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LECTURE 14

DAUBECHIES' FILTER BANKS: GENERAL CONJUGATE QUADRATURE FILTERS

Second member of Daubechies' family:

Highpass filter:

factor of $(1 - z^{-1})^2$

$$= z^{-D} H_0(-z^{-1})$$

where $H_0(Z) =$
corresponding (analysis)
lowpass filter
$\Rightarrow H_0(Z)$ would have
factor $(1+z^{-1})^2$

$H_0(z)$ has 3 roots.
2 already specified
($z = -1$).
3rd to be determined.

The impulse response is orthogonal to even shifts.

$h_0 \quad h_1 \quad h_2 \quad h_3$

$\uparrow$

0

Nontrivial equation (shift by 2):

$$\cdots\ h_0\ \ h_1\ \ h_2\ \ h_3\ \cdots$$
$$\cdots\cdots\ \ h_0\ \ h_1\ \cdots$$

$$\underline{h_0 h_2 + h_1 h_3 = 0}.$$

$$h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3}$$

$$= \underline{c_0}\left(1 + z^{-1}\right)^2\left(1 + B_0 z^{-1}\right)$$

Compare coeff on both sides

$$(1+z^{-1})^2(1+B_0 z^{-1})$$

$$(1+2z^{-1}+z^{-2})(1+B_0 z^{-1})$$

$$1+2z^{-1}+z^{-2}$$

$$B_0 z^{-1}+2B_0 z^{-2}+B_0 z^{-3}$$

Except to within constant c_0,

h_0	h_1	h_2	h_3
1	$2+B_0$	$1+2B_0$	B_0

$$h_0 h_2 + h_1 h_3 = 0$$

$$\Rightarrow 1 + 2B_0 + (2 + B_0) B_0 = 0 .$$

$$\Rightarrow 1 + 4B_0 + B_0^2 = 0 .$$

Solving the equation

$$B_0 = \frac{-4 \pm \sqrt{16-4}}{2}$$

$$= \frac{-4 \pm 2\sqrt{3}}{2} = \underline{\underline{-2 \pm \sqrt{3}}}$$

Solution

$B_0 = \sqrt{3} - 2$ lies inside unit circle

$B_0 = -\sqrt{3} - 2$ lies outside unit circle.

We take the "minimum phase" solution
(B_0 inside unit circle)

$B_0 = \sqrt{3} - 2$

$|B_0| < 1$

Impulse response of Daubechies lowpass filter:

1

$\uparrow$

0

$$2+B_0 = \sqrt{3} \qquad 1+2B_0 = 1-4+2\sqrt{3} = -3+2\sqrt{3} \qquad B_0 = \sqrt{3}-2$$

to within constant C_0!

$$K_0(z) = H_0(z) H_0(\bar{z}^{-1})$$

$$K_0(z) + K_0(-z)$$

$$= \underline{\text{Constant}}$$

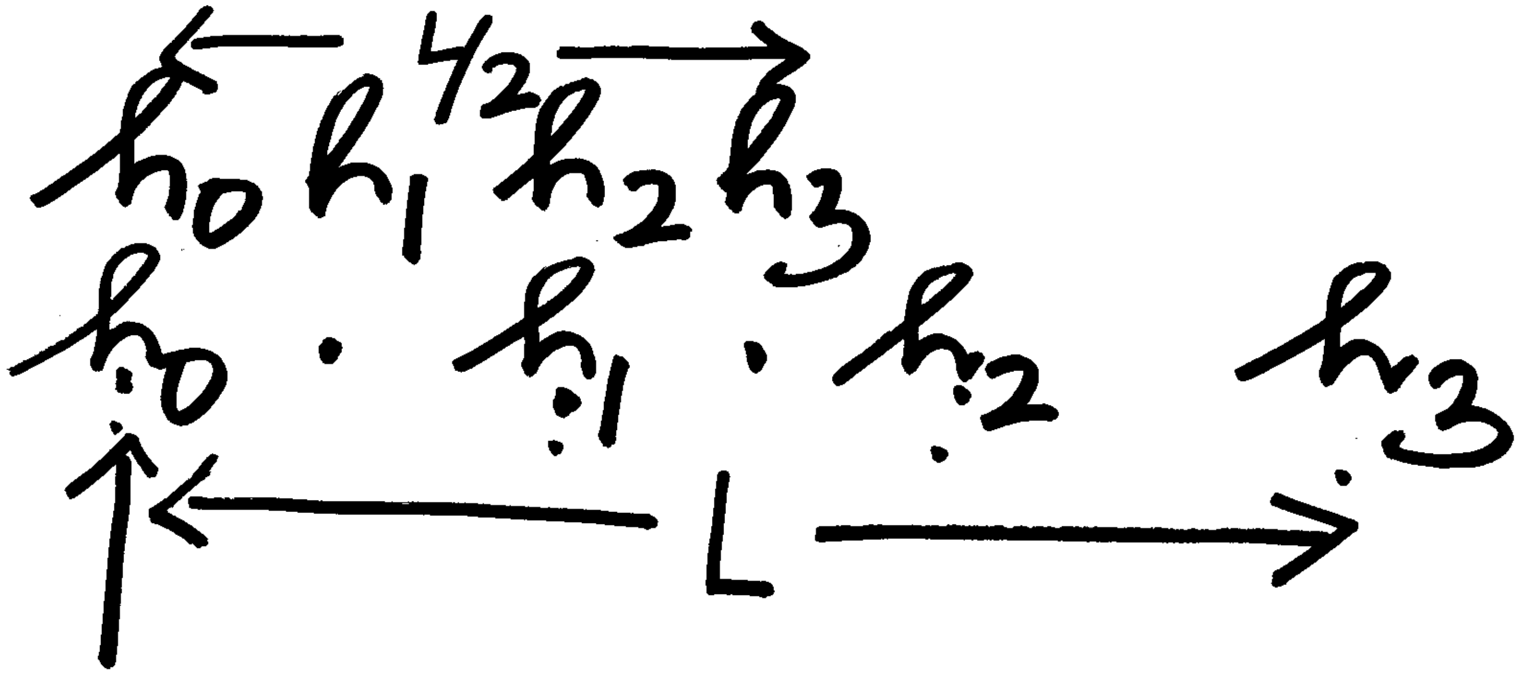
Choice of C_0 means choosing this constant.

The sequence corresponding to $K_0(z)$, at 0^{th} location is essentially the norm^2 in $l_2(\mathbb{Z})$ of $\underset{\substack{\uparrow \\ 0}}{h_0}\, h_1\, h_2\, h_3$

Choose c_0 so that

$$c_0^2 \left\{ h_0^2 + h_1^2 + h_2^2 + h_3^2 \right\} = 1$$

with $h_0 \cdots h_3$ as chosen
(without c_0).



Construction of $\Phi(t)$
(scaling function or
"father wavelet")

$h_0\, h_1\, h_2\, h_3$ $\qquad$ $L + L/2 + L/4 \cdots$

$\longleftarrow L/4 \longrightarrow$

$h_0 \;\; h_1 \;\; h_2 \;\; h_3$ $\qquad$ $L + L/2$

$\longleftarrow L/2 \longrightarrow$

$h_0 \qquad h_1 \qquad h_2 \qquad h_3$

$\longleftarrow \quad L \quad \longrightarrow$

As we carry out this iteration to infinity

length $\equiv$

$$L + \frac{L}{2} + \frac{L}{4} \cdots = 2L$$

geometric series

We converge towards a COMPACTLY SUPPORTED SCALING FUNCTION

The independent variable region over which ....

∴ the (contd)

scaling function is nonzero is

FINITE.

This second member of Daubechies' family: annihilate (do away with, make zero)...

... polynomials of degree 1

i.e. sequences of the form $a_0 + a_1 n$

a_0, a_1 constants

The filters need to obey a property called "regularity" for the iterated convolution to converge.

One guaranteed way of forcing regularity is introduction of factors $(1+z^{-1})$. (zeros at $z=-1$).

Exercise 2:

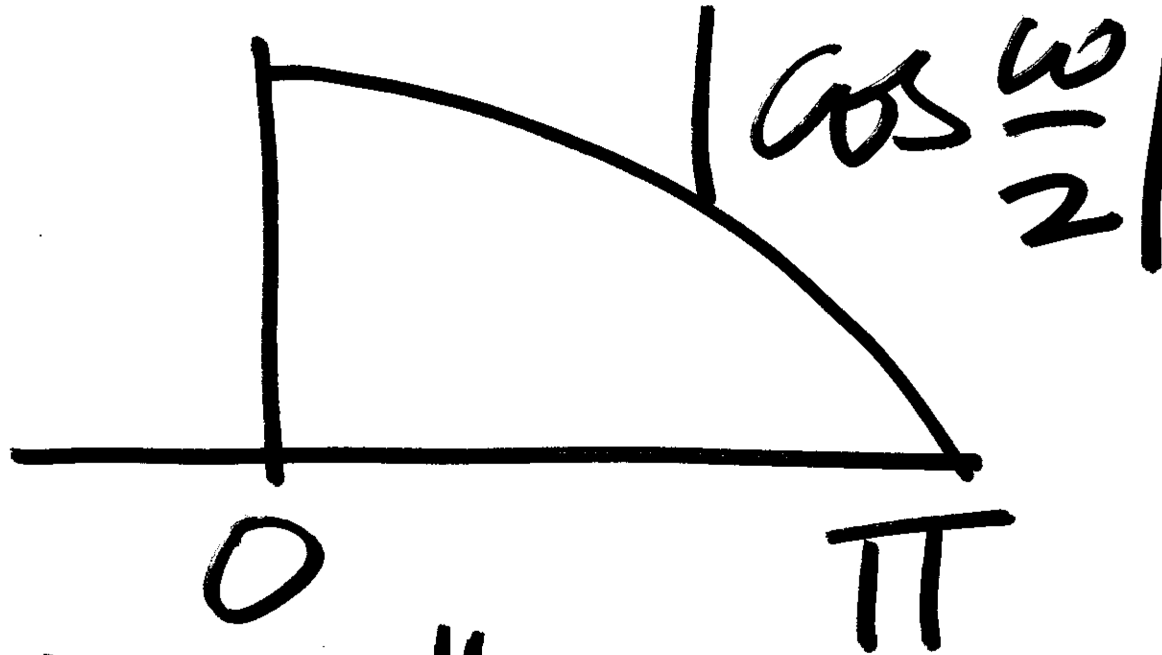
Obtain the frequency response of the "Daub-4" lowpass filter.

i.e. Obtain

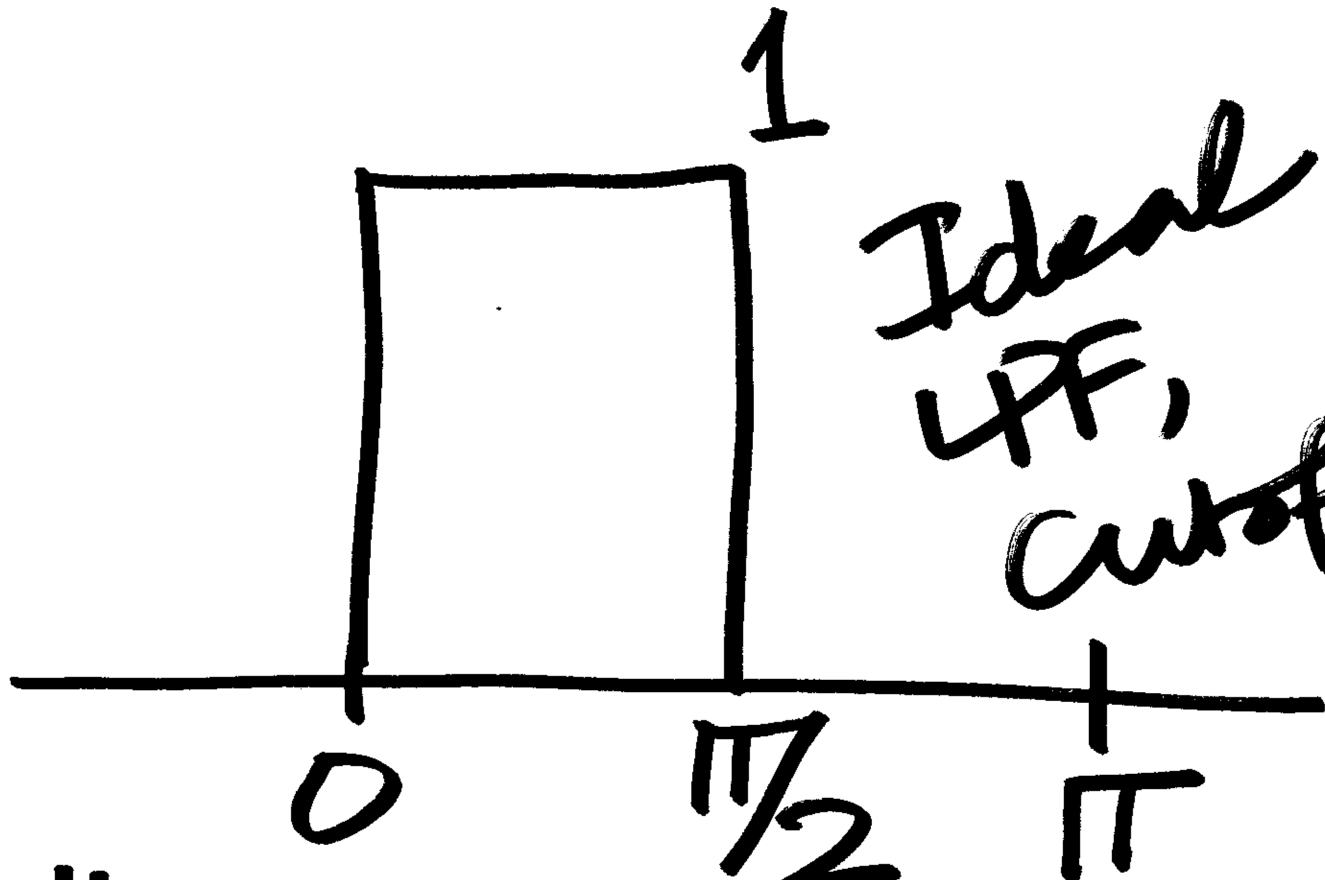
$$h_0 + h_1 e^{-j\omega} + h_2 e^{-j2\omega} + h_3 e^{-j3\omega}.$$

evaluate at many finely spaced ω.

Haar:



In "Daub-4" are we going closer to ideal?



Ideal LPF, cutoff $\frac{\pi}{2}$

"Ideal"

Next member of Daubechies family:

$$H_0(z) = (1 + z^{-1})^3 (1 + \tilde{B}_0 z^{-1})(1 + \tilde{B}_1 z^{-1})$$

Constraints : (nontrivial)
orthogonal to
translation by 2
and 4.

$h_0 \; h_1 \; h_2 \; h_3 \; h_4 \; h_5 \; \cdots$

$\qquad\qquad h_0 \; h_1 \; h_2 \; h_3 \; \cdots$

dot prod $= 0$

$\qquad\qquad\qquad\qquad h_0 \; h_1 \; \cdots$

dot product $= 0$

Minimal requirements of Conjugate Quadrature Filter Bank Design:

Principal equation:

$$K_0(z) + K_0(-z) = \text{constant}$$

$$K_0(z) = H_0(z) H_0(\bar{z}^{\,-1})$$

$$K_0(e^{j\omega})$$

$$= H_0(e^{j\omega}) H_0(e^{-j\omega})$$

With real impulse response,

...

$$|H_0(e^{j\omega})|^2 = K_0(e^{j\omega}).$$

<u>Design problem:</u>

Design $H_0(e^{j\omega})$: nonnegative frequency (lowpass) response $|H_0(e^{j\omega})|^2$

$\pi/2$

$K_0(z)$ corresponds to <u>real, even impulse response</u> with <u>even samples = 0 except 0^{th}</u>.