Prof. D. K. Ghosh


$$\hat{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

$$|\hat{n},+\rangle = e^{i\varphi/2}\begin{pmatrix}\cos\frac{\theta}{2}\, e^{-i\varphi/2} \\ \sin\frac{\theta}{2}\, e^{i\varphi/2}\end{pmatrix}$$

$$|\hat{n},-\rangle = e^{i\varphi/2}\begin{pmatrix}\sin\frac{\theta}{2}\, e^{-i\varphi/2} \\ -\cos\frac{\theta}{2}\, e^{i\varphi/2}\end{pmatrix}$$

$$|z -1\rangle = \begin{pmatrix}0 \\ 1\end{pmatrix}$$

$$= \sin\frac{\theta}{2}|n,+\rangle - \underbrace{\cos\frac{\theta}{2}}_{}|n,-\rangle.$$

$$\cos^2\frac{\theta}{2}$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left[\,|01\rangle - |10\rangle\,\right]$$

$$(\sigma_i^A + \sigma_i^B)|\psi\rangle = 0$$

$$\langle\psi|(\sigma^A\cdot\hat{a})(\sigma^B\cdot\hat{b})|\psi\rangle$$

$$= -\langle\psi|(\sigma^A\cdot\hat{a})(\sigma^A\cdot\hat{b})|\psi\rangle$$

$$= -\sum_{ij}\langle\psi|\sigma_i^A\sigma_j^A|\psi\rangle a_i b_j \,.$$

For terms $i \neq j$ 

$$\cancel{\langle\psi| (\sigma_x^A \sigma_y^A + \sigma_y^B \sigma_y^B}$$

$$-\langle\psi| \underline{\sigma_x^A \sigma_y^A} + \sigma_x^A \sigma_z^A + \sigma_y^A \sigma_z^A |\psi\rangle$$

$$= -\Big[ \big(\langle 01| - \langle 10|\big) \sigma_x^A \underline{\sigma_y}^A \big(|01\rangle - |10\rangle\big)$$

$$+ \big(\langle 01| - \langle 10|\big) \sigma_x^A \sigma_z^A \big(|01\rangle - |10\rangle\big)$$

$$+ \big(\langle 01| - \langle 10|\big) \sigma_y^A \sigma_z^A \big(|01\rangle - |10\rangle\big) \Big].$$

$$= -\Big[ \big(\langle 01| - \langle 10|\big) \big(i|01\rangle + i|10\rangle\big) \qquad \sigma_y|0\rangle = i|1\rangle, \quad \sigma_y|1\rangle = -i|0\rangle$$

$$+ \cdots + \cdots$$

$$= -\big[ (i - i) + 0 + 0 \big] = 0$$

$$-\sum_i \langle \psi | a_i b_i \underbrace{\sigma_i^A \sigma_i^A}_{\sigma_i^2 = 1} | \psi \rangle$$

$$= -\sum_i a_i b_i \langle \psi | \psi \rangle$$

$$= -\vec{a} \cdot \vec{b} = -\cos\theta$$

$$\frac{I + \hat{n}\cdot\vec{\sigma}}{2}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{2}\left[n_x\sigma_x + n_y\sigma_y + n_z\sigma_z\right]\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{2}n_x\begin{pmatrix}0\\1\end{pmatrix} + \frac{1}{2}n_y(i)\begin{pmatrix}0\\1\end{pmatrix} + \frac{1}{2}n_z\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1+\cos\theta\\ \sin\theta\cos\phi + i\sin\theta\sin\phi\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1+\cos\theta\\ \sin\theta\, e^{i\phi}\end{pmatrix} = e^{i\phi}\begin{pmatrix}\cos^2\frac{\theta}{2}\\ \frac{\sin\theta}{2}\end{pmatrix}$$

$n_x = \sin\theta\cos\phi$

$n_y = \sin\theta\sin\phi$

$n_z = \cos\theta$

$$\frac{I + \hat{n}.\vec{\sigma}}{2}\begin{pmatrix}1\\0\end{pmatrix} = \cos\frac{\theta}{2} e^{i\varphi/2} |n, +\rangle$$

$$P^A(n,+)\, P^B(m,+)\, \underbrace{|\psi^-\rangle}$$

$$= \frac{1}{\sqrt{2}} P^A(n,+)\, P^B(m,+)\left[\,|01\rangle - |10\rangle\,\right]$$

$$= \frac{1}{\sqrt{2}}.\Big[\cos\frac{\theta_1}{2} e^{i\varphi/2} \sin\frac{\theta_2}{2} e^{-i\varphi/2} |n+, m+\rangle$$

$$- \sin\frac{\theta_1}{2} \cos\frac{\theta_2}{2} |n+, m+\rangle\Big]$$

$$= \frac{1}{\sqrt{2}} \sin\underbrace{\left(\frac{\theta_2 - \theta_1}{2}\right)} |n+, m+\rangle$$

$$= \frac{1}{\sqrt{2}} \sin\frac{\Omega}{2} |n+, m+\rangle$$

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Alice – particle 1
Bob – particle 2.

Alice's measurement $S_z = \frac{\hbar}{2}$
$(\sigma_z = +1)$

Bob's measurement $S_z = -\frac{\hbar}{2}$
$\sigma_z = -1$

$$\hat{n} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

$$|\hat{n},+\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix} \equiv e^{i\varphi/2}\begin{pmatrix} \cos\frac{\theta}{2}\, e^{-i\varphi/2} \\ \sin\frac{\theta}{2}\, e^{+i\varphi/2} \end{pmatrix}$$

$$|\hat{n},-\rangle = e^{i\varphi/2}\begin{pmatrix} \sin\frac{\theta}{2}\, e^{-i\varphi/2} \\ -\cos\frac{\theta}{2}\, e^{i\varphi/2} \end{pmatrix}$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left[|01\rangle - |10\rangle\right]$$

$$(\sigma_i^A + \sigma_i^B)|\psi^-\rangle = 0$$

$$(\sigma_x^A + \sigma_x^B)\cdot\frac{1}{\sqrt{2}}\left[|01\rangle - |10\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[(|11\rangle - |00\rangle) + (|00\rangle - |11\rangle)\right]$$

$$= 0$$

$$(\sigma_z^A + \sigma_z^B)\cdot\frac{1}{\sqrt{2}}\left[|01\rangle - |10\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[(|01\rangle + |10\rangle) + (-|01\rangle - |10\rangle)\right]$$

$$= 0$$

$$\langle \psi | (\vec{\sigma} \cdot \hat{a})(\vec{\sigma} \cdot \hat{b}) | \psi \rangle$$