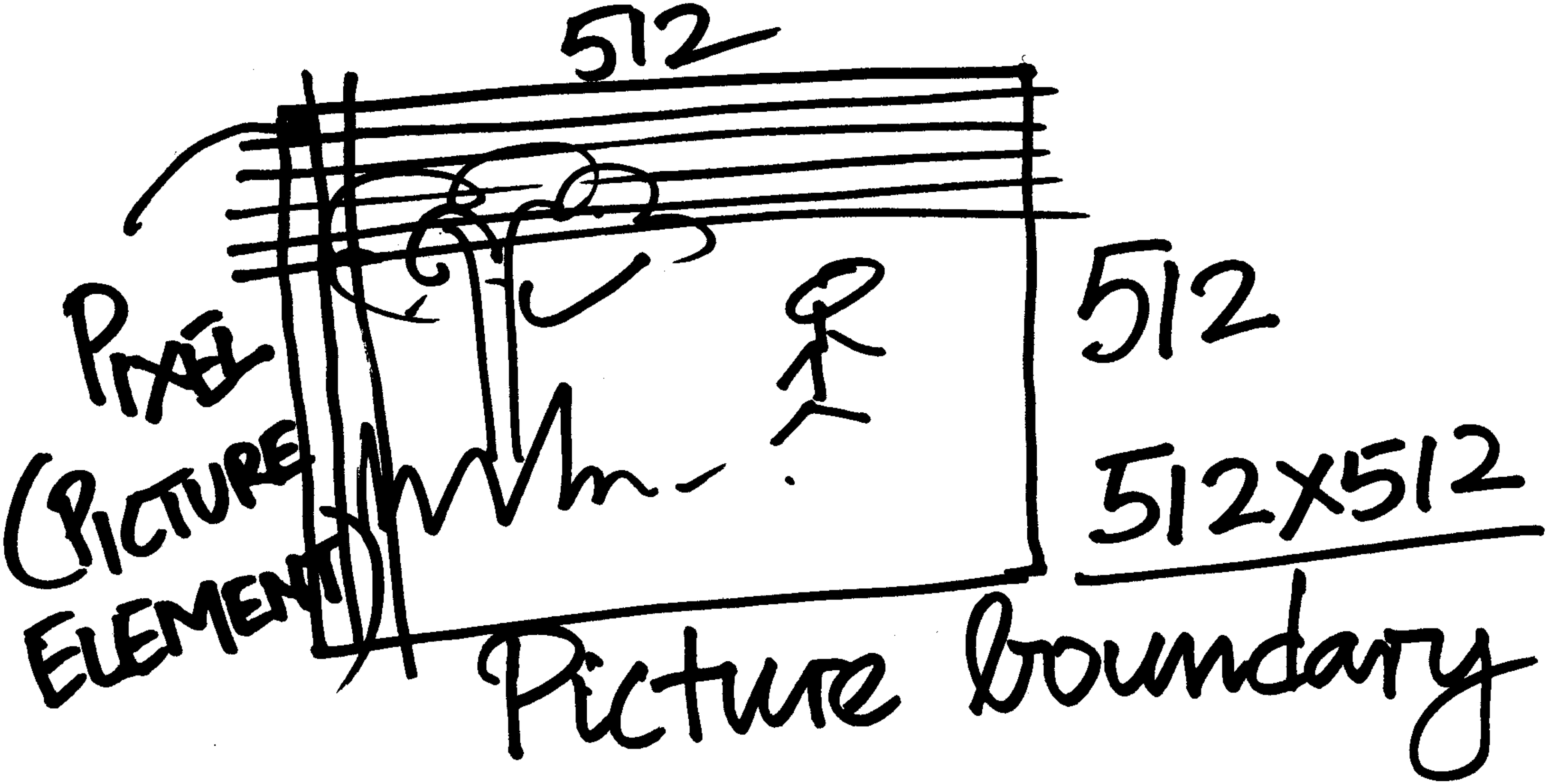
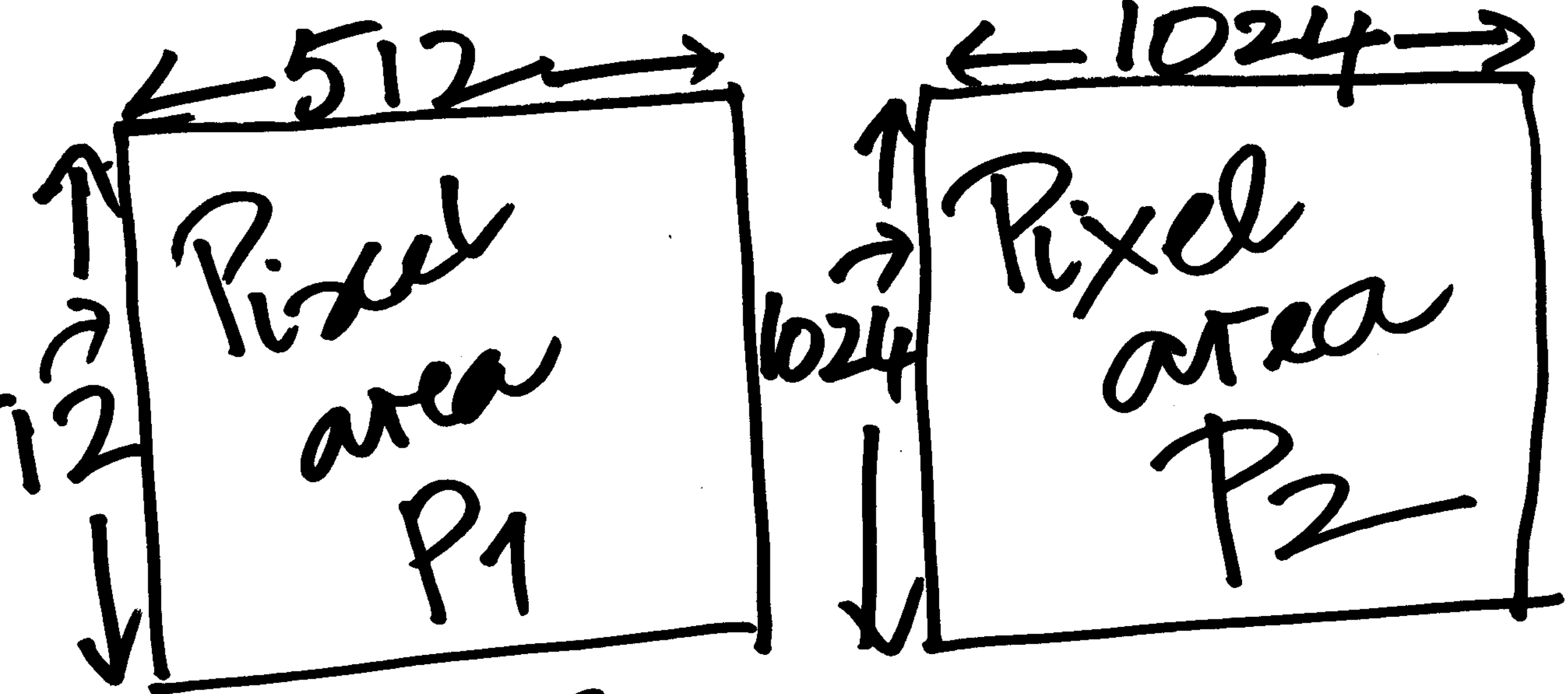
Prof. V. Gadre.
Lec. No. 2
Date: 7/1/2010

The Haar Wavelet

(7.1.2010)


512
512
512X512
PIXEL
(PICTURE
ELEMENT)
Picture boundary

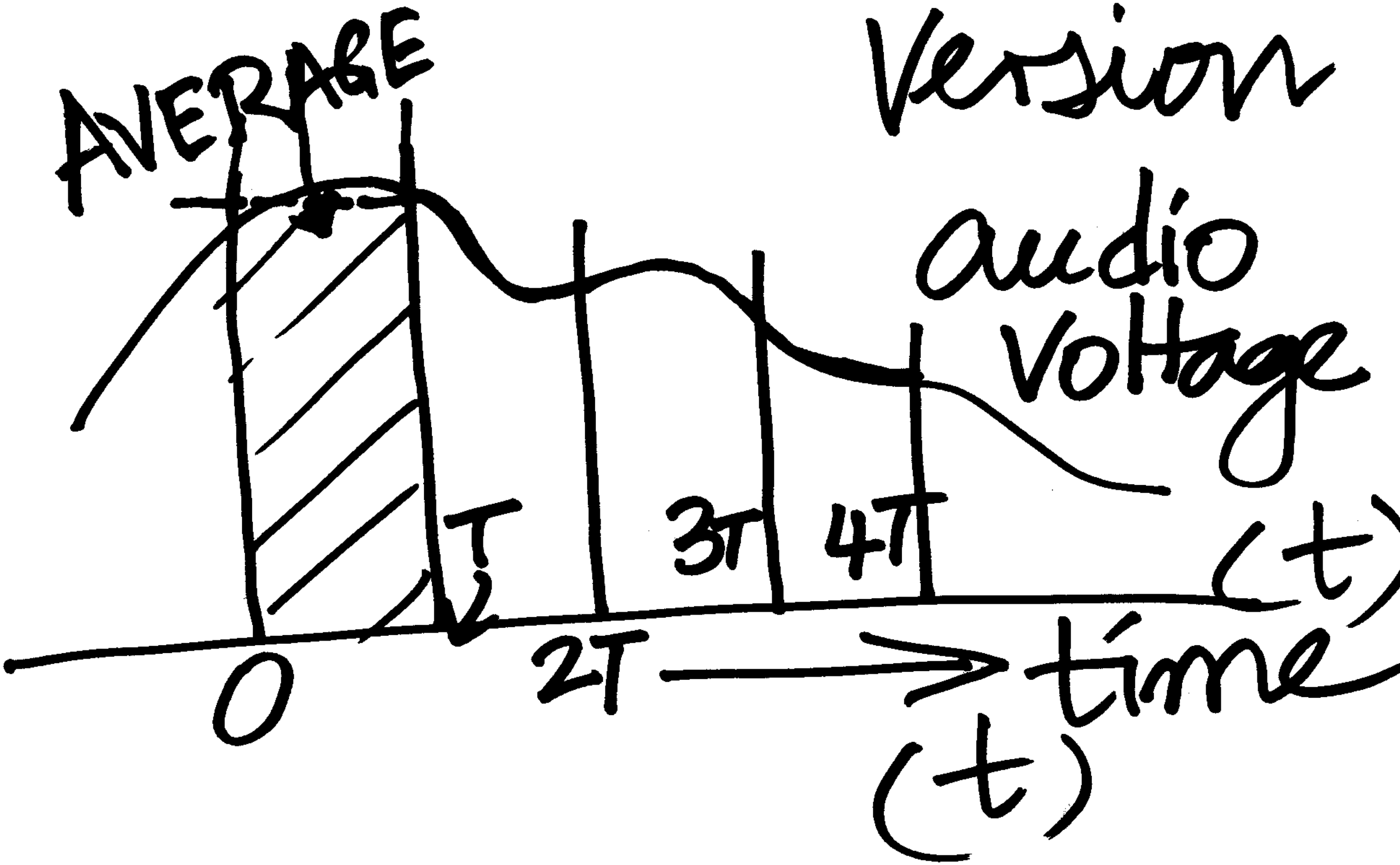
Piecewise constant
representation :
one constant for
each "piece": <u>pixel</u>



$P_2 = \frac{1}{4} P_1$. Same picture

The smaller pixel area: gives <u>more</u> <u>resolution</u>

One-dimensional Version



$x(t)$: function

'Good' piecewise

Constant

representation

Over $]0, T[$

$$\frac{1}{T}\int_0^T x(t)\,dt = \text{"average"}$$

On any particular interval of size T,

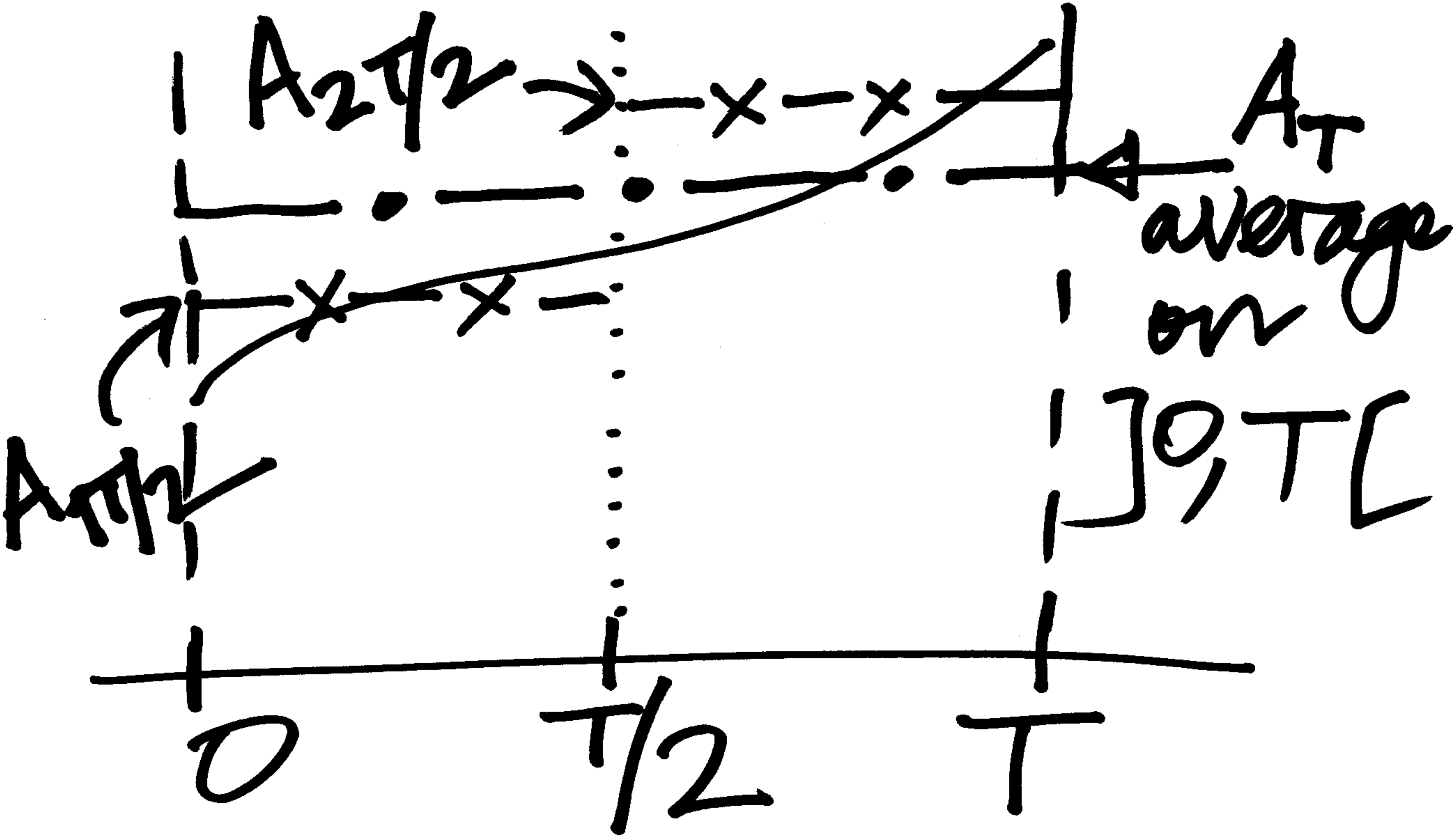
$$\frac{1}{T} \int_{(T)} x(t)\, dt$$

that interval

Over an interval of size $T/2$,

Similarly

$$\frac{1}{T/2} \int_{(T/2)} x(t)\, dt$$



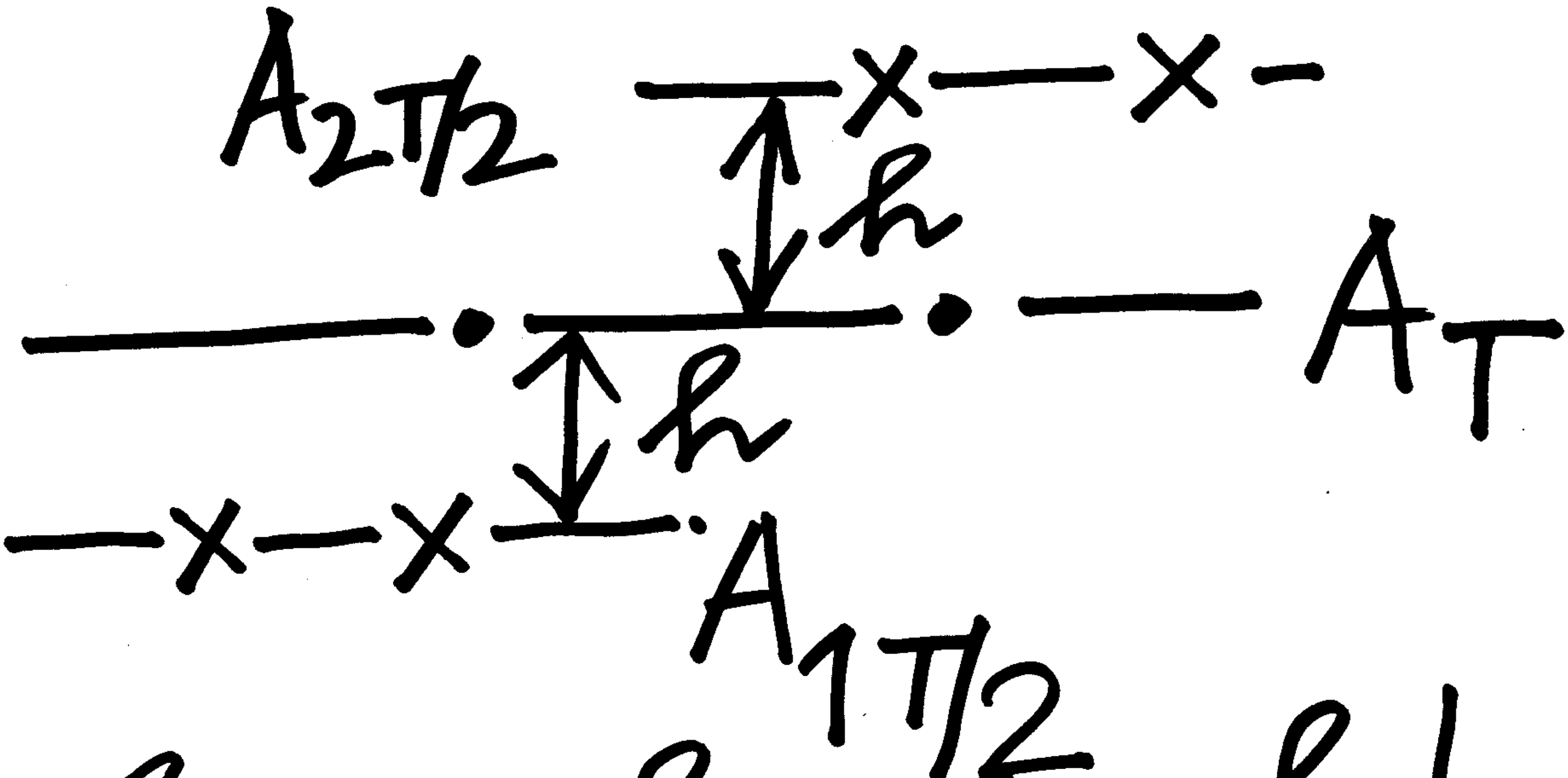
A2T/2
AT
average
on
]0,T[
AT/2
0
T/2
T

$$A_T = \frac{1}{T}\int_0^T x(t)\,dt$$

$$A_{1\pi/2} = \frac{1}{\pi/2}\int_0^{\pi/2} x(t)\,dt$$

$$A_{2\pi/2} = \frac{1}{\pi/2} \int_{\pi/2}^{T} x(t)\,dt$$

$$A_T = \frac{1}{2}\left\{A_{1\,T/2} + A_{2\,T/2}\right\}$$

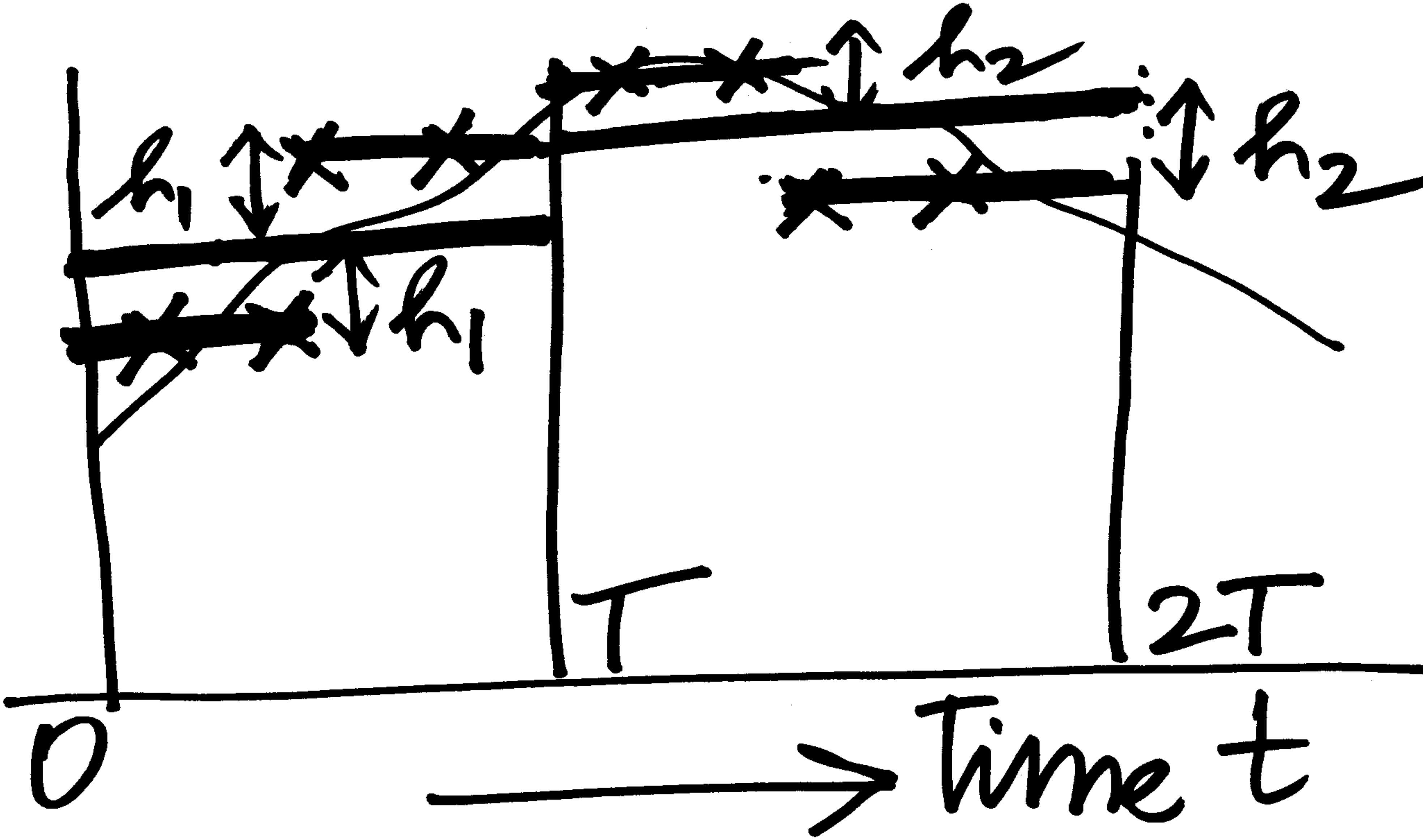


Same heights h!

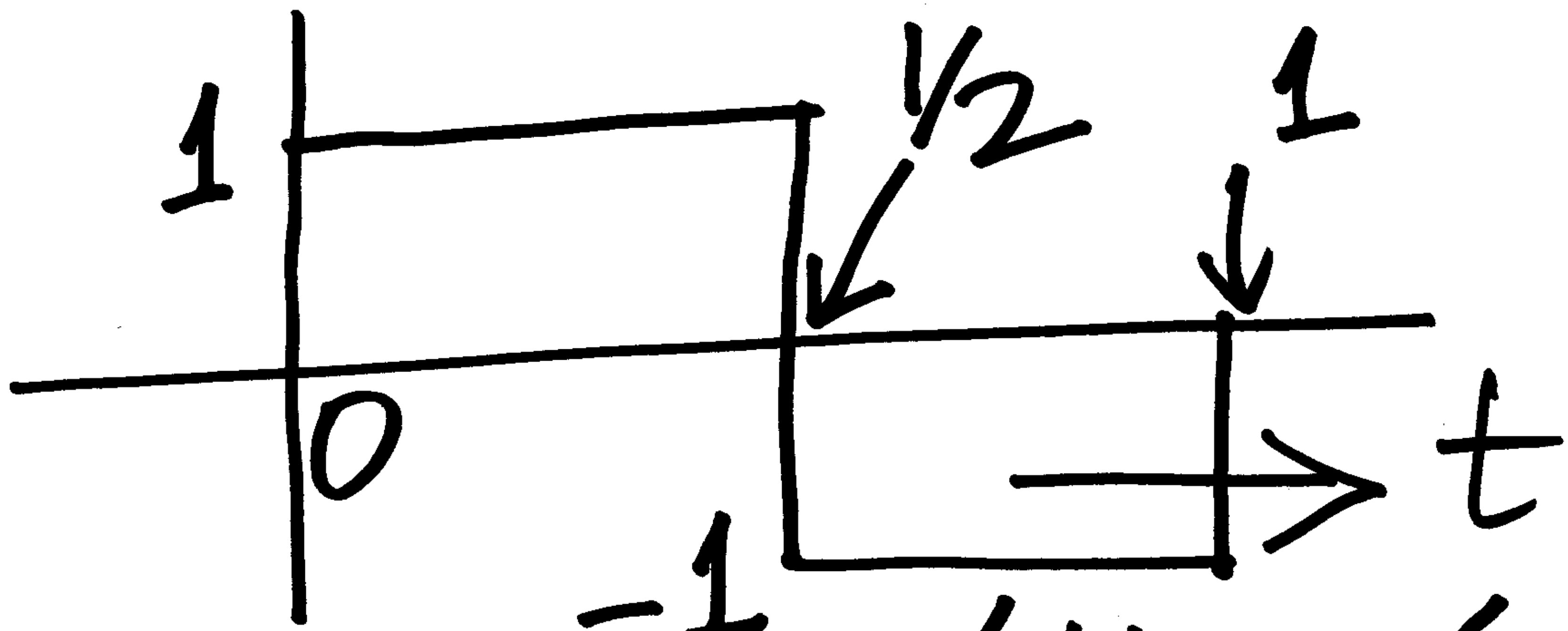
$f_1(t)$

$f_2(t)$

$f_2(t) - f_1(t)$: "ADDITIONAL INFO"

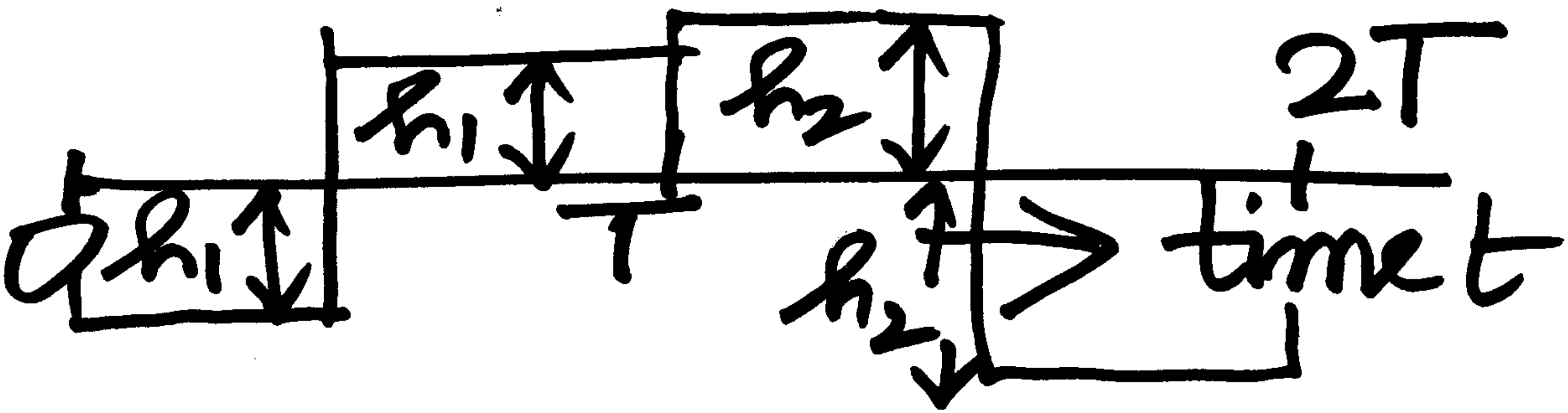


h1
h1
h2
h2
T
2T
O
Time t



$\psi(t)$ 'HAAR' WAVELET

$$f_2(t) - f_1(t)$$



$$f_2(t) - f_1(t)$$
$$= h_1 \psi(t/T)$$
$$+ h_2 \left(\psi\left(\frac{t-T}{T}\right)\right)$$

$$\psi\left(\frac{t-\tau}{s}\right)$$

s positive real

τ : real

τ : translation index

δ : dilation index

Dilates and translates of $\psi(t)$ 'capture' additional information $(f_2 - f_1)$

"Proponent":
We can go arbitrarily close to $x(t)$

Arbitrarily close?

$x_a(t)$: approximation

$x(t)$: function being approximated

$$x_e(t) = x(t) - x_a(t)$$

$$\mathcal{E}_e = \int_{-\infty}^{+\infty} |x_e(t)|^2 \, dt$$

"Adversary or Opponent":
Bring ε to
the "small value" ε_0

"Proponent":

Certainly! Here is the m, such that $T/2^m$ is OK!

We shall focus on functions with finite <u>energy</u>

$x(t)$

Energy $= \int_{-\infty}^{+\infty} |x(t)|^2 \, dt$

FINITE

'$\mathcal{L}_2$ norm' of x

$$= \left\{ \int_{-\infty}^{+\infty} |x(t)|^2 \, dt \right\}^{1/2}$$

$\mathcal{L}_2(\mathbb{R})$:

Space of functions whose $\mathcal{L}_2$ norm FINITE!

"$\mathcal{L}_1$ norm" of x:

$$\int_{-\infty}^{+\infty} |x(t)|\, dt$$

'$\mathcal{L}_p$ norm' of x

$$= \left\{ \int_{-\infty}^{+\infty} |x(t)|^p \, dt \right\}^{1/p}$$

$p > 0$

p REAL

"$\mathcal{L}_\infty$ norm of x"

$$\left\{ \int_{-\infty}^{+\infty} |x(t)|^p \, dt \right\}^{1/p}$$

$p \to \infty$??

"DYADIC"
WAVELET
(powers of 2)