Prof V. Gadre. Date. 23

LECTURE 33

THE LATTICE STRUCTURE FOR ORTHOGONAL FILTER BANKS

'LATTICE':
homogeneous (modular) repetitive pattern/structure.

Haar Analysis Filter Bank:

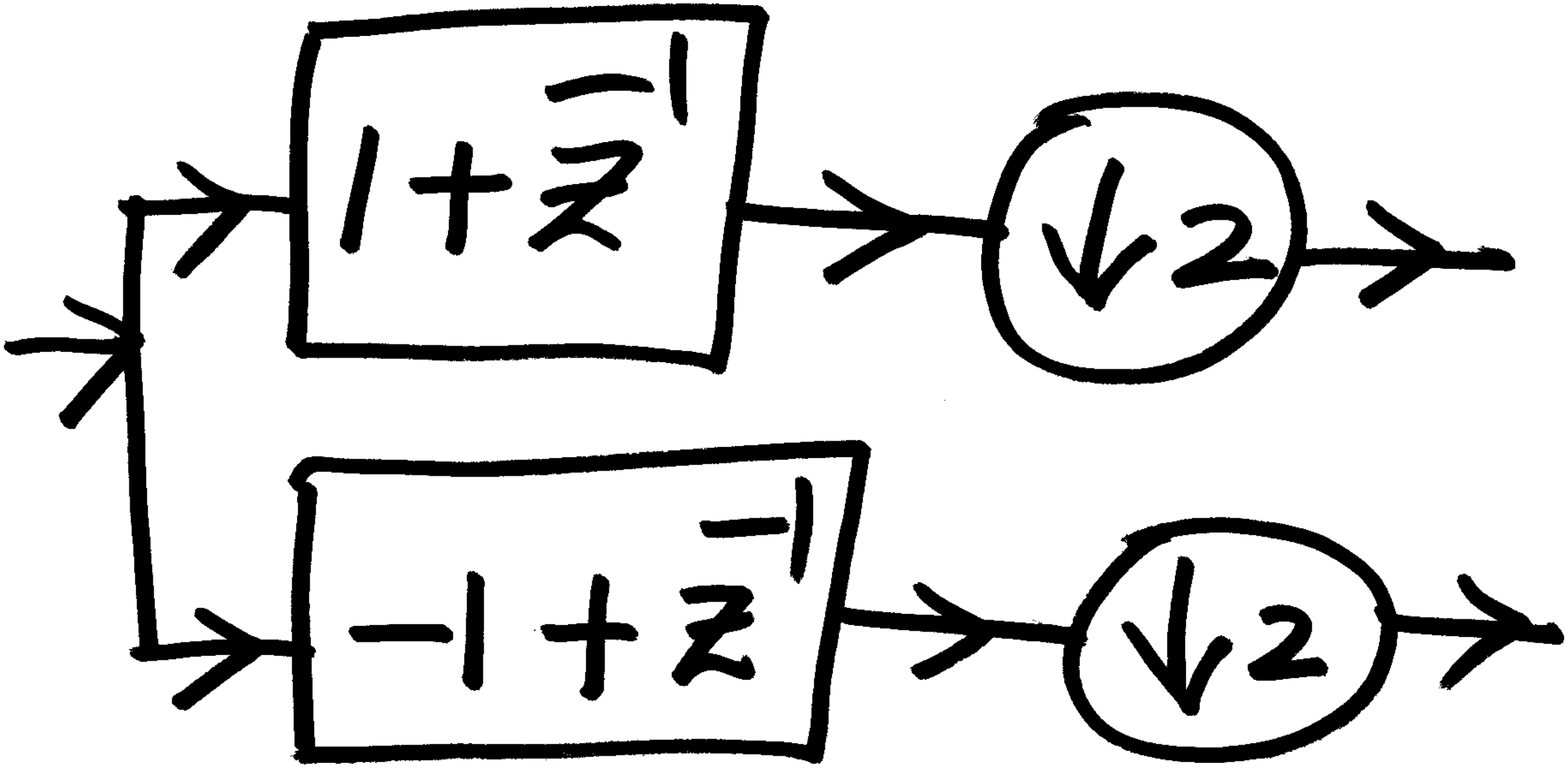
Lowpass $= 1 + z^{-1}$

$H_{Low}(z)$

Highpass: $z^{-1} H_{Low}(-z^{-1})$

$$= z^{-1}(1 - z)$$

$$= -1 + z^{-1}$$

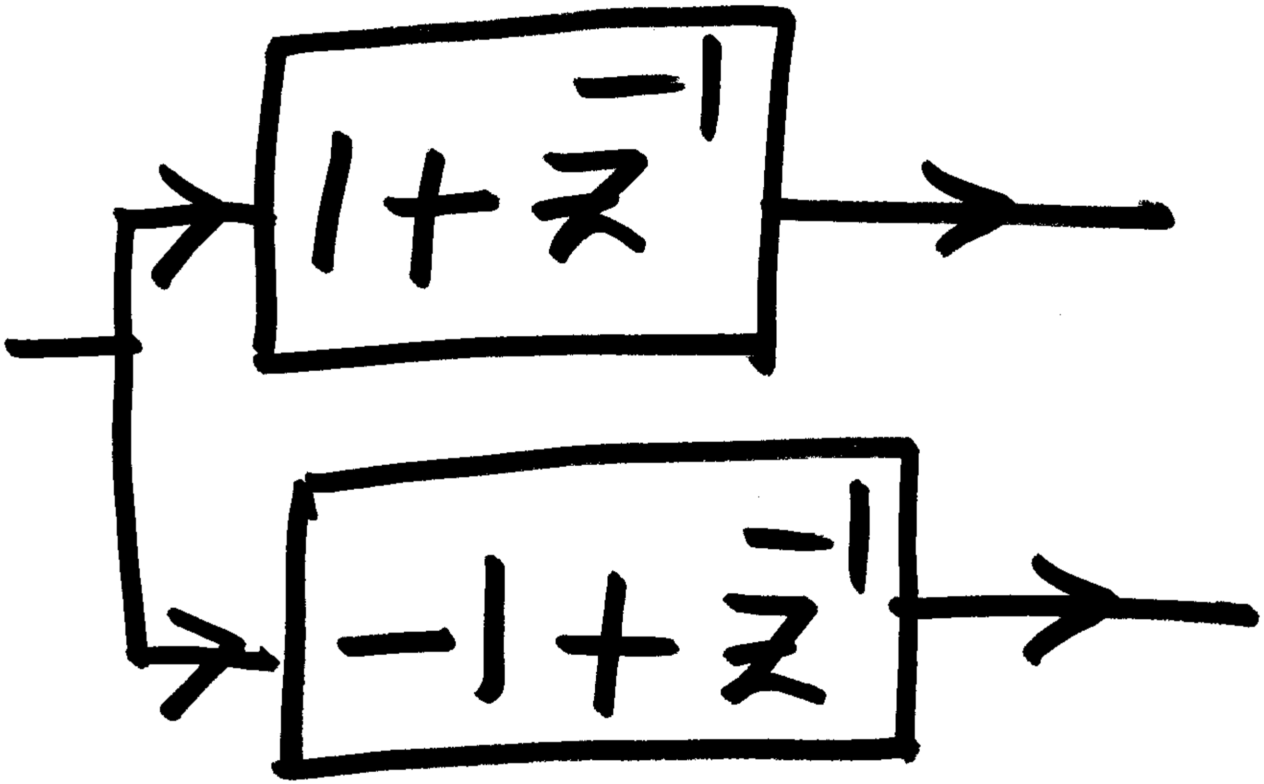


$\downarrow 2$: "wasting" half computation!

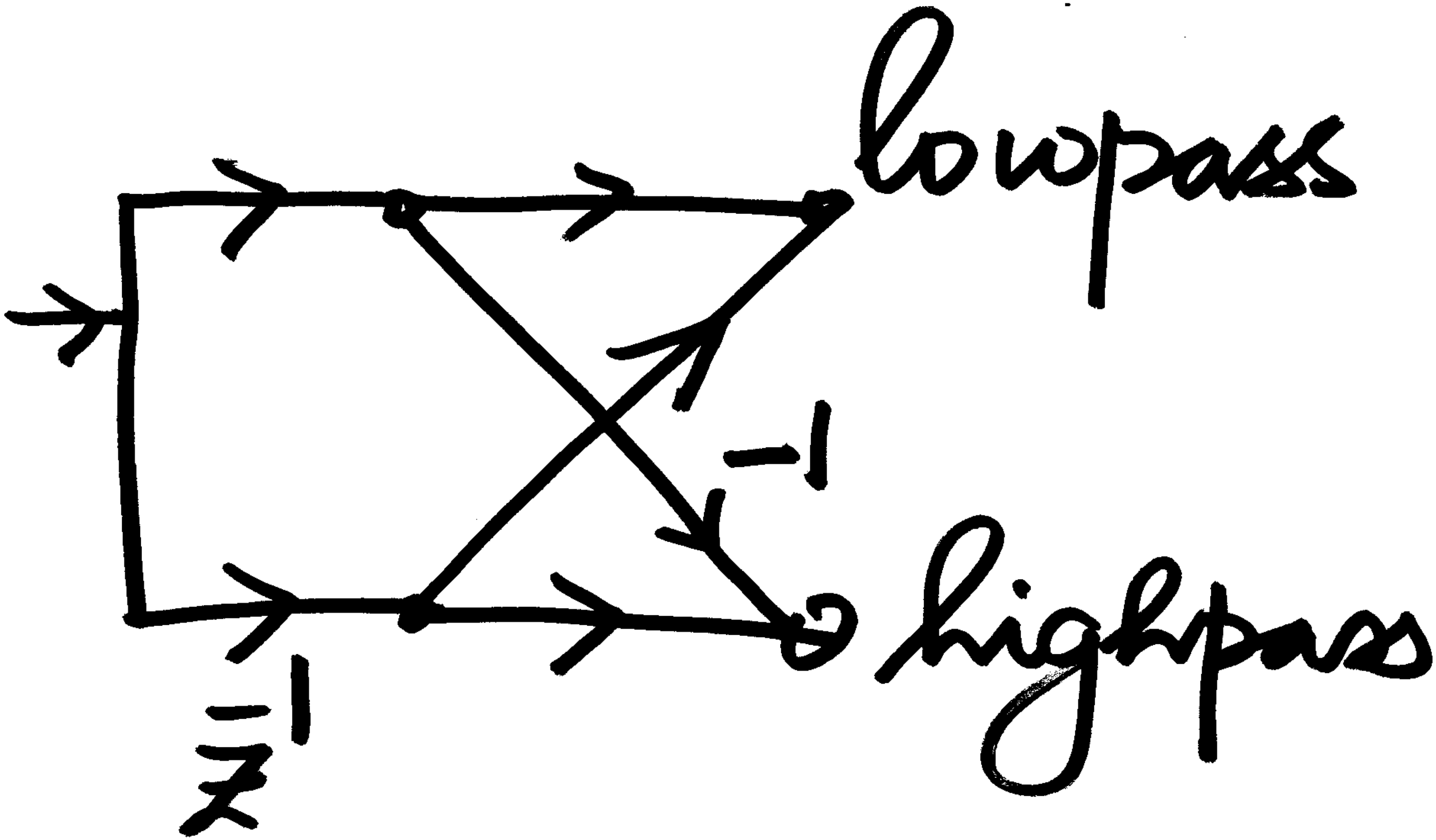
We must look for a more efficient structure:

$\downarrow 2$ first, then compute

We must invoke the "Noble" Identities.
We need to recast the analysis filters!

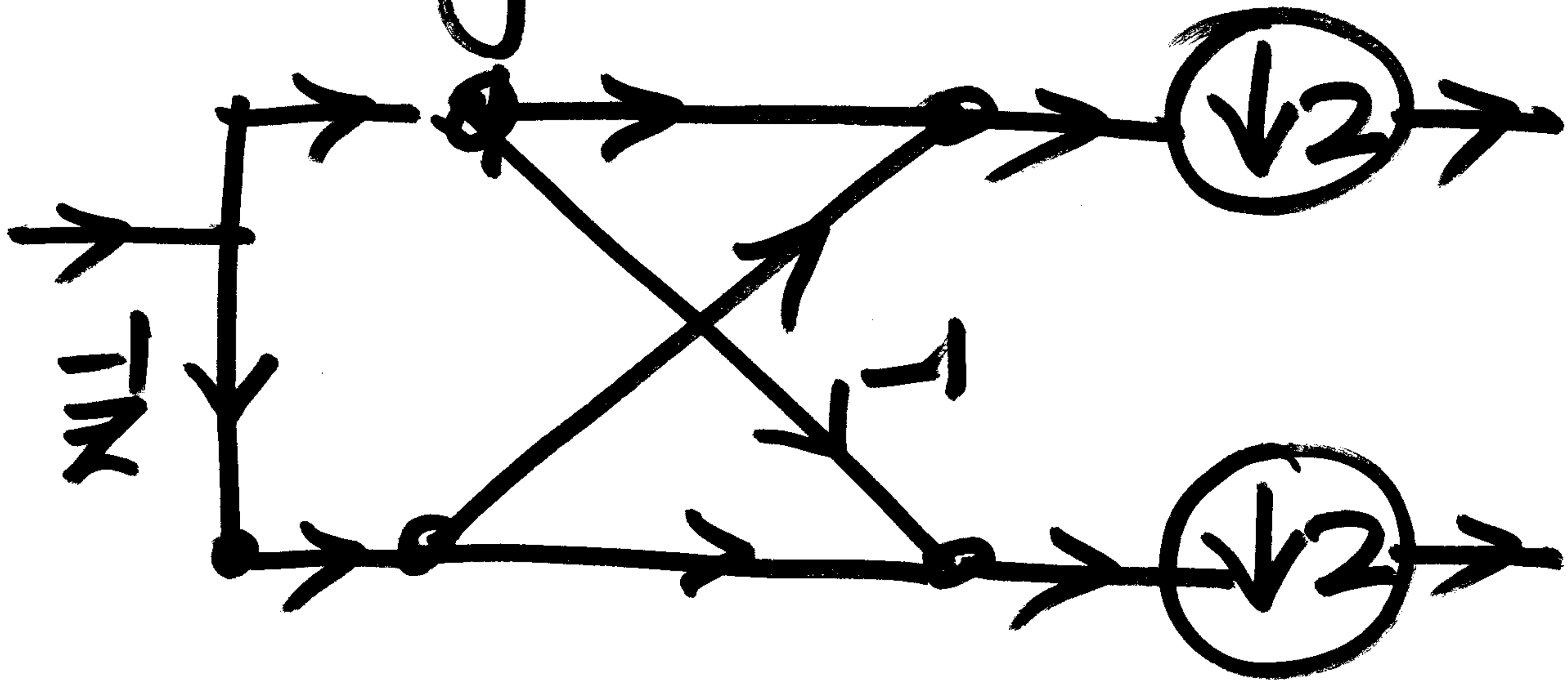


$\Downarrow$ redraw/recast!



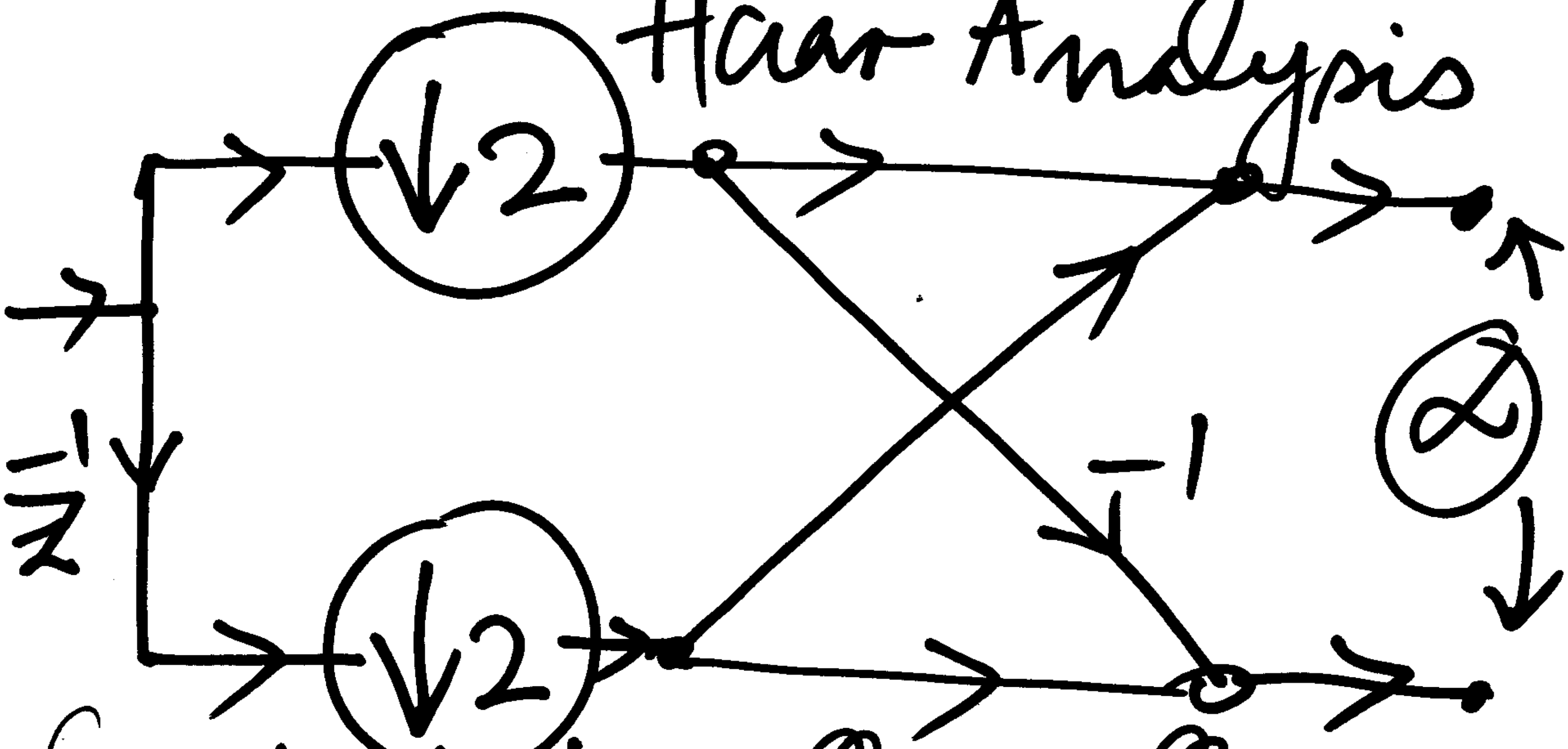
lowpass
highpass
-1
z^{-1}

Analysis filter bank:



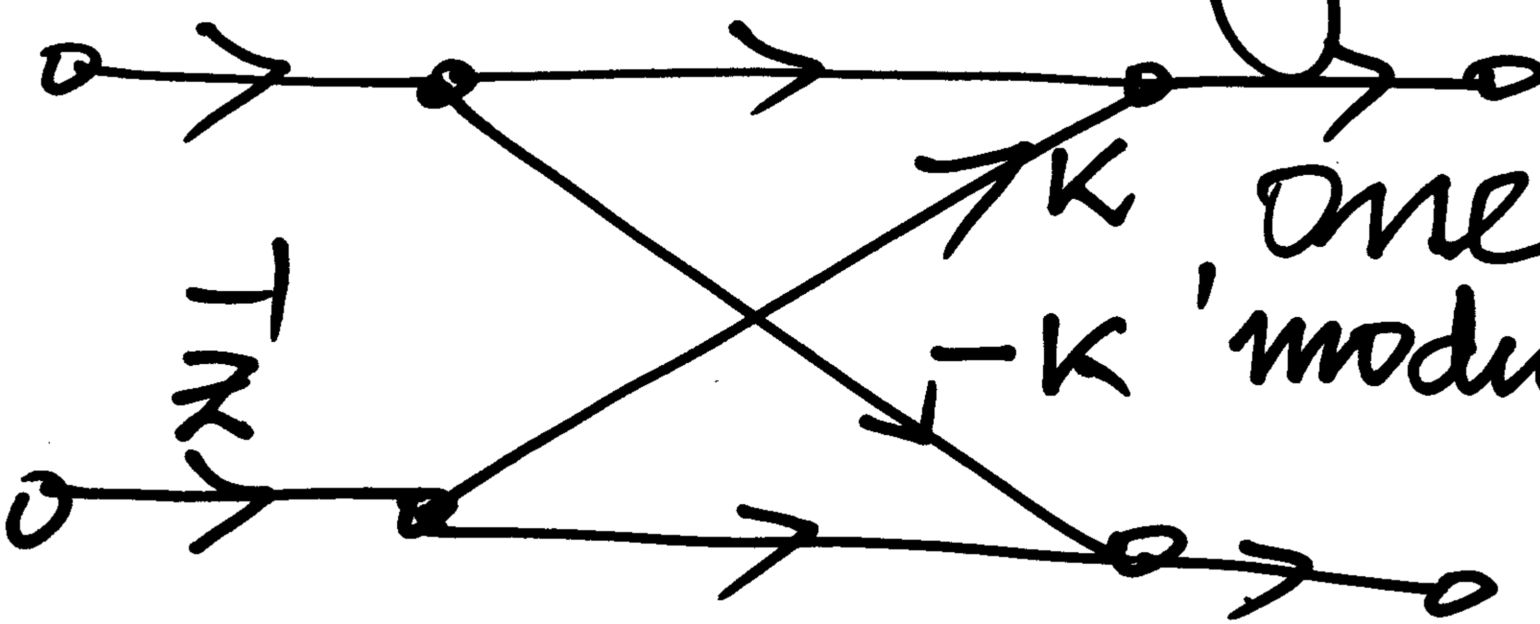
The downsamplers $\downarrow 2$ can "jump" across adders, constant multipliers and branch points!

Haar Analysis



Computationally efficient

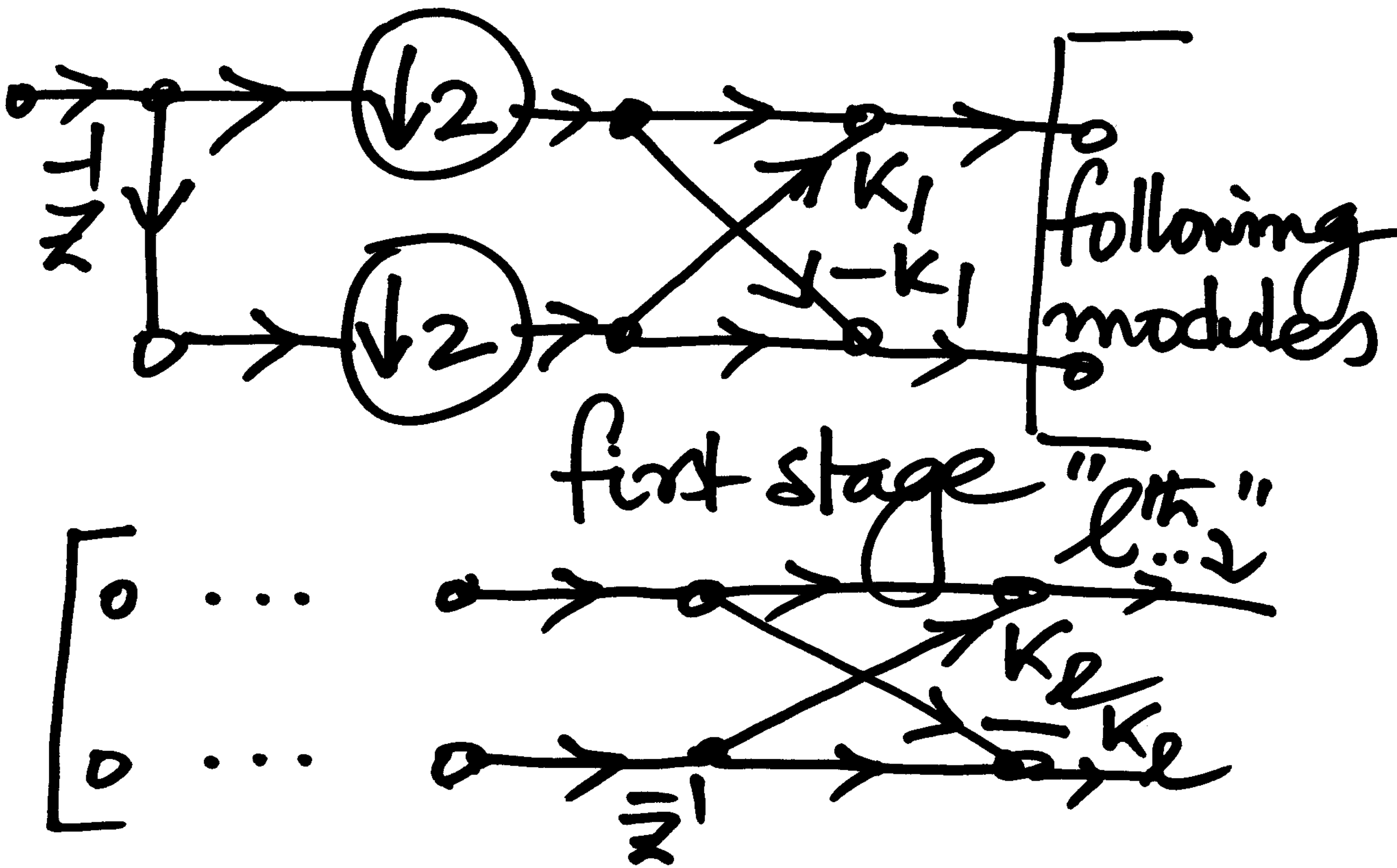
At ② let us envisage:



one 'module'

Let us build an inductive argument to analyze this structure.

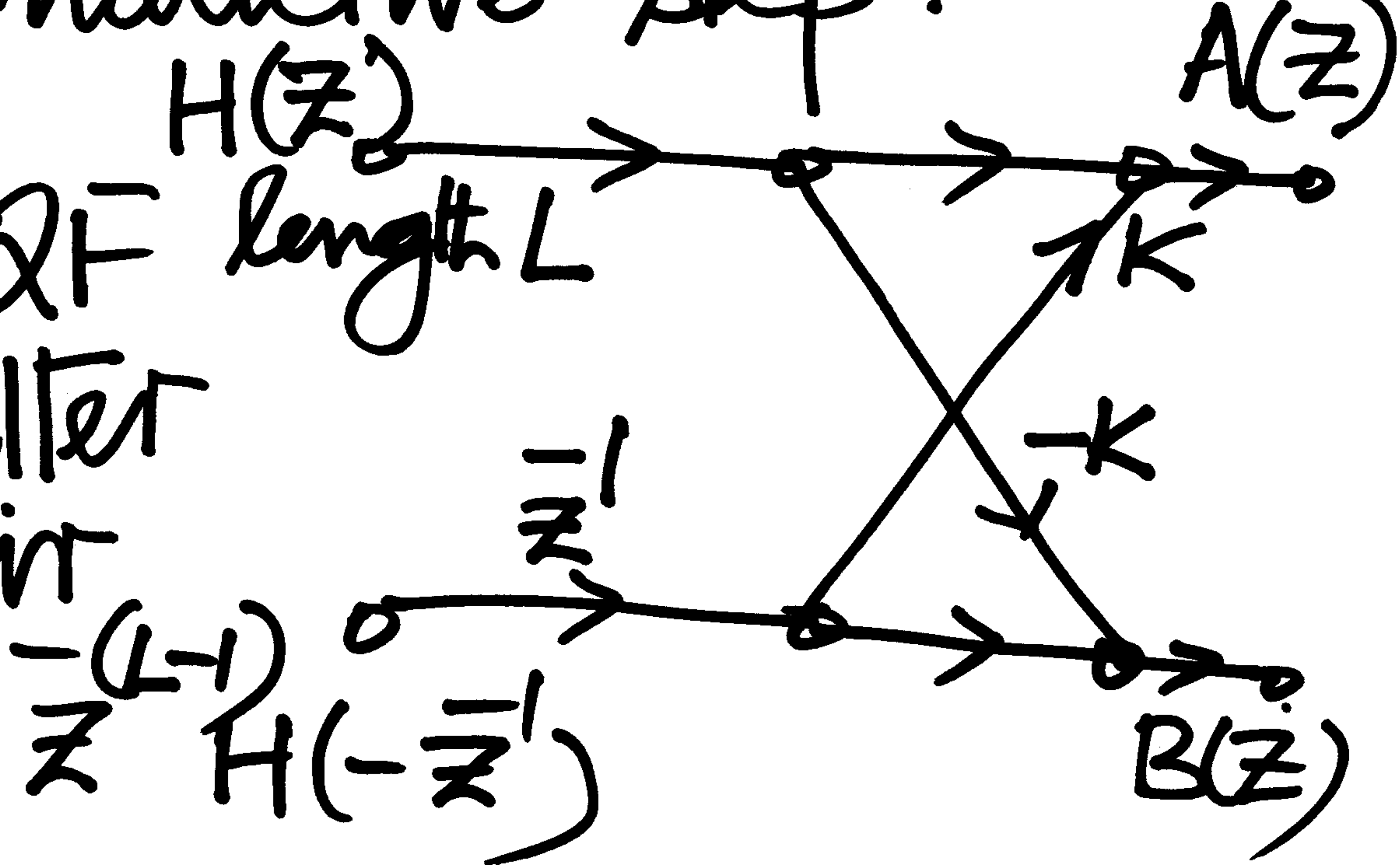
Inductive assumption:
A Conjugate Quadrature Pair has been created at the input of this module!



z^{-1}
$\downarrow 2$
$\downarrow 2$
K_1
$-K_1$
following modules
first stage
"l^{th}"
K_l
$-K_l$
z^{-1}

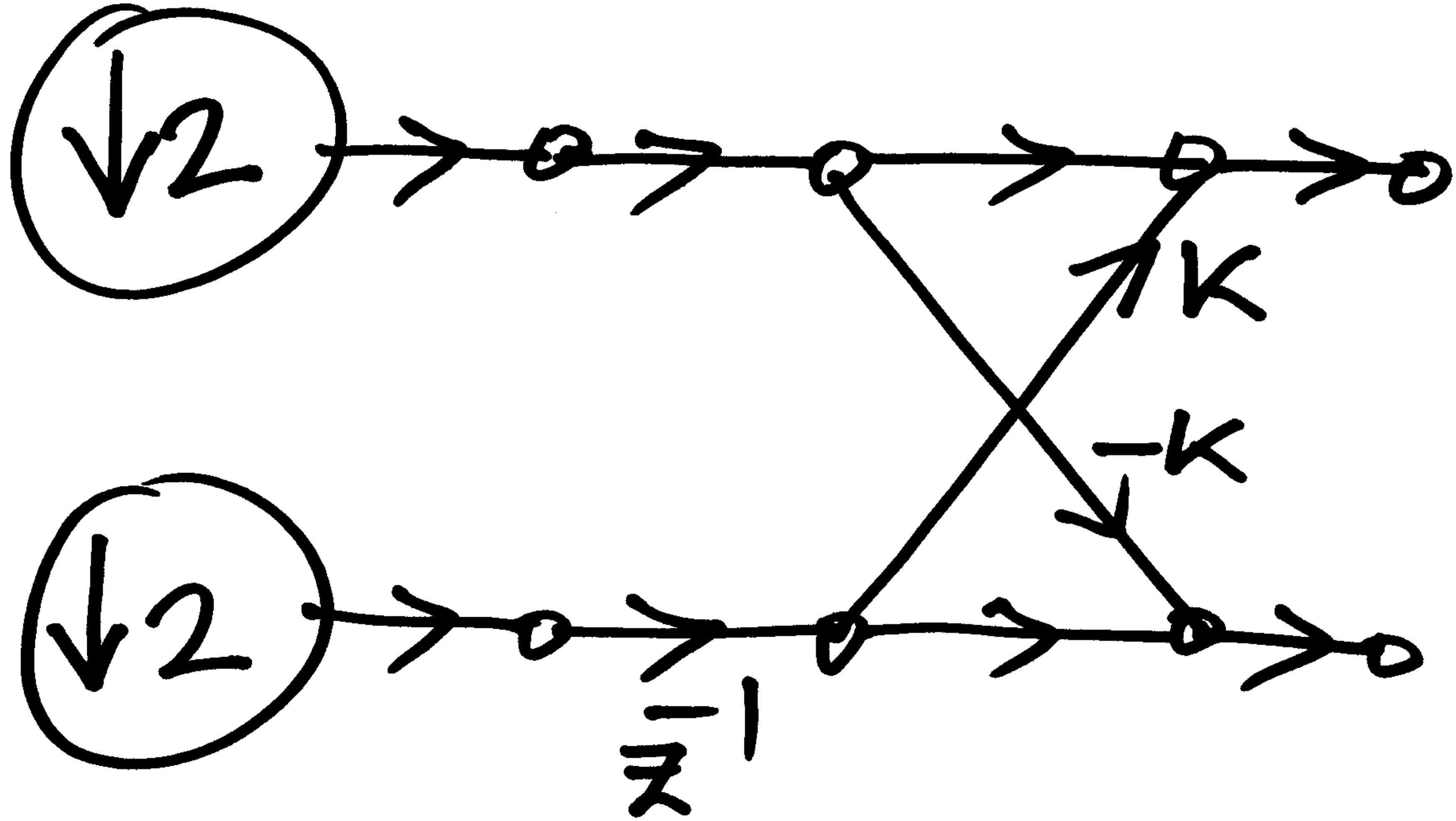
Inductive step:

CQF length L Filter Pair

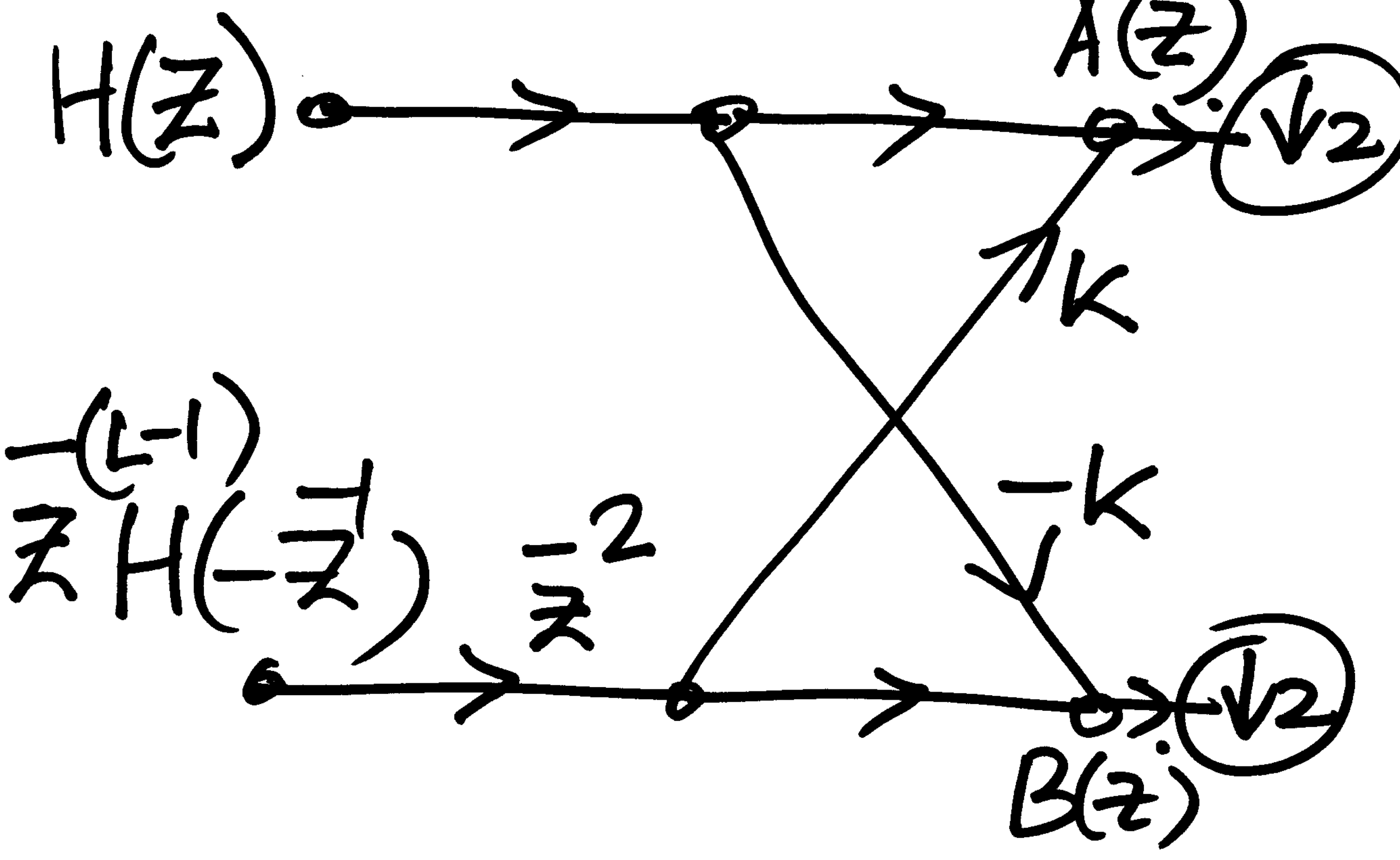


The inductive step is essentially to prove:

$$B(z) = z^{-(L+2-1)} A(-z^{-1})$$



↓2
↓2
K
-K
z^{-1}


$H(z)$
$z^{-(L-1)} H(-z^{-1})$
z^{-2}
K
$-K$
$A(z)$
$B(z)$
$\downarrow 2$
$\downarrow 2$

$$A(z) = H(z) + K \cdot z^{-2} z^{-(L-1)} H(-z^{-1})$$

$$B(z) = -K H(z) + z^{-2} \cdot z^{-(L-1)} H(-z^{-1})$$

We essentially need to consider

$$z^{-(l+2-1)} A(-z^{-1})$$

Which is: ...

$$\cdots z^{-(L+2-1)}\left\{\cdots z^{-2-L+1} \cdot H(-z^{-1}) + K \cdot (-z) H(z)\right\}$$

L even

$$z^{-(L+2-1)} A(-z^{-1})$$

becomes

$$- - -$$

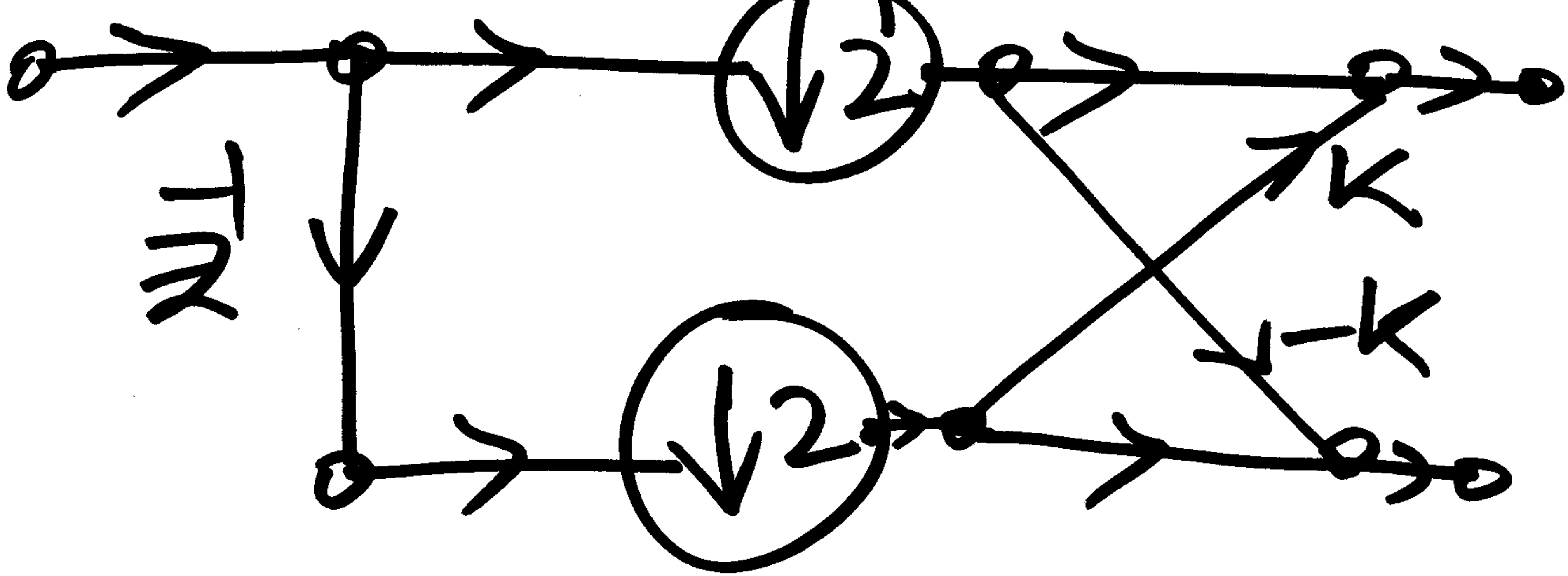
$$z^{-(L+2-1)} H(-z^{-1}) + K(-1) H(z)$$

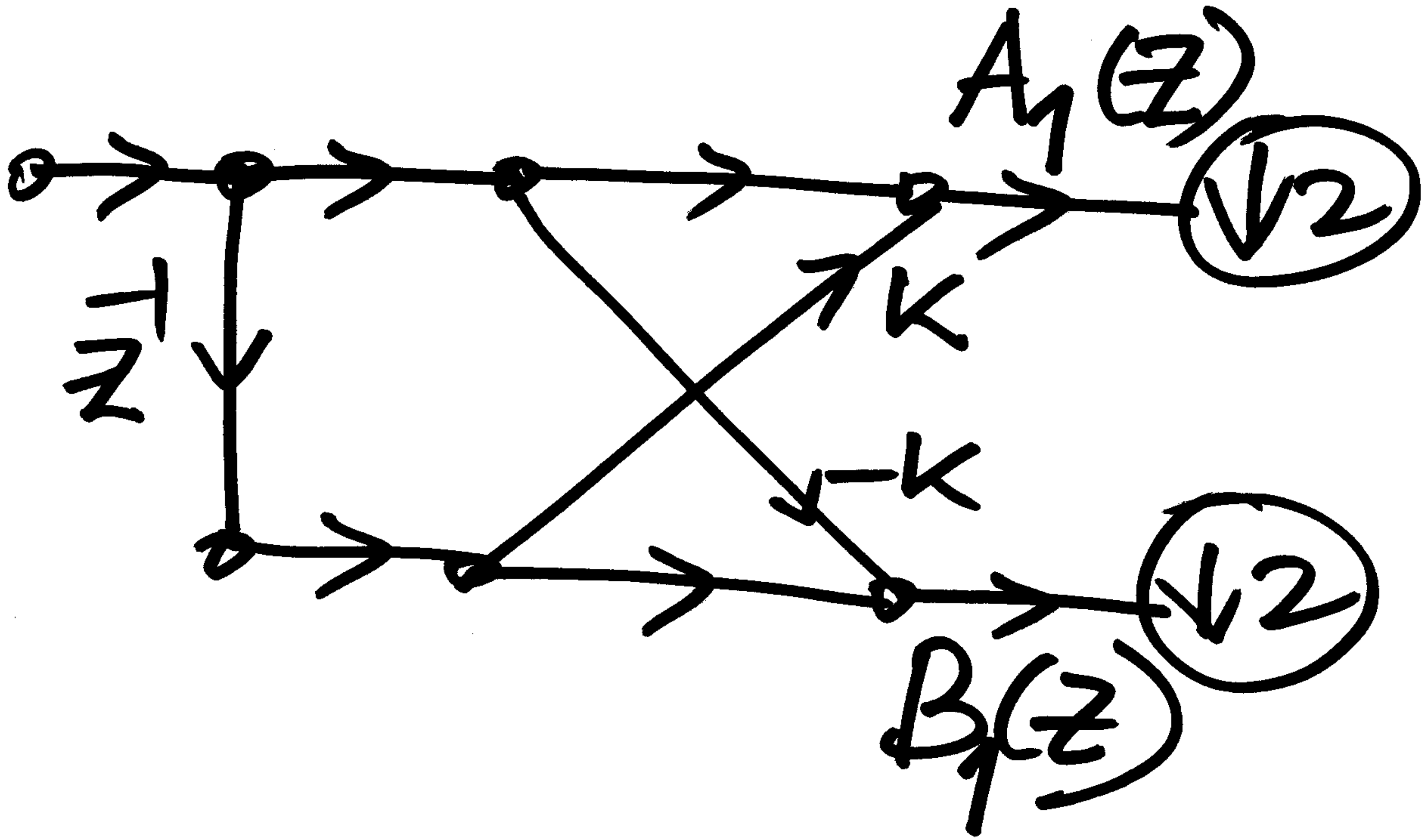
Same as $B(z)$!

The inductive step is complete!

$$B(z) = z^{-(L+2-1)} A(-z^{-1})$$

Basis step:





$A_1(z)$
$\downarrow 2$
z^{-1}
K
$-K$
$\downarrow 2$
$B_1(z)$

$$A_1(z) = 1 + Kz^{-1}$$

$$B_1(z) = -K + z^{-1}$$

Indeed: $B_1(z) = z^{-1} A_1(-z^{-1})$

Basis step also complete:
Proved by mathematical induction!

Question: Given $H(z)$ and $z^{-(L-1)} H(-z^{-1})$

Construct such a structure!

$(m+1)^{th}$ stage:

$$H_m(z)$$

$$\tilde{H}_m(z) = z^{-(2m-1)} H_m(-z^{-1}) \implies \begin{cases} H_{m+1}(z) \\ \tilde{H}_{m+1}(z) \end{cases}$$

$$H_{m+1}(z)$$

$$= H_m(z) + K \cdot z^{-2} \tilde{H}_m(z)$$

$$= H_m(z) + K \cdot z^{-2} \cdot z^{-(2m-1)} H_m(-z^{-1})$$

$$\tilde{H}_{m+1}(z) = -KH_m(z) + z^{-2} \cdot z^{-(2m+1)} H_m(-z^{-1})$$

Construction means:

go from

$H_{m+1} \Longrightarrow H_m$

$\tilde{H}_{m+1}$ to $\tilde{H}_m$

$$H_{m+1}(z) = H_m(z) + -(2m+2-1) K \cdot z \quad H_m(-z')$$

$H_m(z) :=$

$h_0 + h_1 z^{-1} + \cdots + h_{2m-1} z^{-(2m-1)}$

$\tilde{H}_m(z)$ is essentially:

$$\cdots h_{2m-1} + h_{2m-2} z^{-1} + \cdots + h_0 z^{-(2m-1)}$$

$h_0 K$ is the coefficient of the highest power of z^{-1} in $H_{001}(z)$