

Trap door Function.

f : Easy problem
(Polynomial time)

f^{-1} : Hard Problem.

RSA : f : Multiplication of 2 Large primes.

f^{-1} : Given the result of multiplication, to find prime factors.

$$p; a \in \mathbb{Z} \quad a \not\equiv 0 \pmod{p^2}$$

$$\boxed{a^{p-1} = 1 \pmod{p}}$$

$$S: 1, 2, 3, 4, \dots, (p-1)$$

$$s \in S$$

$$r \in S$$

$$ar = as \pmod{p}$$

$$a(r-s) = 0$$

$\underbrace{\quad}_{r-s}$ is divisible by p .

$$S: 1, 2, 3, \dots, p-1$$

$$S': a, 2a, 3a, \dots, a(p-1) \pmod{p}$$

$$1 \cdot 2 \cdot 3 \cdot \dots \cdot (p-1) = a^{p-1} \cdot (1 \cdot 2 \cdot \dots \cdot p-1)$$

$$\boxed{a^{p-1} = 1}$$

Fermat's
Little Theorem

Euler's Theorem

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$$\phi(18) = 6$$

1, 5, 7, 11, 13, 17

If n is a prime

$$\underline{\phi(n) = n - 1}$$