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$$\langle A \rangle = \mathrm{Tr}(A\rho)$$

$\mathrm{Tr}\,\rho = 1 \Rightarrow$ Pure system

$\rho^2 = \rho.$

For a pure system $S(\rho) = 0$

$$S(\rho) = -\mathrm{Tr}[\rho \log_2 \rho]$$

$$S(\rho) = -\sum_i \lambda_i \log \lambda_i$$

$$S(\rho) = -\sum_i \lambda_i \log \lambda$$

For a pure system

$\lambda_i = 1$ for specific i

$= 0$ everything else

$$S(\rho) = 0$$

$$S(\rho) = \log D.$$

$$S(\rho_{AB}) \leq S(\rho_A) + S(\rho_B)$$

Equality $\rho_{AB} = \rho_A \otimes \rho_B$

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad S(\rho) = 0$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\rho = |\psi\rangle\langle\psi|$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2}\left[\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad S(\rho) = 0.$$

$$\rho = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$S(\rho) = -\log_2\left(\frac{1}{2}\right)$$

$$= \log_2 2 = 1.$$

$$|\psi\rangle = \cos\theta\, |\overset{A}{0}\overset{B}{0}\rangle + \sin\theta\, |11\rangle$$

$$\rho^A = Tr_B(\rho_{AB})$$

$$\rho_{AB} = (\cos\theta\, |00\rangle + \sin\theta\, |11\rangle)$$
$$(\cos\theta\, \langle 00| + \sin\theta\, \langle 11|)$$

$$= \cos^2\theta\, |00\rangle\langle 00| + \sin^2\theta\, |11\rangle\langle 11|$$
$$+ \cos\theta \sin\theta\, (|00\rangle\langle 11| + |11\rangle\langle 00|)$$

$$Tr_B\, \rho_{AB} = \cos^2\theta\, |0\rangle\langle 0|$$
$$+ \sin^2\theta\, |1\rangle\langle 1|$$

$$\rho_A = \begin{pmatrix} \cos^2\theta & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$

$$S^A = -\cos^2\theta \log(\cos^2\theta) - \sin^2\theta \log(\sin^2\theta)$$

$$= -2\cos^2\theta \log(\cos\theta) - 2\sin^2\theta \log(\sin\theta)$$

$$S^B = -2\cos^2\theta \log(\cos\theta) - 2\sin^2\theta \log \sin\theta.$$

$$S^A + S^B = -4\left[\cos^2\theta \log(\cos\theta) + \sin^2\theta \log(\sin\theta)\right]$$

$$> 0$$