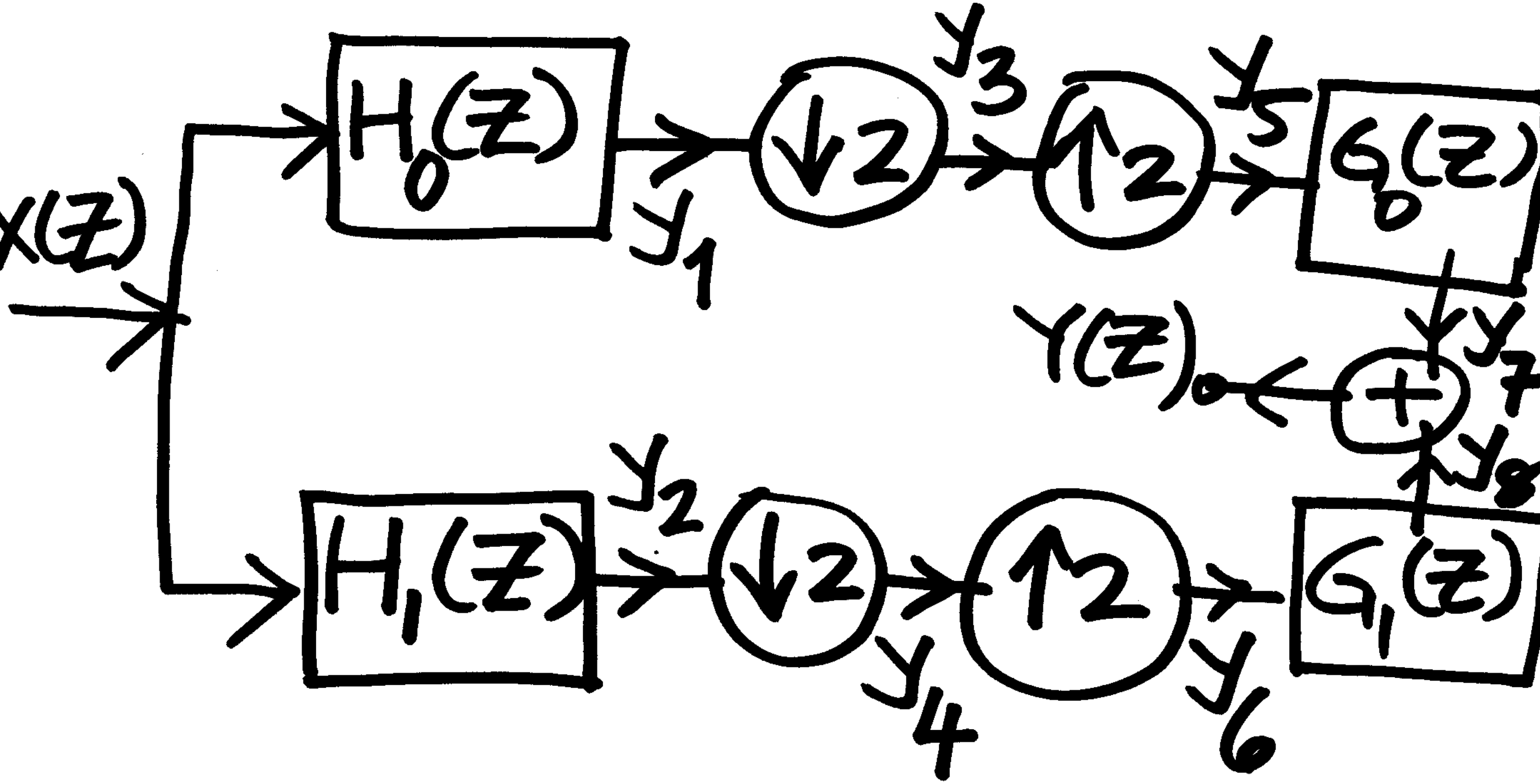
Prof. V. Gadre
Lec. No. 11
Date: 25|1|10

LECTURE 11
TWO CHANNEL FILTER BANK


X(Z)
H0(Z)
H1(Z)
y1
y2
↓2
↓2
y3
y4
↑2
↑2
y5
y6
G0(Z)
G1(Z)
y7
y8
+
Y(Z)

Upsampler:

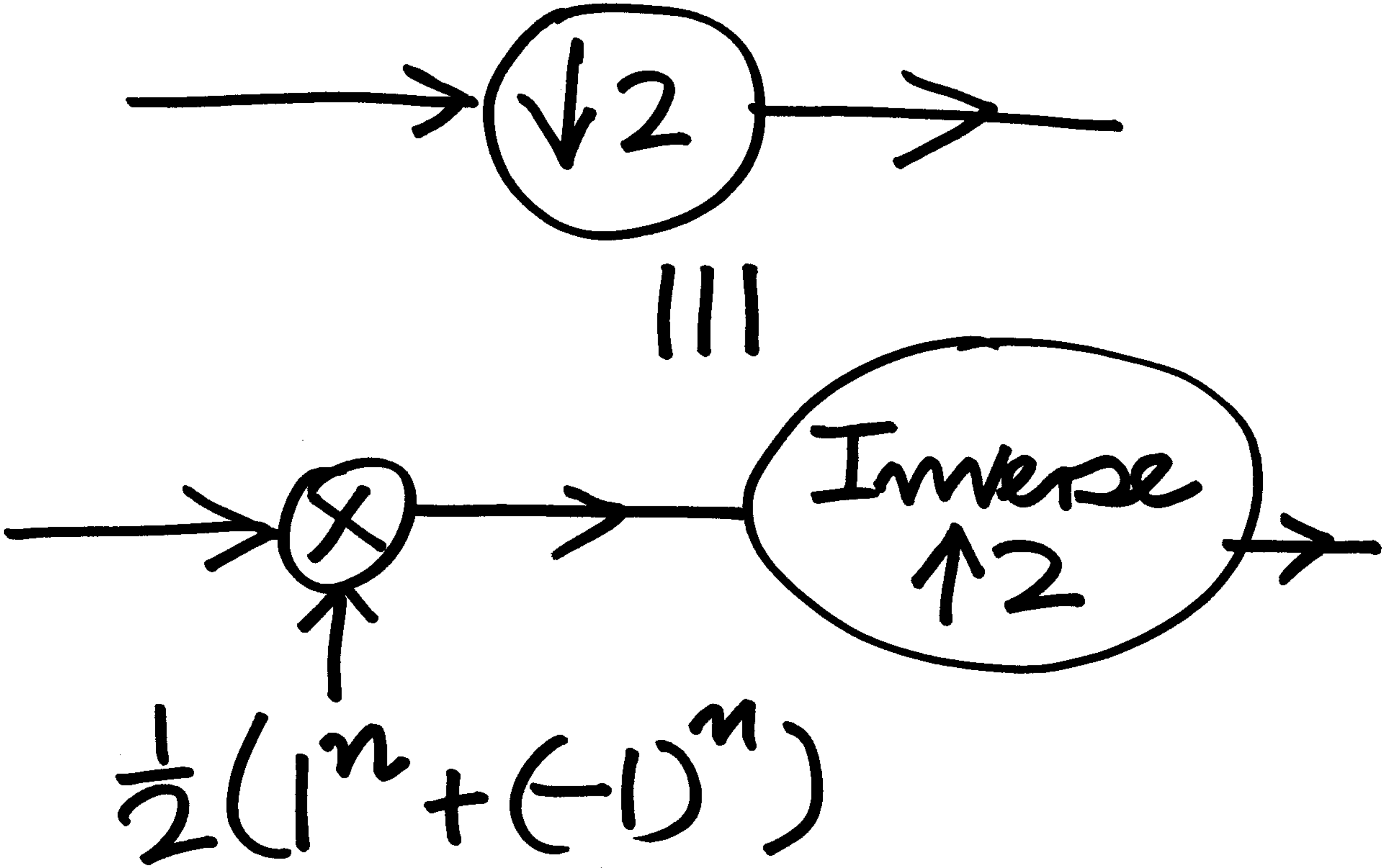
$$x_{in}[n] \longrightarrow \uparrow 2 \longrightarrow x_{out,U}[n]$$

$$X_{out,U}(Z) = X_{in}(Z^2)$$

Downsampler:

$$x_{in}[n] \longrightarrow \downarrow 2 \longrightarrow x_{out,D}[n]$$

$$X_{out,D}(z) = \frac{1}{2}\left\{ X_{in}\left(z^{1/2}\right) + X_{in}\left(-z^{1/2}\right) \right\}$$


↓2
≡
×
Inverse
↑2
$\frac{1}{2}(1^n+(-1)^n)$

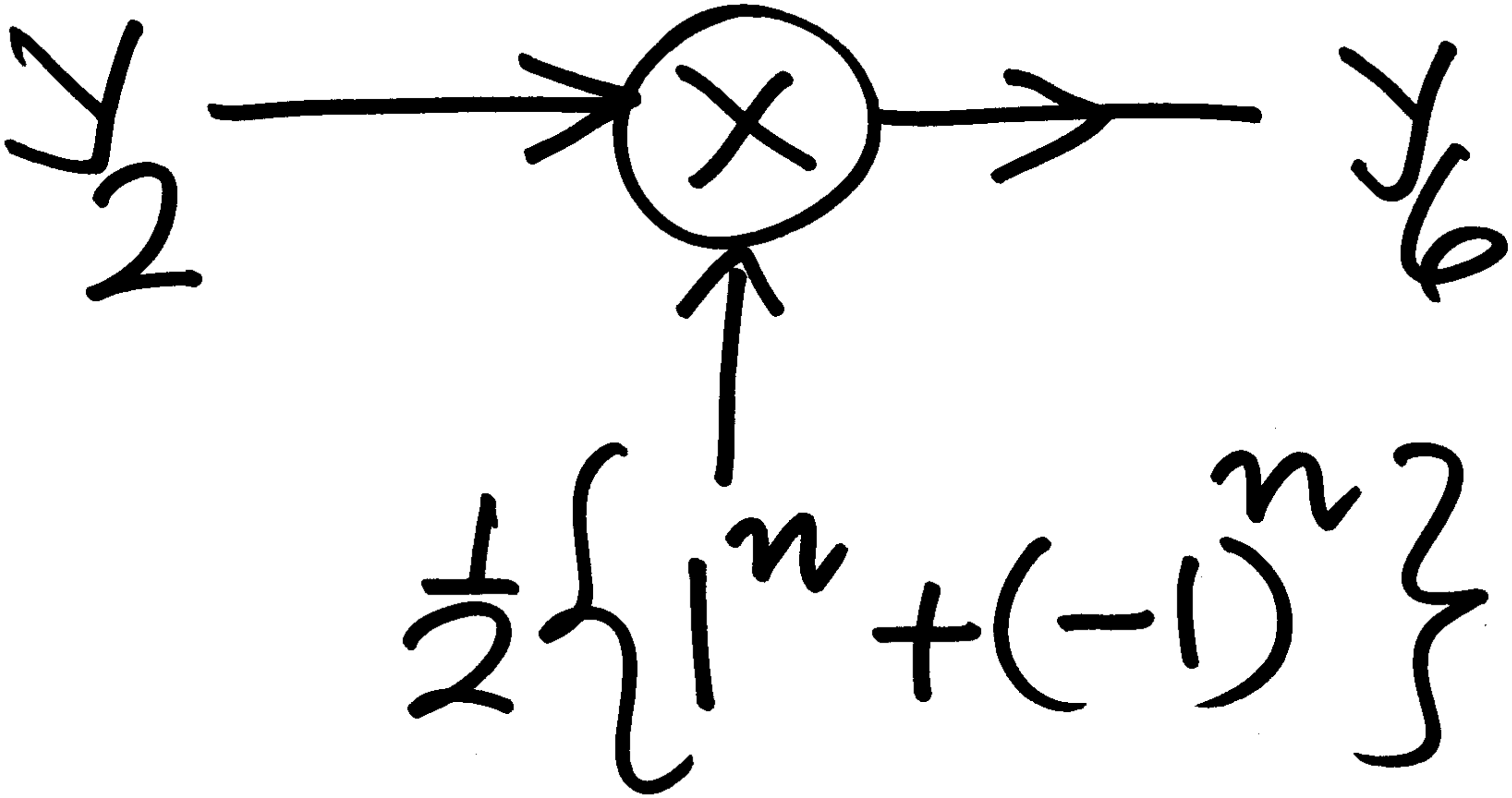
$$Y_1(z) = H_0(z)X(z)$$

$$Y_2(z) = H_1(z)X(z)$$

$$
\begin{tikzcd}
y_1 \arrow[r] & \otimes \arrow[r] & y_5 \\
& \frac{1}{2}\left\{1^n + (-1)^n\right\} \arrow[u] &
\end{tikzcd}
$$

$$Y_5(z)$$

$$= \frac{1}{2}\left\{ Y_1(z) + Y_1(-z) \right\}$$


y_2
$\times$
y_6
$\frac{1}{2}\{1^n + (-1)^n\}$

$$Y_6(z) =$$

$$\frac{1}{2}\left\{Y_2(z) + Y_2(-z)\right\}$$

$$Y_7(z) = Y_5(z)\, G_0(z)$$

$$Y_8(z) = Y_6(z)\, G_1(z)$$

$$Y(z) = Y_7(z) + Y_8(z)$$

$=$ (let us consider Y_8 first)

$$Y_8(z) = Y_6(z)\, G_1(z)$$

$$Y_6(z) = \frac{1}{2}\left\{ Y_2(z) + Y_2(-z) \right\}$$

$$Y_2(z) = X(z)H_1(z)$$

$$Y_2(-z) = X(-z)H_1(-z)$$

In total

$$Y(z) = \underline{T_0(z)} X(z) + \underline{T_1(z)} X(-z)$$

$$J_0(z) = \frac{1}{2}\left\{ G_0(z)H_0(z) + G_1(z)H_1(z) \right\}$$

$$T_1(z) = \frac{1}{2}\left\{ G_0(z)H_0(-z) + G_1(z)H_1(-z) \right\}$$

$Y(z) =$ (in z domain) linear combination of $X(z)$ and $X(-z)$??

What does $X(-z)$ do?

$$z \leftarrow e^{j\omega}$$

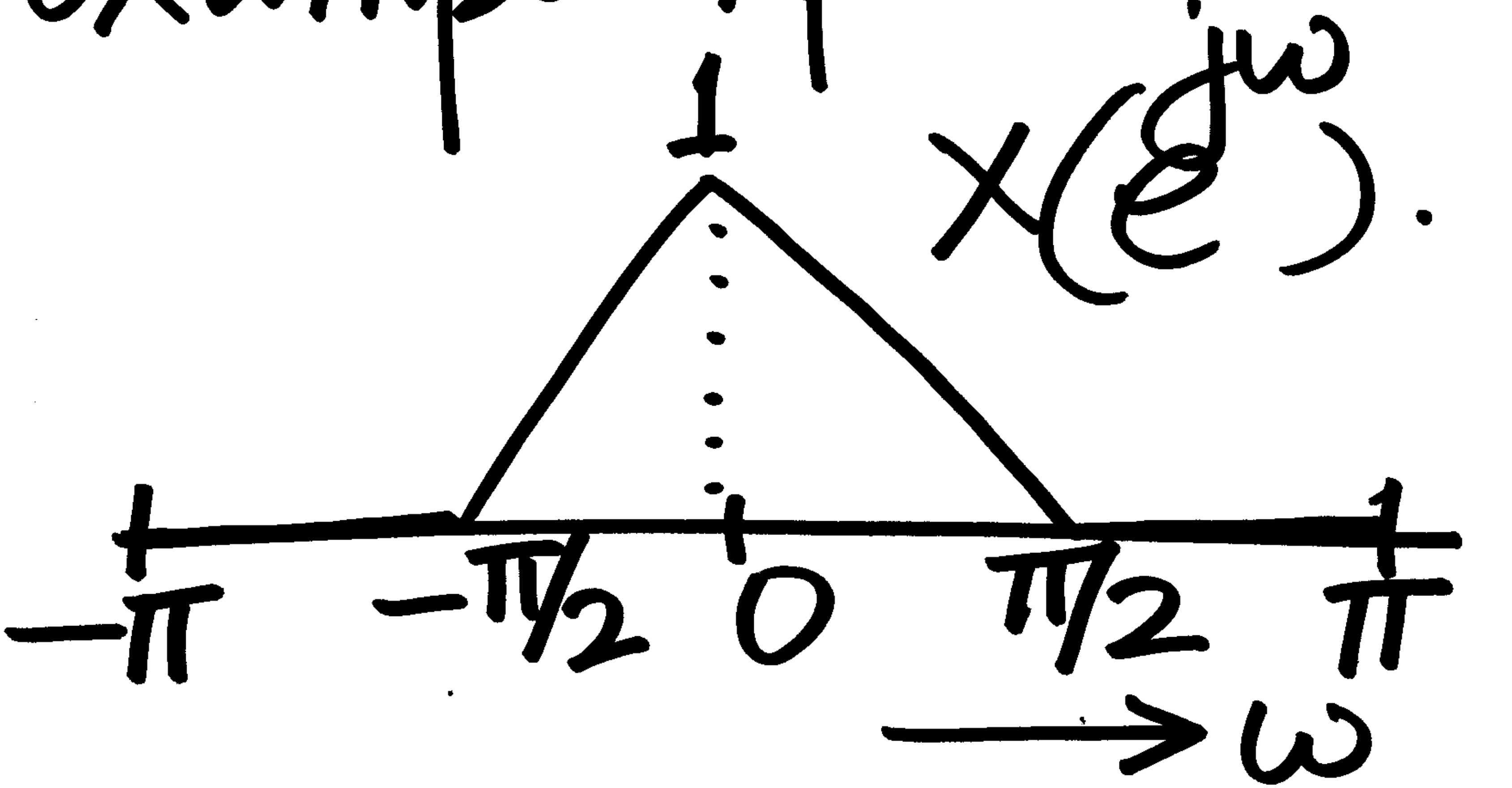
$$(-z) = e^{\pm j\pi} e^{j\omega}$$

for $z = e^{j\omega}$

Replacing $e^{j\omega}$

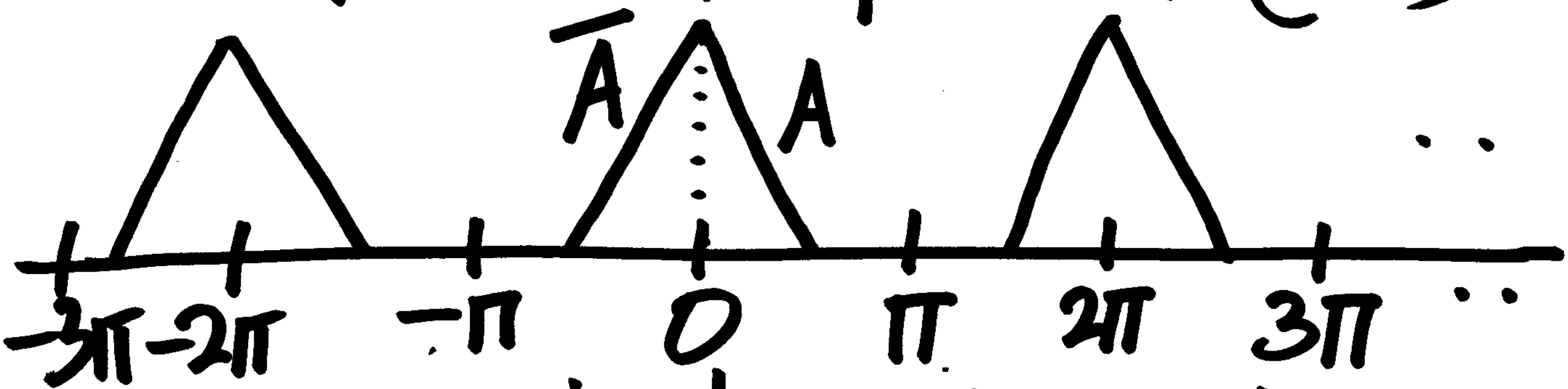
by $e^{j(\omega \pm \pi)}$

Example spectrum:



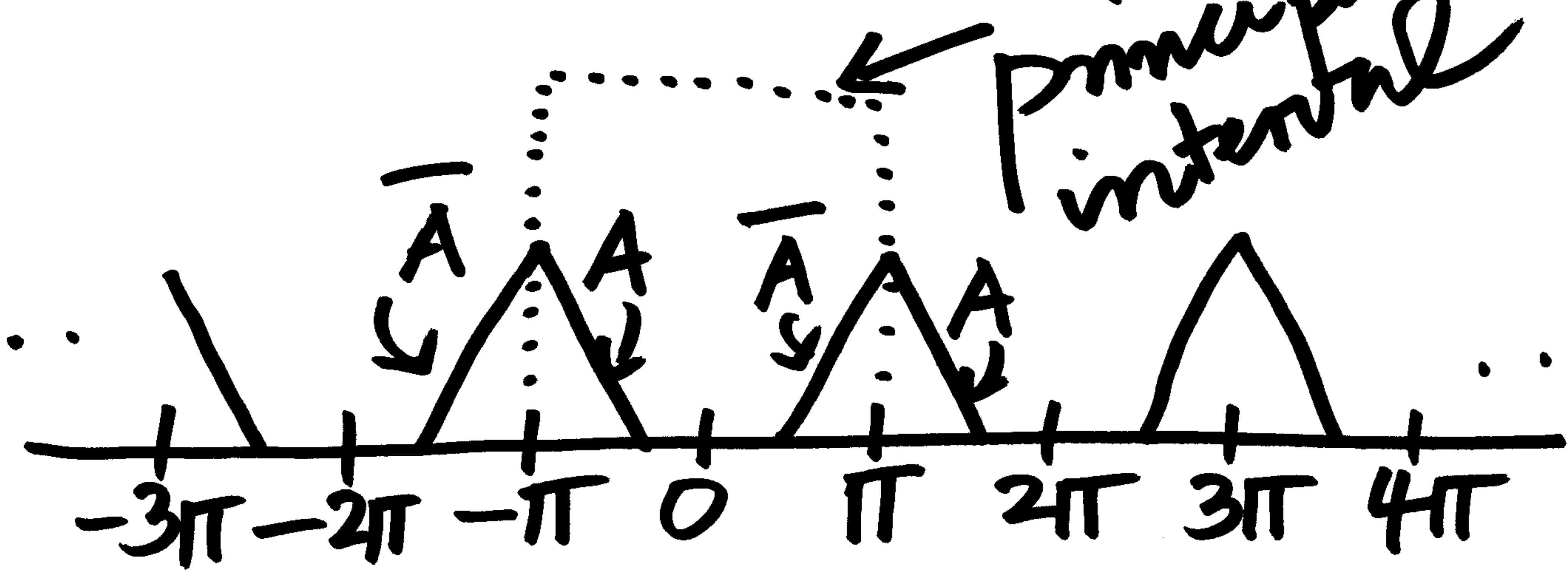
Actually:

periodic repeat: $X(e^{j\omega})$

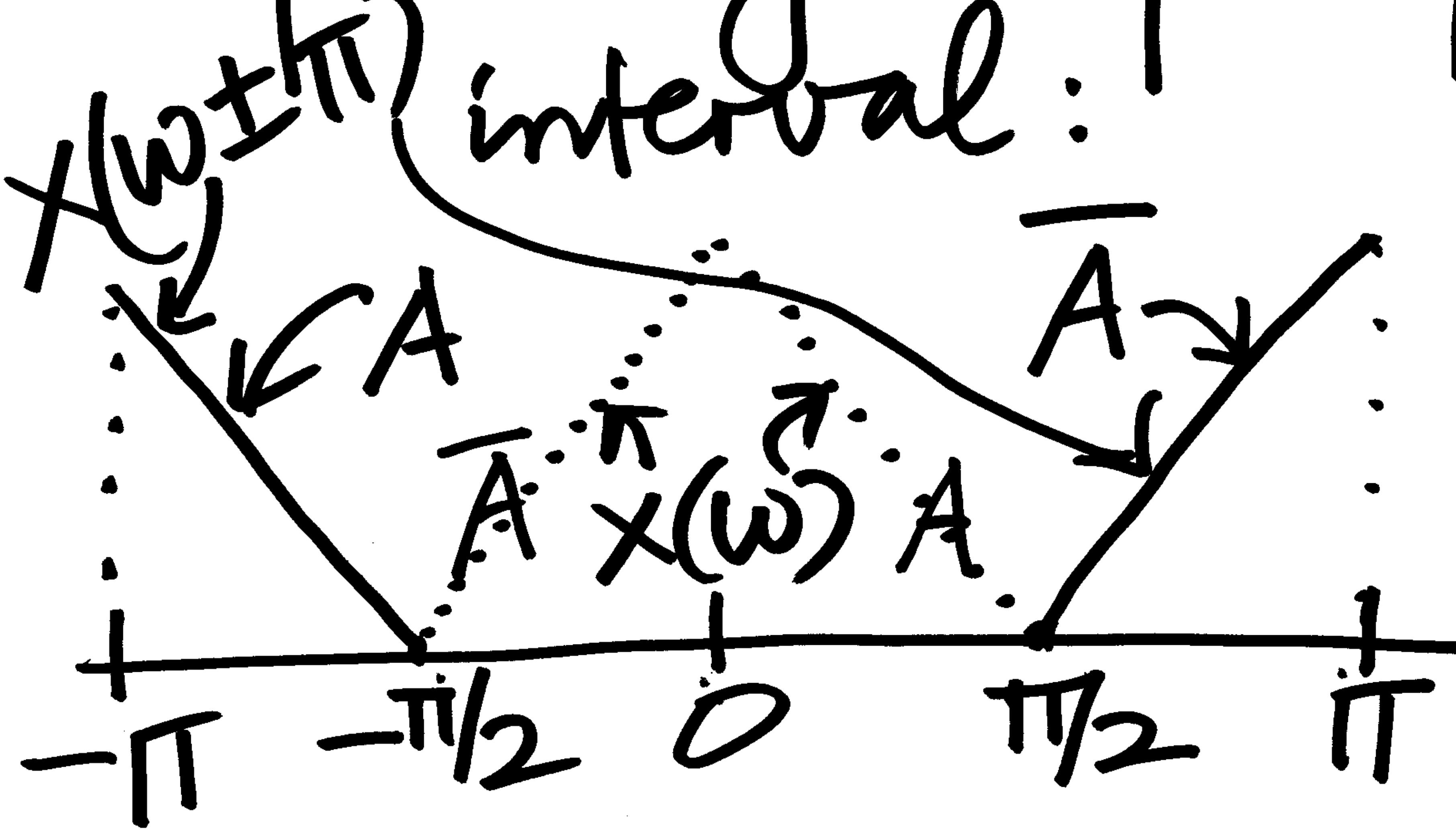


(period 2π) $\longrightarrow \omega$

$X(e^{j(\omega \pm \pi)})$:



Expanding the principal interval:



$X(-z)$ term or equivalently $X(-e^{j\omega})$ ...

or $X(e^{j(\omega \pm \pi)})$

term is the consequence of ALIASING

Aliasing does two things:

(i) one frequency manifested as another

(ii) increasing actual frequency → decreasing apparent!

If we want $Y(Z)$ to reconstruct $X(Z)$ we must first do away with ALIASING!

i.e. $T_1(z) = 0$

$$G_0(z)H_0(-z) + G_1(z)H_1(-z) = 0$$

$$\frac{G_1(z)}{G_0(z)} = -\frac{H_0(-z)}{H_1(-z)}.$$

Very simple choice:

$$G_1(z) = \pm H_0(-z)$$

$$G_0(z) = \mp H_1(-z)$$

More generally:

$$G_1(z) = \pm R(z) H_0(-z)$$

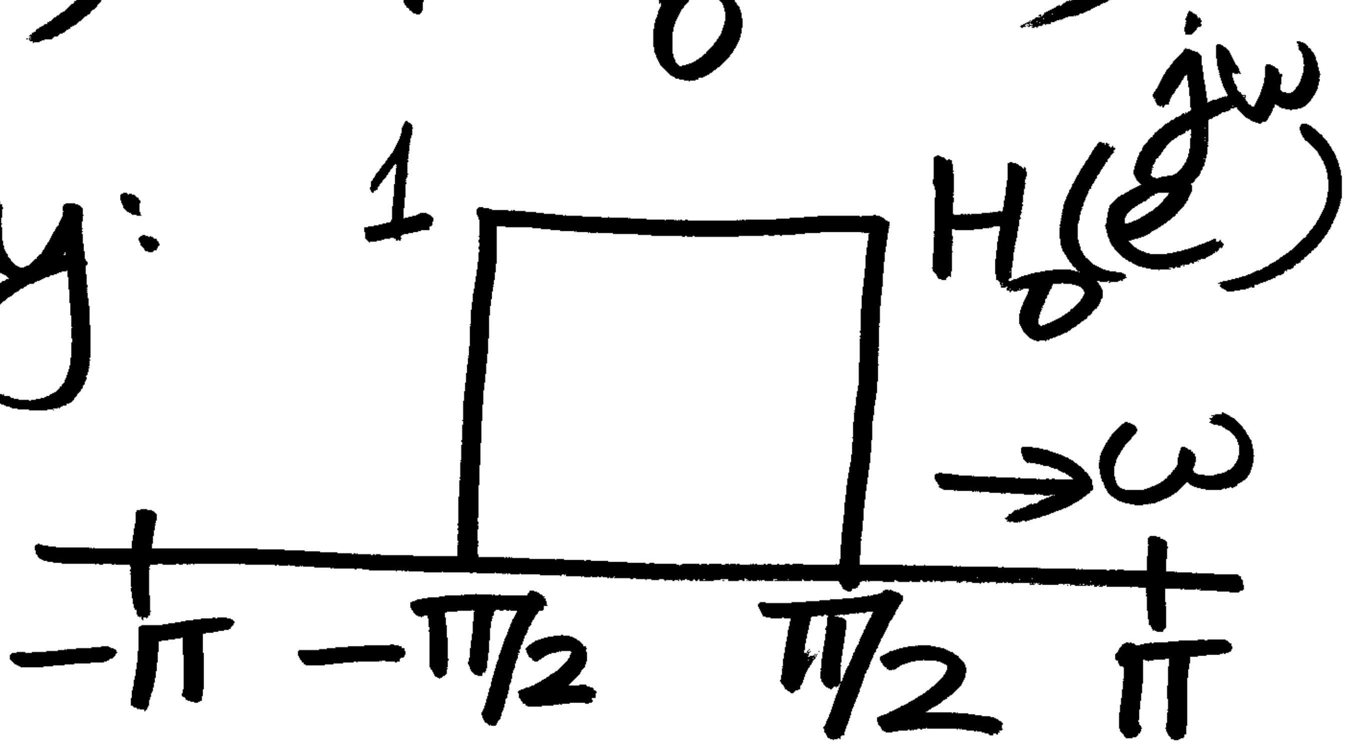
$$G_0(z) = \mp R(z) H_1(-z)$$

$R(z)$: the factor "cancelled"

Interprete

$$G_1(z) = +H_0(-z)$$

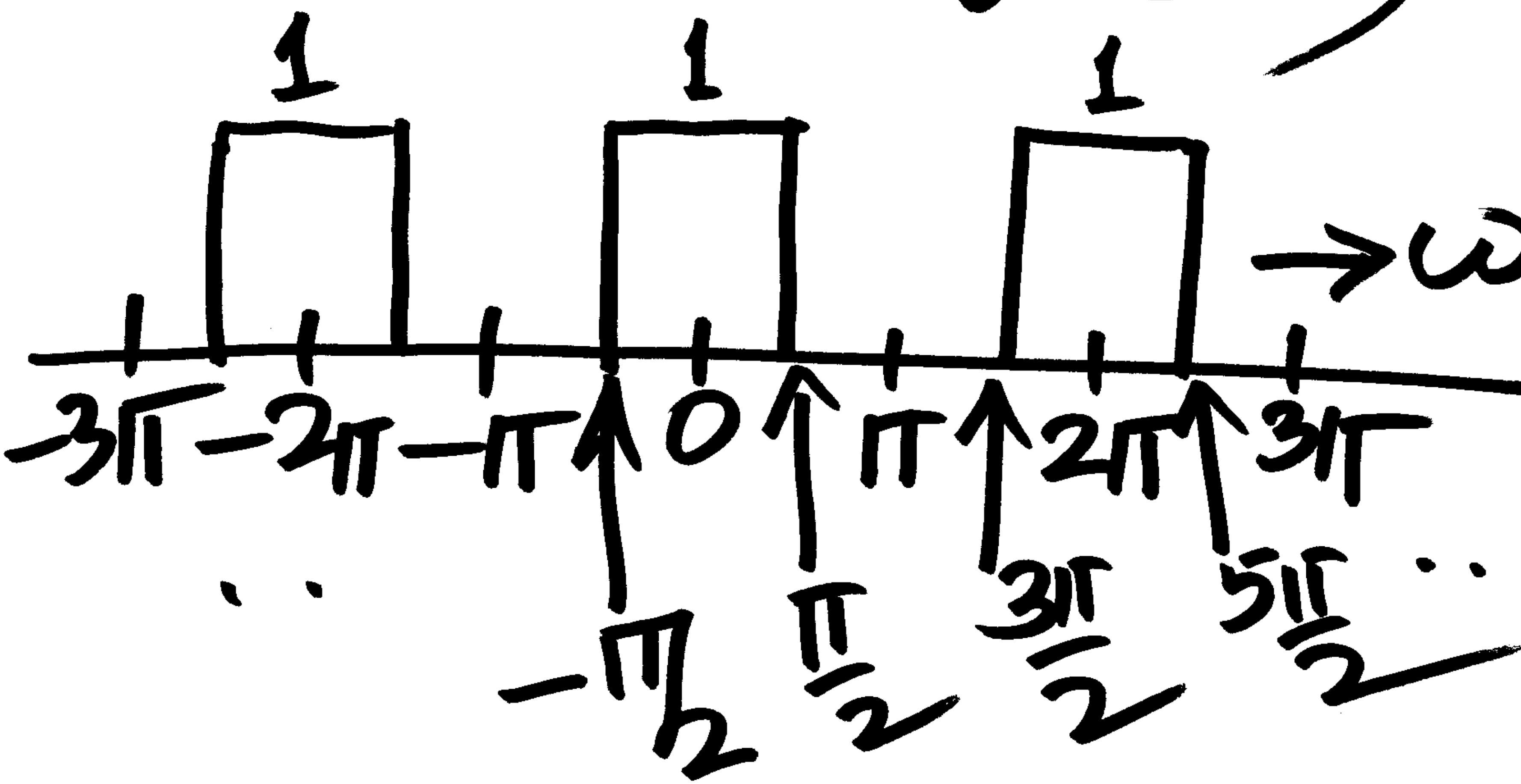
Ideally:



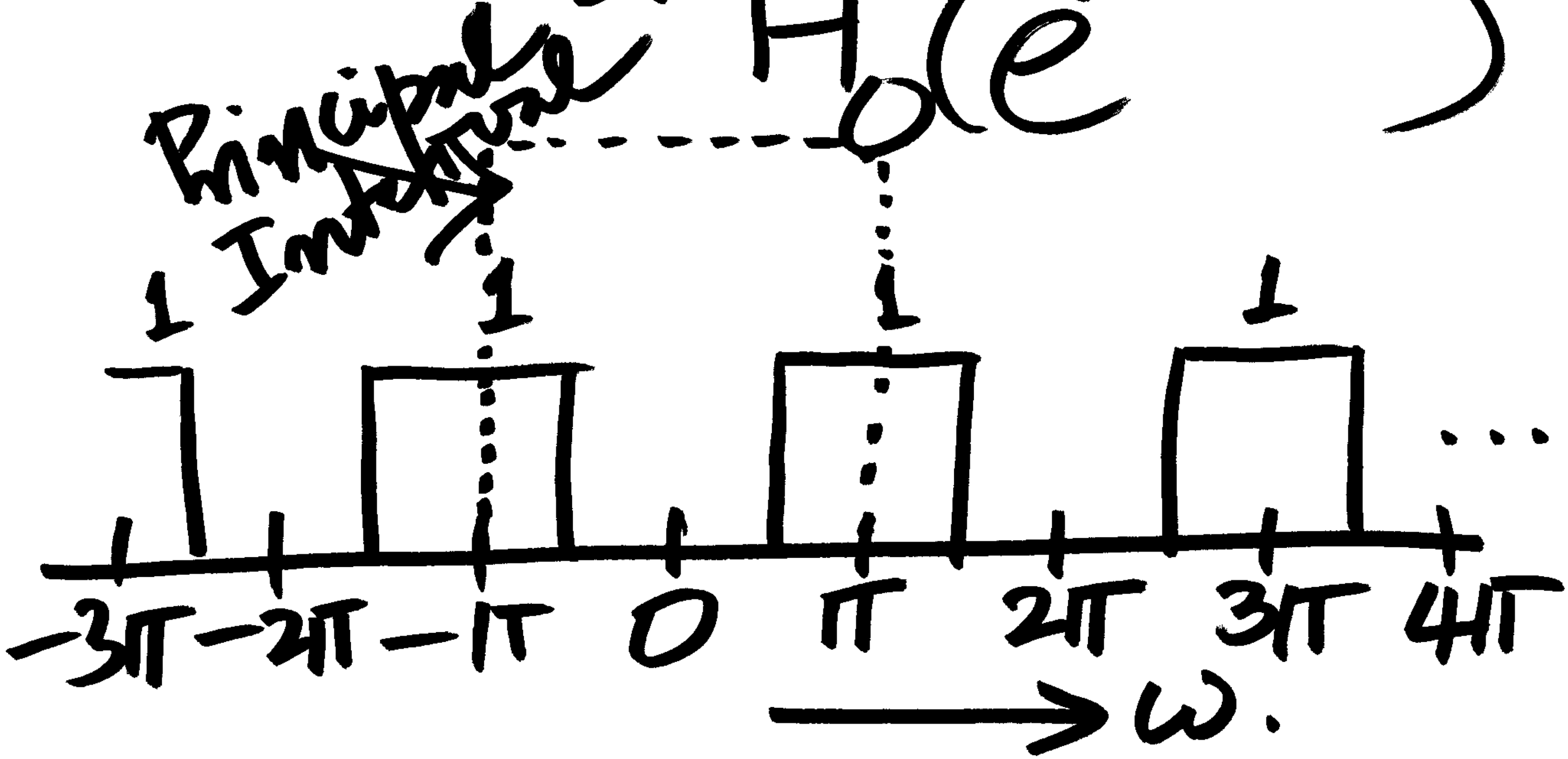
$$H_0(-z)\Big|_{z \leftarrow e^{j\omega}}$$

$$= H_0\left(e^{j(\omega \pm \pi)}\right)$$

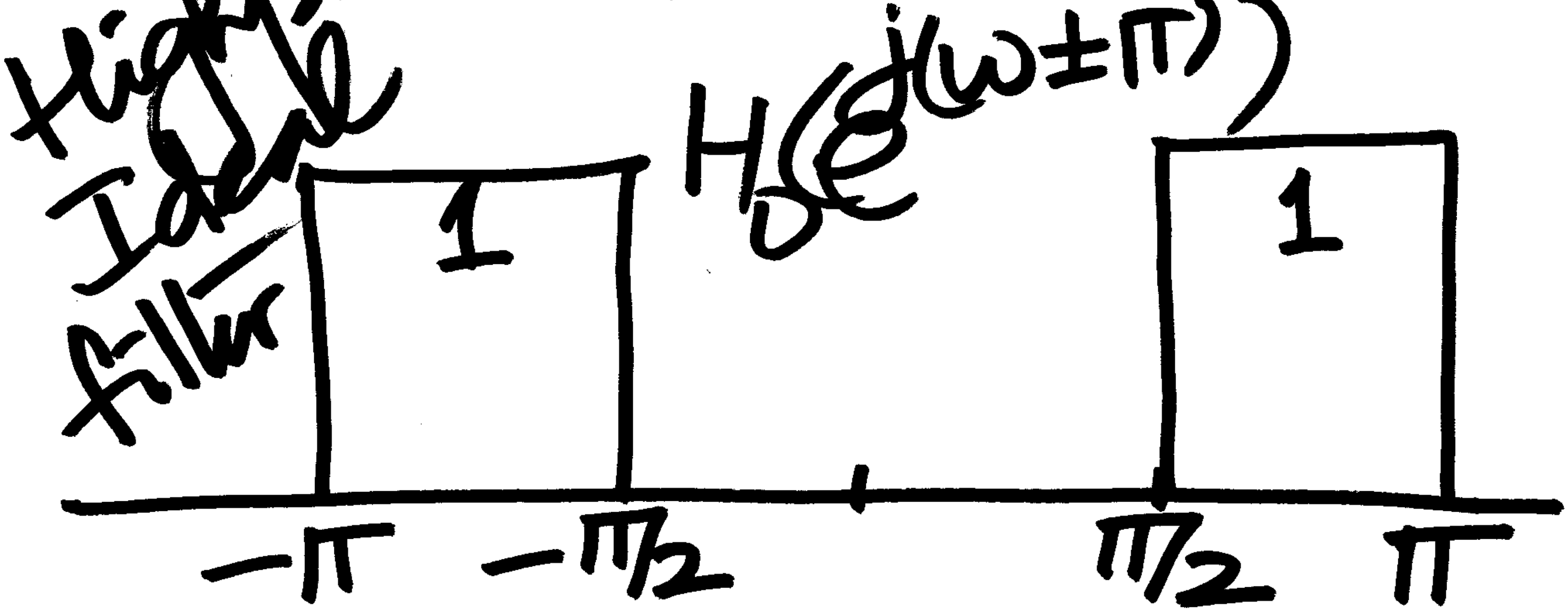
"Zoomed" $H_0(e^{j\omega})$



"Zoomed" $H_0(e^{j(\omega \pm \pi)})$



Showing only principal interval



$G_1(z) = H_0(-z)$

makes a lot of sense for the ideal filter!

Exercise: Show, similarly that

$$\text{ideal HPF}, \frac{\pi}{2} \xrightarrow[z \leftarrow -z]{} \text{Ideal LPF}, \pi/2$$