



$W$  = Number of microstates  
associated with a given  
macrostate

$n_1$  : Cell 1

$n_2$  : Cell 2

$\vdots$

$n_L$  : Cell L

$$= \frac{N!}{n_1! n_2! \dots n_L!}$$

$$\sum_L n_i = N$$

Stirling approximation

$$\log N! \cong N \log N - N.$$

$$W = \frac{N!}{n_1! n_2! \dots n_L!}$$

$$\sum_i n_i = N$$

$$\log W = (N \log N - N) - \sum_i (n_i \log n_i - n_i)$$

$$= N \log N - \sum_i n_i \log n_i$$

$$p_i = \frac{n_i}{N}$$

$$\begin{aligned} \log W &= N \log N - \sum_i n_i \log n_i \\ &= N \log N - \sum_i (N p_i) \log (N p_i) \\ &= N \log N - \sum_i (N p_i) [\underline{\log N + \log p_i}] \\ &= N \log N - N \log N \sum_i p_i - N \sum_i p_i \log p_i \\ &= -N \sum_i p_i \log p_i \end{aligned}$$

Average entropy

$$S = -\sum_i p_i \log p_i$$

Let  $L \approx 10^6$

$p_i = 1$  for a particular  $i$   
 $= 0$  for all others.

$$S = 0$$

6.

No. of configuration (for equal population in 2 cells)

$$\frac{10^6!}{2! (10^6 - 2)!} = \frac{10^6 (10^6 - 1)}{2}$$

$$\approx 5 \times 10^{11}$$

$$\frac{5 \times 10^{11}}{10^6 + 5 \times 10^{11}} \approx 1 - 10^{-5}$$

A = 00 — 40%.

C = 01 — 30%.

G = 10 — 15%.

T = 11 — 15%.

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2 bits per letter on  
average.

$$A = 0 \quad - 40\%$$

$$C = 10 \quad - 30\%$$

$$G = 110 \quad - 15\%$$

$$T = 1110 \quad - 15\%$$

$$0.4 \times 1 + 0.3 \times 2 + 0.15 \times 3 + 0.15 \times 4$$

$$= 1.9 \text{ bits per letter.}$$



A	0.4
C	0.3
T	0.15
G	0.15

$$-\sum_i p_i \log_2 p_i$$

$$= -0.4 \log_2(0.4) - 0.3 \log_2(0.3) \\ - 0.3 \log_2(0.15)$$

$$= 1.871$$