LECTURE 28

JPEG 2000 5/3 FILTER BANK AND SPLINE MRA

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Lec. 28
Date : 05/3/20

$$\phi_0(t) = \phi_0(2t) + \phi_0(2t-1)$$

$$\Big\downarrow \; * \text{ with itself}$$

$$\phi_1(t) = \frac{1}{2}\phi_1(2t) + \phi_1(2t-1) + \frac{1}{2}\phi_1(2t-2)$$

$$\frac{1}{2}(1+z^{-1})^2$$

$\Phi_1(t)$ is piecewise linear.

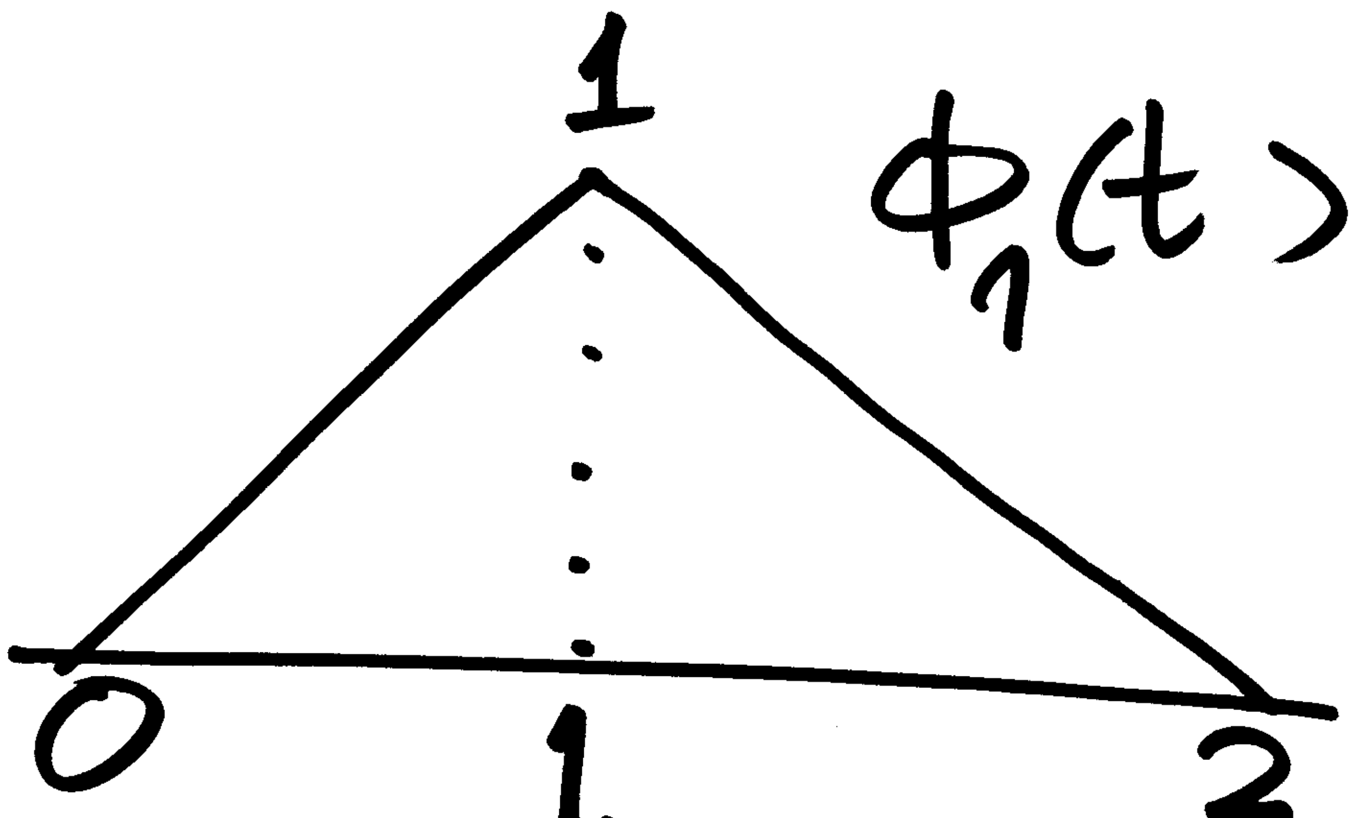
'Splines' are
polynomial pieces
or
piecewise polynomial
interpolants.

'Shortcoming' of

$\phi_1(t)$:

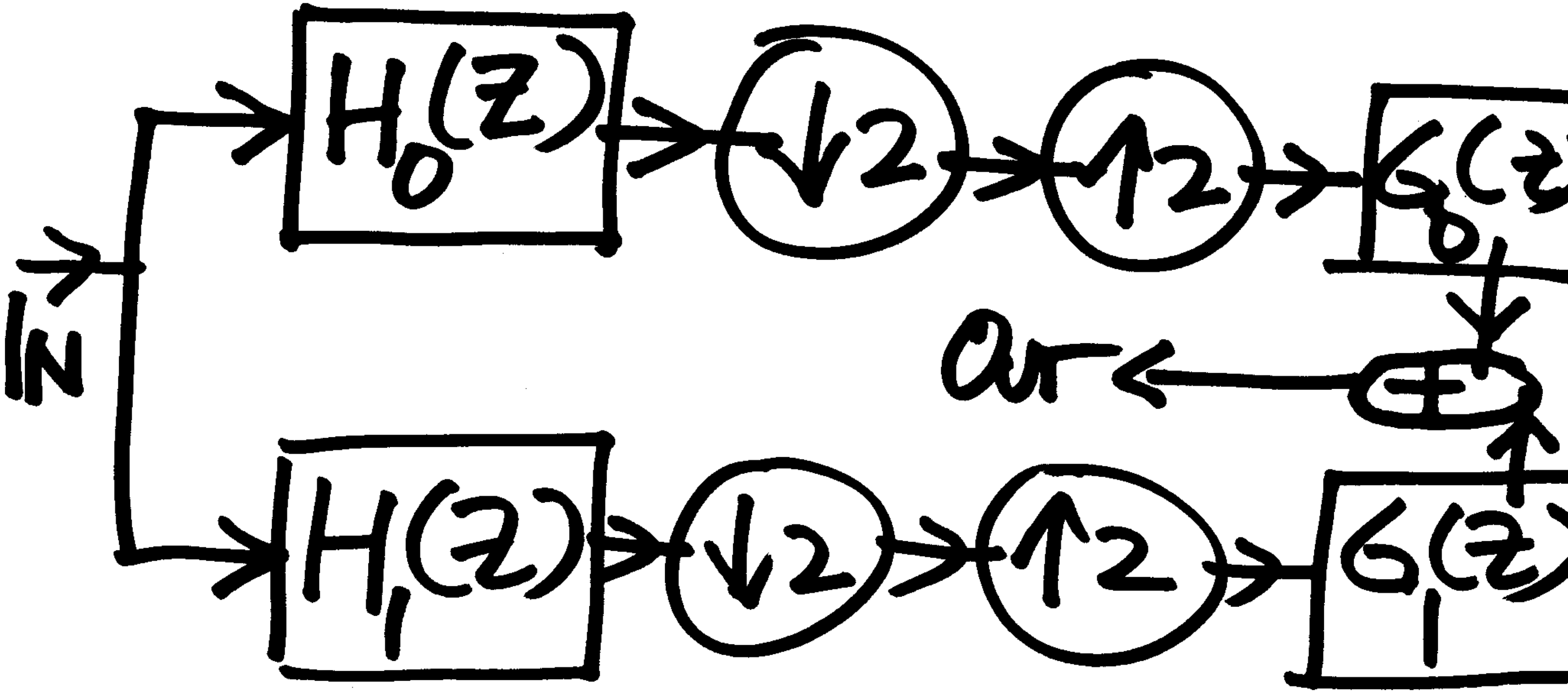
<u>not</u> orthogonal to

<u>all</u> integer translates



orthogonal to $\phi_1(t-m)$

$m > 2$.



Let without loss of gen

$$G_0(-z) = (1+z^{-1})^2$$

Alias cancellation:

$$G_0(z)H_0(-z) + G_1(z)H_1(-z) = 0.$$

Replace $z \leftarrow -z$

$$H_0(z)G_0(-z) + H_1(z)G_1(-z) = 0$$

Perfect reconstruction condition

$$G_0(z)H_0(z) + G_1(z)H_1(z) = c_0 z^{-D}$$

$$G \longleftrightarrow H$$

Once perfect reconstruction conditions satisfied in a 2-band filter bank
– – –

the -- analysis and synthesis filters can be <u>exchanged</u> to give another PR 2-band filter bank

$$G_0(z) = (1 + z^{-1})^2$$

Alias cancellation:

$$G_0(z) H_0(-z) + G_1(z) H_1(-z) = 0$$

$$\frac{G_0(z)}{G_1(z)} = -\frac{H_1(-z)}{H_0(-z)}$$

One simple choice

num = num

den = den

$$G_0(z) = -H_1(-z)$$

$$G_1(z) = H_0(-z).$$

Here:

$$-H_1(-z) = (1+z^{-1})^2$$

$$H_1(z) = -\left\{\left(1 - z^{-1}\right)^2\right\}$$

$$G_0(z)H_0(z) + G_1(z)H_1(z) = c_0 z^{-D}$$

$$G_0(z)H_0(z) + H_0(-z)H_1(z) = C_0 z^{-D}.$$

$$H_1(z) = -G_0(-z)$$

$$G_0(z)H_0(z) + H_0(-z)(-G_0(-z)) = c_0 z^{-D}$$

$$G_0(z)H_0(z)$$

$$= K_0(z)$$

$$K_0(z) - K_0(-z)$$

$$= c_0 z^{-\Delta}$$

$$K_0(z) - K_0(-z) \rightarrow = c_0 z$$

1 Kill even samples in Inv z-transf of $K_0(z)$ - - -

② Out of the remaining odd samples, only one is nonzero.

D^{th} sample $= C_0$.

We shall choose $H_0(z)$ also to have 2 zeros at $z = -1$

$H_0(z)$ to have a factor $(1+z^{-1})^2$

Parsimoniously ("stingily") extending $H_0(z)$ to retain impulse response symmetry — —

introduce a factor

$$\left(1 + h_0 z^{-1} + z^{-2}\right)$$

only one degree of freedom

$$H_0(z) =$$

$$(1+z^{-1})^2 (1+h_0 z^{-1} + z^{-2})$$

Let us now consider $G_0 H_0$

$$G_0(z)H_0(z)$$

$$= (1+z^{-1})^2 (1+z^{-1})^2 (1+h_0 z^{-1} + z^{-2})$$

$$(1+z^{-1})^4$$

$$= 1 + 4z^{-1} + 6z^{-2}$$

$$+ 4z^{-3} + z^{-4}$$

$$
\begin{tikzcd}
1 & h_0 & 1 \\
0 \arrow[u] & &
\end{tikzcd}
$$

1 4 6 4 1

$\uparrow$

0

Convolved with

1 h_0 1

$\uparrow$

0

$$
\begin{array}{ccccccc}
1 & 4 & 6 & 4 & 1 & & \\
 & h_0 & 4h_0 & 6h_0 & 4h_0 & h_0 & \\
0 & & 1 & 4 & 6 & 4 & 1 \\
\hline
1 & 4+h_0 & 1+4h_0+6 & 8+6h_0 & 1+4h_0+6 & 4+h_0 & 1
\end{array}
$$

If we make

$$4 + h_0 = 0$$

$$h_0 = -4$$

we have retained only ONE ODD SAMPLE!

Therefore we choose

$$h_0 = -4$$

$$H_0(z) = (1+z^{-1})^2(1-4z^{-1}+z^{-2})$$

$$H_0(z) =$$

$$\begin{array}{ccc} 1 & 2 & 1 \\ \uparrow & & \\ 0 & & \end{array} \quad * \quad \begin{array}{ccc} 1 & -4 & 1 \\ \uparrow & & \\ 0 & & \end{array}$$

$$
\begin{array}{ccccc}
1 & 2 & 1 & & \\
 & -4 & -8 & -4 & \\
 & & 1 & 2 & 1 \\
\hline
1 & -2 & -6 & -2 & 1 \\
\hline
\end{array}
$$

0

$H_0(z) =$

$1 - 2z^{-1} - 6z^{-2} - 2z^{-3}$

$+ z^{-4}.$

length $= 5$

$$G_0(z)$$

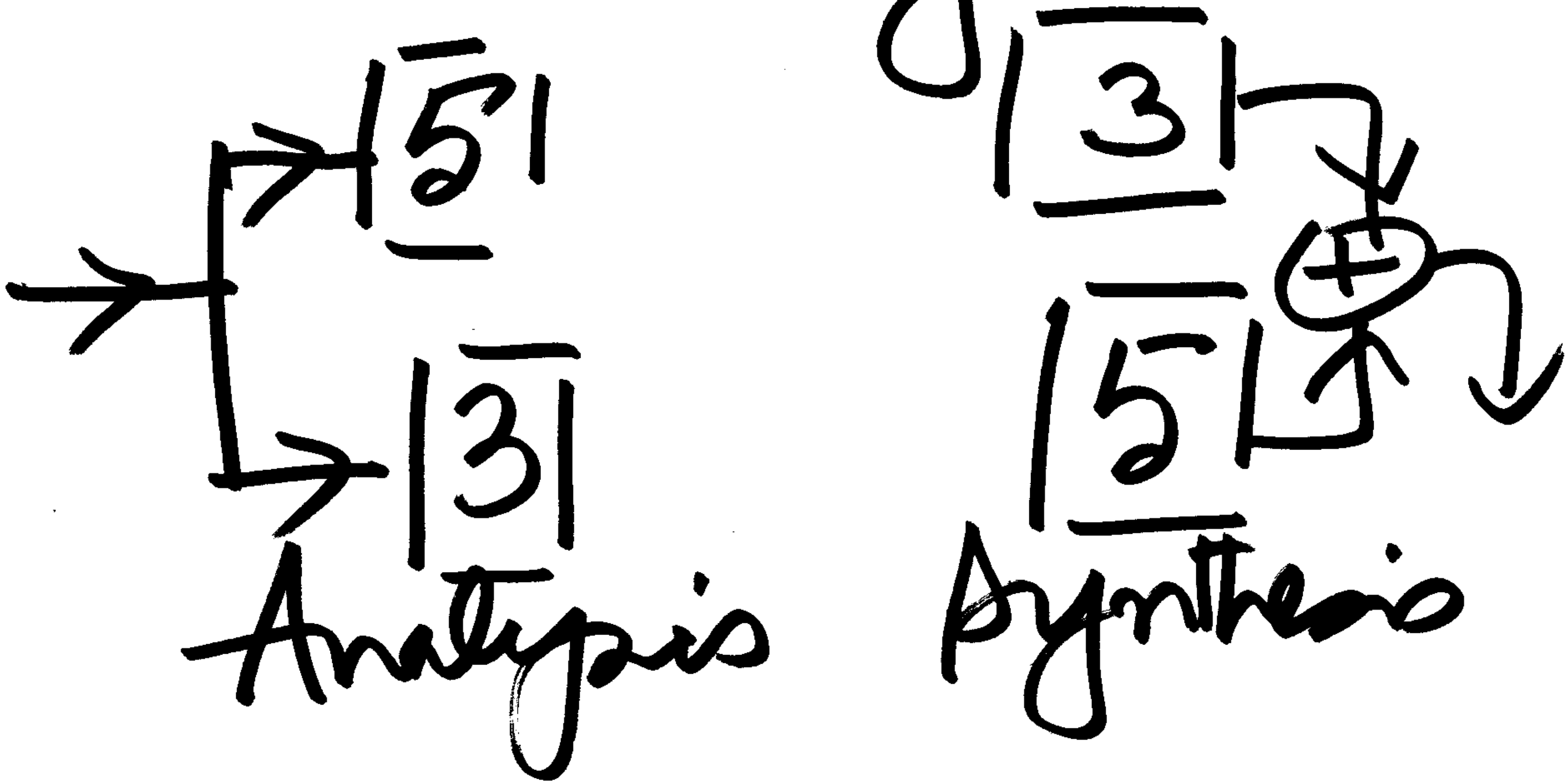
$$= \left(1 + z^{-1}\right)^2$$

$$= 1 + 2z^{-1} + z^{-2}$$

length $= \underline{3}$

This is the celebrated 5/3 filter bank in JPEG-2000.

5/3 filter bank lengths:



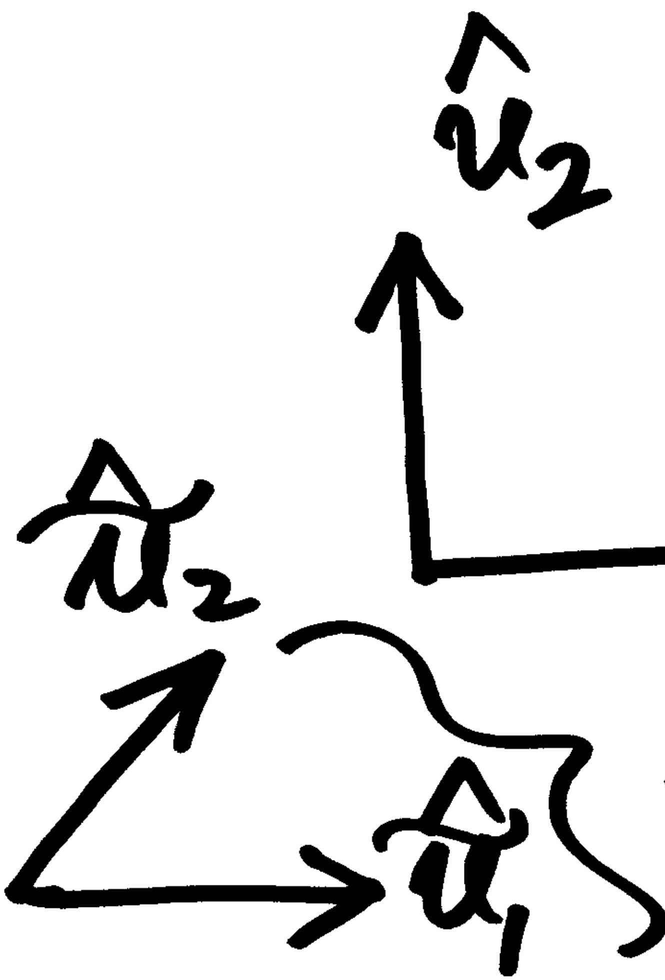
$$G_0(z) = -H_1(-z)$$

$$H_1(z) = -G_0(-z)$$

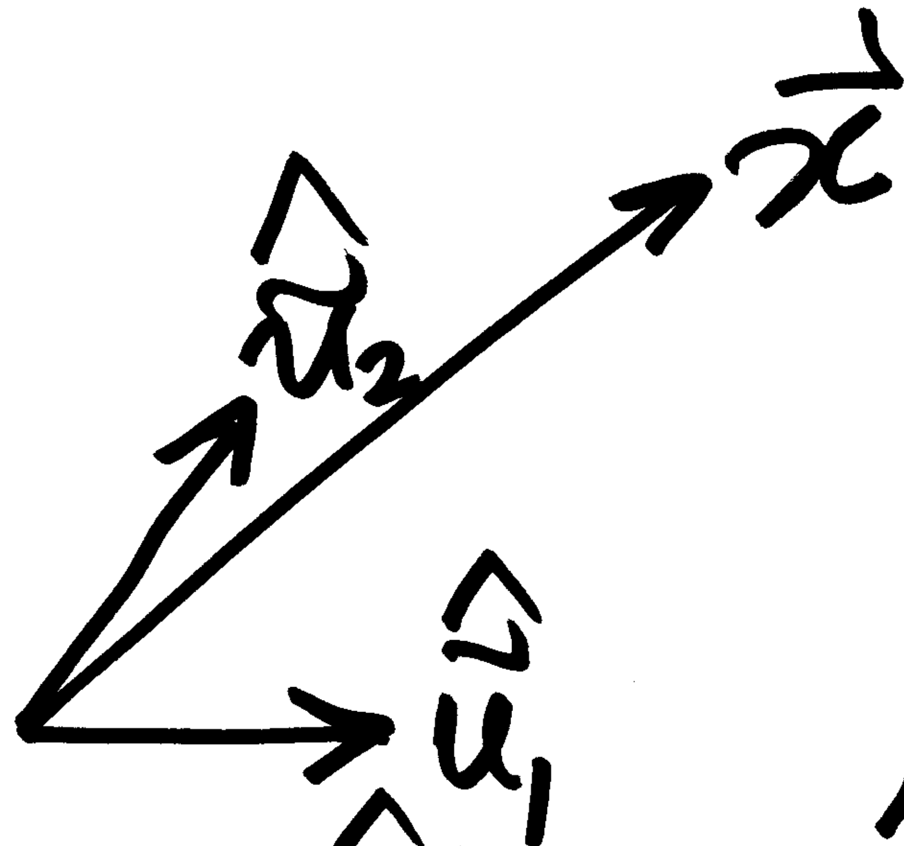
$$= -(1 - z^{-1})^2$$

$$G_1(z) = H_0(-z)$$

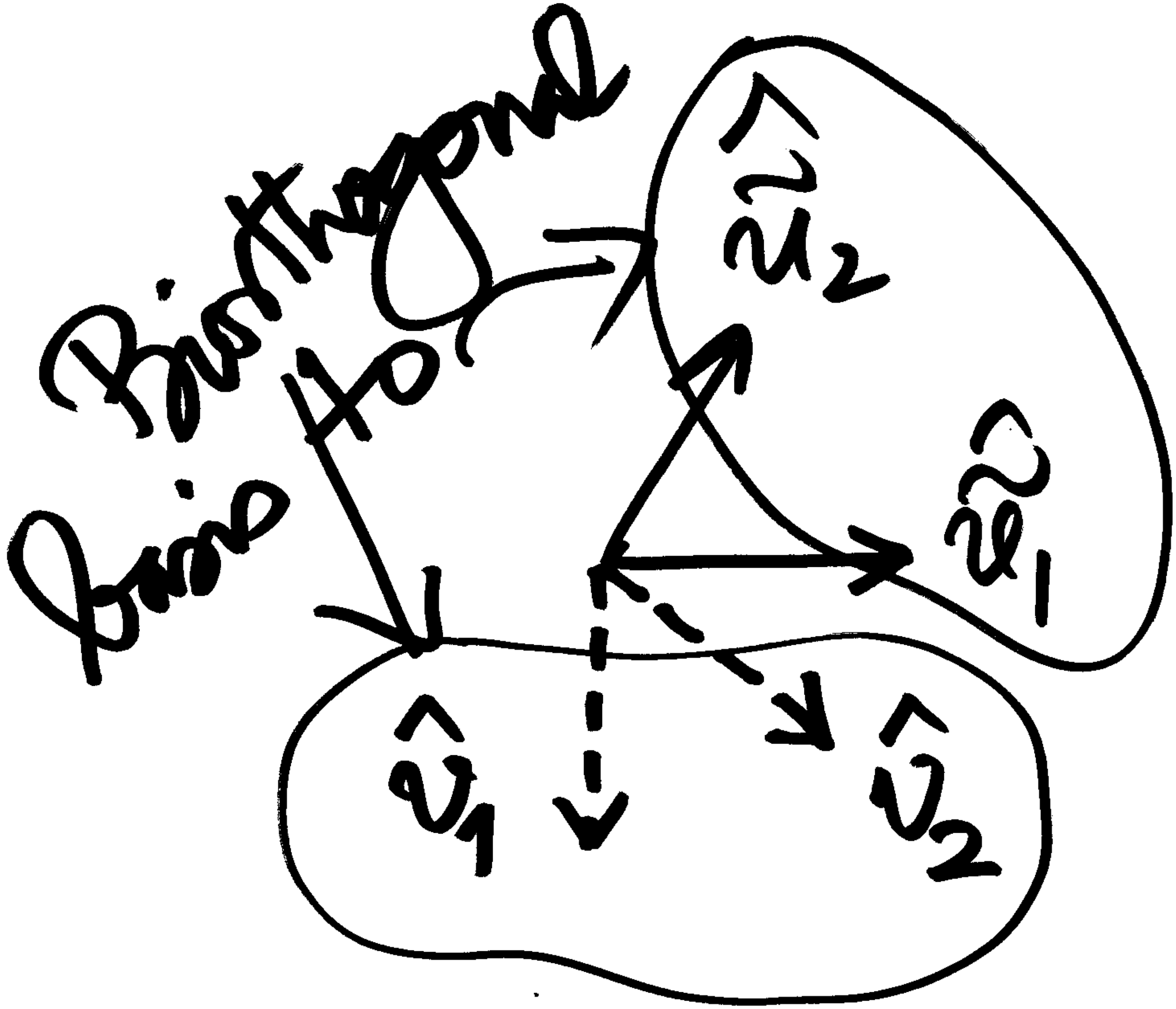
$$= 1 + 2z^{-1} - 6z^{-2} + 2z^{-3} + z^{-4}$$



non orthogonal but form basis



$$\vec{x} = c_1 \hat{u}_1 + c_2 \hat{u}_2$$


Biorthogonal
Basis
Ho
$\hat{\tilde{u}}_2$
$\hat{\tilde{u}}_1$
$\hat{v}_1$
$\hat{v}_2$