$$H(p_1, p_2, \ldots, p_M)$$

$$f(M) = H\left(\frac{1}{M}, \frac{1}{M}, \cdots \frac{1}{M}\right)$$

$f(M) > f(M')$ if $M > M'$.

$$f(1) = 0$$

$$f(MN) = f(M) + f(N)$$

Grouping Theorem.

$\boxed{C \log M}$

$$f(M^2) = f(M * M) = 2f(M)$$

$$f(M^k) = k f(M)$$

$$f(M) = f\left[(M^{1/n})^n\right] = n f(M^{1/n})$$

$$f(M^{l/n}) = \frac{l}{n} f(M)$$

For any real number a

$$\boxed{f(M^a) = a f(M).}$$

2. $f(1) = 0$

Let $M > 1$ Let r be a +ve integer.

$$M^k \leq 2^r \leq M^{k+1}$$

$$4^k \leq 4 \leq 4^{k+1} \quad M = 4 \quad r = 2$$

$\underline{k = 1}$

$$f(M^k) \leq f(2^r) \leq f(M^{k+1})$$

$$k\, f(M) \leq r\, f(2) \leq (k+1) f(M)$$

$$\boxed{\frac{k}{r} \leq \frac{f(2)}{f(M)} \leq \frac{k+1}{r}}$$

$$\frac{k}{r} \le \frac{\log 2}{\log M} \le \frac{k+1}{r}.$$

$$\frac{\log 2}{\log M} = \frac{f(2)}{f(M)}$$

$$\boxed{f(M) = C \log M}$$

Choose $C = 1$

$\log_2$. Base (2)

$H(p, 1-p)$

Grouping Theorem

$$\underline{H\left(\frac{1}{s}, \frac{1}{s}, \cdots \frac{1}{s}\right)} - H\left(\frac{r}{s}, \frac{s-r}{s}\right)$$

$$= \frac{r}{s} H\left(\underline{\frac{1}{s}, \frac{1}{s} \cdots}\right) + \frac{s-r}{s} H\left(\frac{1}{s}, \cdots \frac{1}{s}\right)$$

$$f\cancel{X}(s) = H\left(\frac{r}{s}, \frac{s-r}{s}\right) + \frac{r}{s} f(r) + \frac{s-r}{s} f(s-r)$$

$\cancel{f(s)} -$

$$f(s) = \underline{H\left(\frac{r}{s}, \frac{s-r}{s}\right)} + \frac{r}{s} f(r) + \frac{s-r}{s} f(s-r)$$

$$f(M) = \log M$$

$$H(p, 1-p) = -\left[p \log r + (1-p) \log(s-r) - \log s \right]$$

$$= -\left[p \log r - p \log s + p \log s + (1-p) \log(s-r) - \log s \right].$$

$$= -p \log p + (1-p) \log(1-p)$$

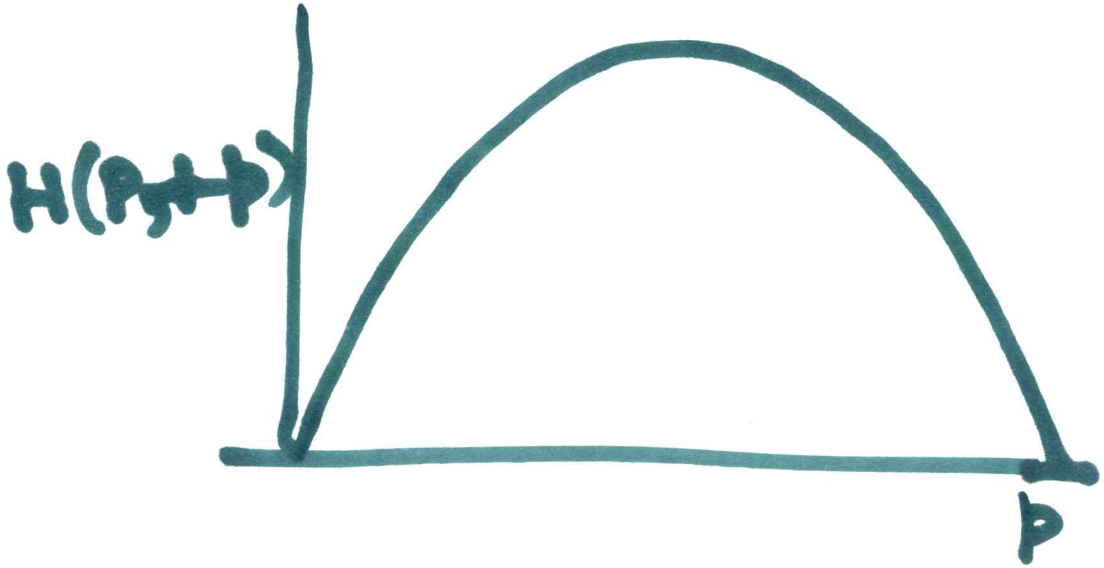
$$\boxed{H(\{p_i\}) = -\sum_i p_i \log p_i}$$

Coin Toss.

$$H\left(\frac{1}{2},\frac{1}{2}\right) = -\frac{1}{2}\log\frac{1}{2} + \left(1-\frac{1}{2}\right)\log\left(1-\frac{1}{2}\right)$$

$$= 1$$

1 bit of uncertainty

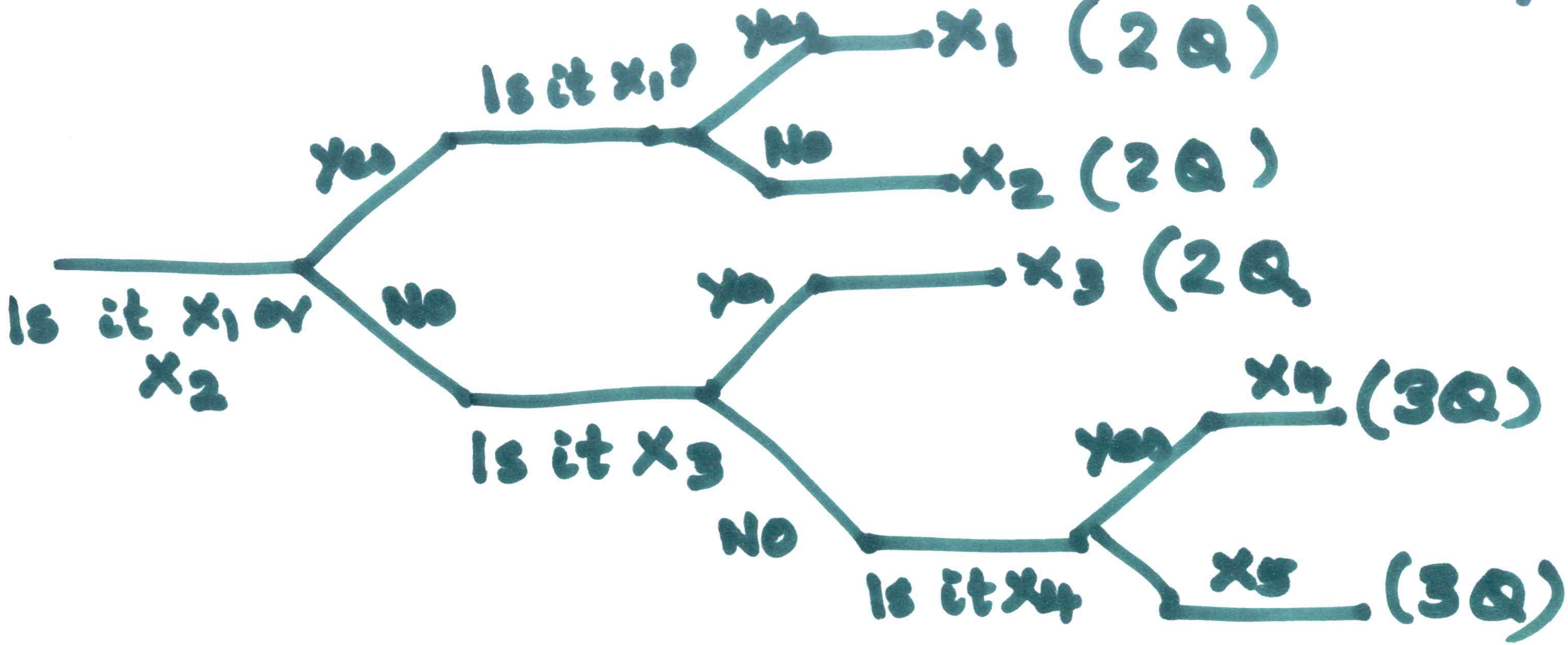


X	x_1	x_2	x_3	x_4	x_5
	0·3	0·2	0·2	·15	·15

$$
\begin{aligned}
H(\{P_i\}) = & -0.3 \log(0.3) - 0.2 \log(0.2) \\
& - 0.2 \log(0.2) - 0.15 \log(0.15) \\
& - 0.15 \log(0.15)
\end{aligned}
$$

$$= 2.27$$

Average uncertainty 2.27 bits.



$$0.3 \times 2 + 0.2 \times 2 + 0.2 \times 2 + 0.15 \times 2 + 0.15 \times 2$$

$$= 2.3$$