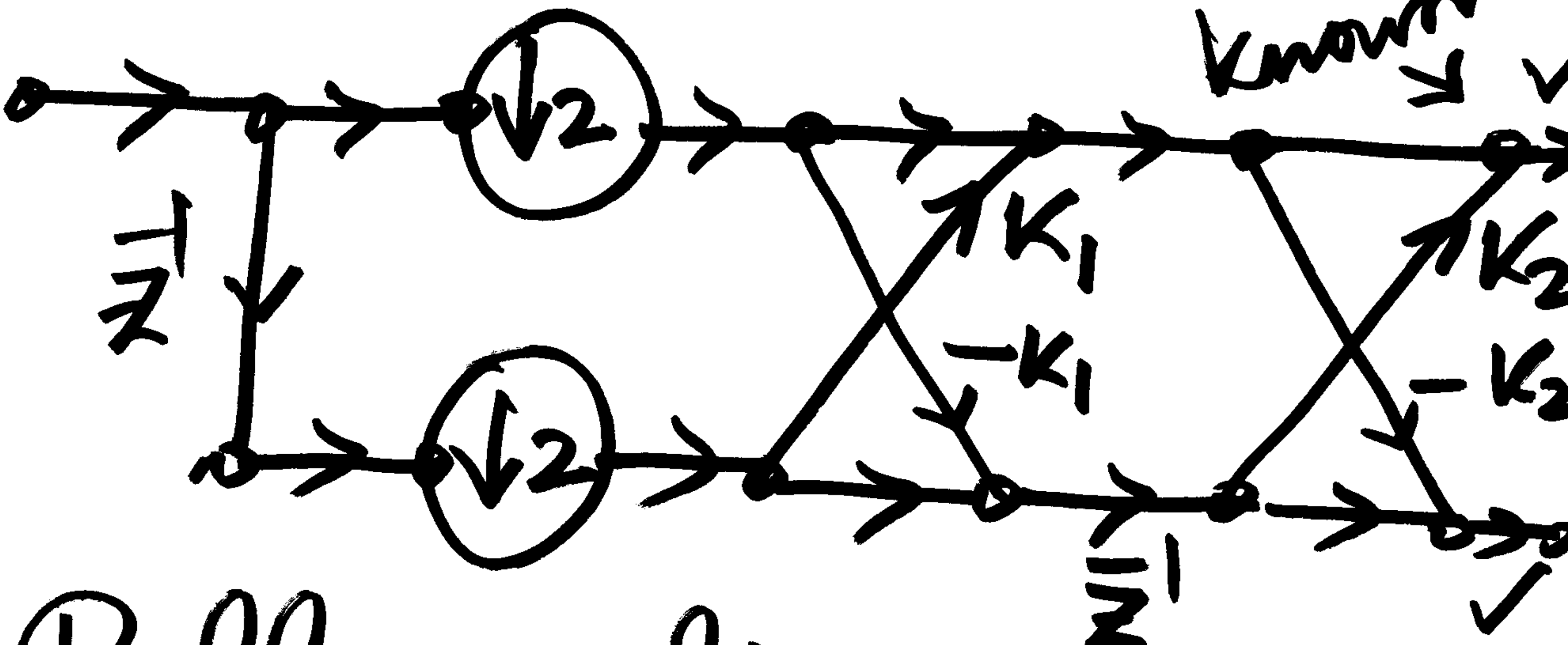
$$\frac{H_{m+1}(z) - K_{m+1}\tilde{H}_{m+1}(z)}{1 + K_{m+1}^2}$$

gives $H_m(z)$.

Example:

length 4 filter:

$$1 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3}$$



Problem: obtain K_2, K_1!

Given the length 4 filter

$$1 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3}$$

This is k_2!

$$H_2(z) = 1 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3}$$

$$\tilde{H}_2(z) = z^{-3} H_2(-z^{-1})$$

$$= z^{-3}\left\{1 - h_1 z + h_2 z^2 \pm h_3 z^3\right\}$$

$$\tilde{H}_2(z) =$$
$$-h_3 + h_2 z^{-1} - h_1 z^{-2} + z^{-3}.$$

Consider numerator of "notional" $H_1(z)$!

$$H_1(z) = \frac{H_2(z) - K_2 \tilde{H}_2(z)}{1 + K_2^2}$$

$$H_2(z) - K_2 \tilde{H}_2(z) \qquad K_2 = h_3$$

$$1 + h_1 z^{-1} + \cancel{h_2 z^{-2}} + \cancel{h_3 z^{-3}}$$

$$+ h_3^2 - h_3 h_2 z^{-1} + \cancel{h_1 h_3 z^{-2}} - \cancel{h_3 z^{-3}}$$

0 because: 0

$1 \quad h_1 \quad h_2 \quad h_3$

orthogonal to $\left\{ 1 \quad h_1 \quad h_2 \quad h_3 \right.$

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means:

$$h_2 + h_3 h_1 = 0$$

Remaining:

$$(1+h_3^2) + (h_1 - h_2 h_3) z^{-1}$$

The length has reduced by 2.

$$H_1(z) = \frac{(1+h_3^2) + (h_1 - h_2 h_3)\, z^{-1}}{(1+h_3^2)}$$

$$= 1 + \frac{h_1 - h_2 h_3}{1 + h_3^2} z^{-1}$$

K_1

The backward recursion

$$H_m(z) = \frac{H_{m+1}(z) - K_{m+1}\tilde{H}_{m+1}(z)}{1 + K_{m+1}^2} \quad ---$$

-- is effected by:

1. K_{m+1} is the coefficient of highest negative power of z^{-1} (in $H_{m+1}(z)$)

② As long as this coefficient is real, the denominator:

$\left(1 + K^2_{m+1}\right)$ poses no problem!

3 In the numerator

$$H_{m+1}(z) - K_{m+1}\tilde{H}_{m+1}(z)$$

there is a length reduction of $\underline{\quad 2 \quad}$.

Reduction by 1 is easy:

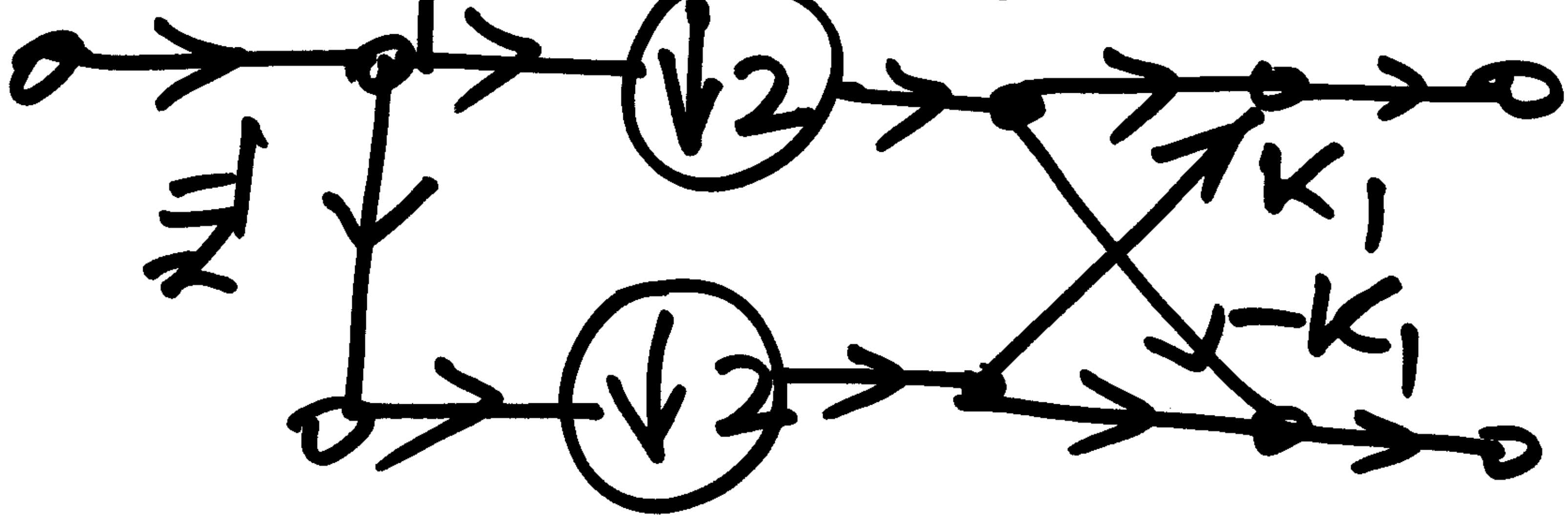
if occurs by cancellation of highest power of z^{-1}

Additional reduction in degree by 1: attributed to orthogonality to even translates

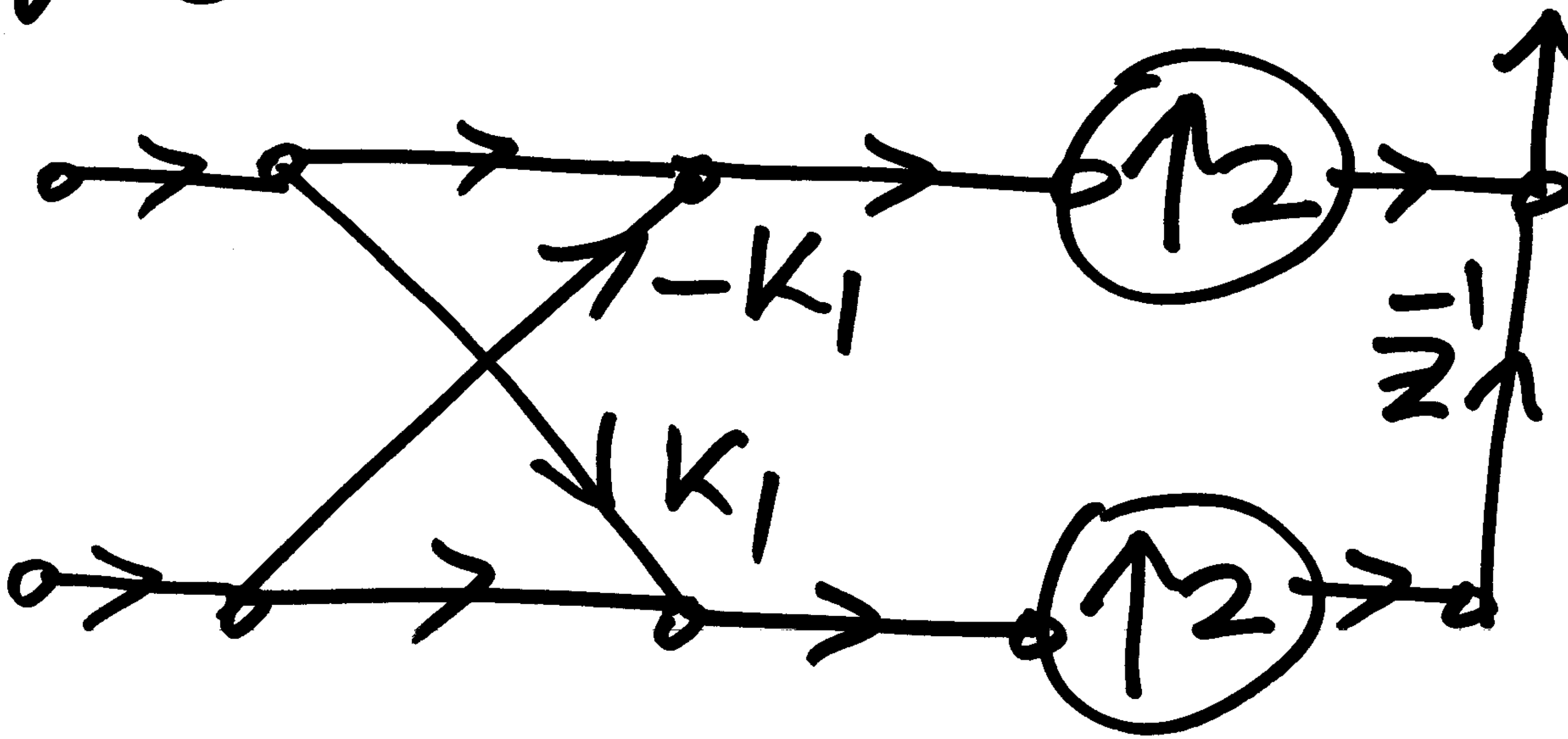
Synthesis variant
essentially the transpose of the analysis lattice.

Length 2 synthesis:

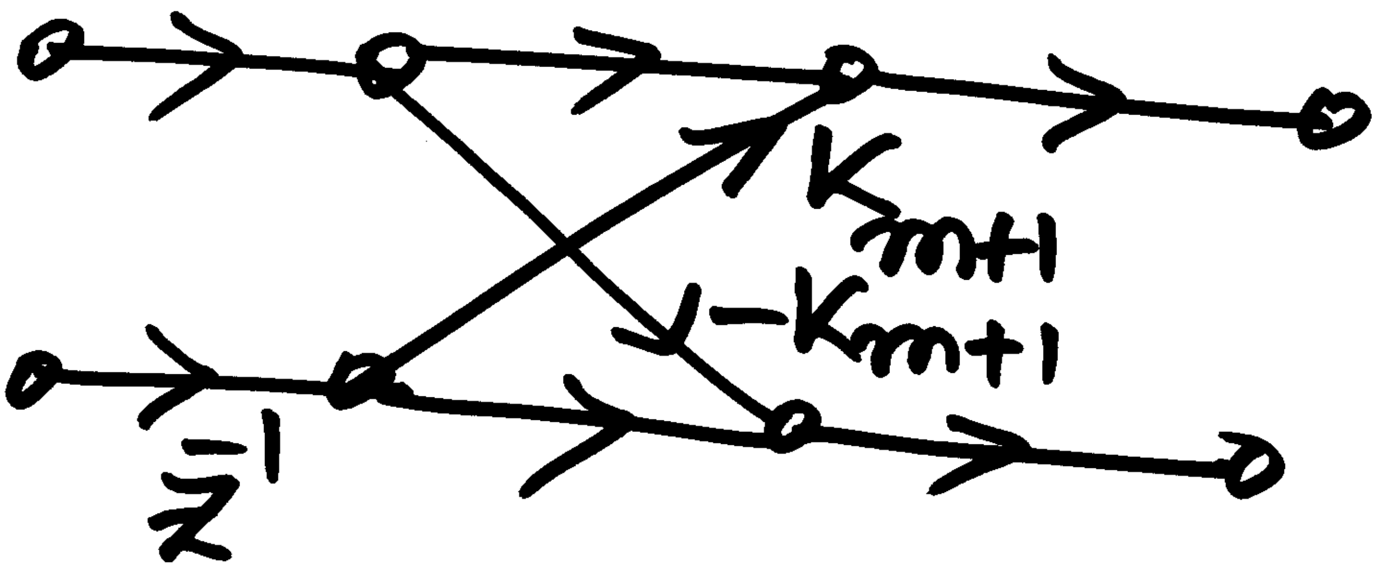
Transpose of:



i.e.:



Inductive Analysis module:



Inductive Synthesis module: