## WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING

Lecture 21: STFT and Wavelet Transform

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## Self Evaluation Quizzes

**Q** 1. What is the significance of  $\Omega_0$  and  $\tau_0$  in the STFT expression?

**Ans.** In the STFT expression  $\tau_0$  represents translation along time axis and the quantity  $\Omega_0$  represents dilation. We choose  $\tau_0$  and  $\Omega_0$  and place the window, according to time and frequency around which we want to take Fourier transform.

**Q** 2. Explain filter bank interpretation of CWT.

**Ans.** Continuous Wavelet Transform(CWT) of a function is a dot product of the function x(t) with dilated and translated versions of the wavelet function  $\Psi(t)$ .

$$W_s^{\Psi} x(\tau, s) = \frac{1}{\sqrt{s}} \int x(t) \Psi(\frac{t-\tau}{s}) dt$$

The wavelet function  $\Psi(t)$  must obey admissibility condition to analyze and perfectly reconstruct the signal without any loss of information *i.e.* 

$$\int \frac{|\Psi(\omega)|^2}{|\omega|} \, d\omega < +\infty$$

This implies that the Fourier transform of the wavelet function goes to zero at 0 and  $\infty$  frequency, hence wavelet function must have a bandpass nature. Filter bank implementation of CWT:

At a particular time  $\tau$ , the dot product can be interpreted as series of filter banks dilated by



 $s^x \omega$  as shown in the figure above. Here since  $s \in \mathbb{R}$  hence the number of filters required will

be infinite.

Though CWT provides high time resolution for high frequencies and low time resolution for low frequencies, it cannot be realized and cannot be used on real data due to its infinite size. In order to achieve above it is necessary to discretize the dilation parameter.