

$$\begin{aligned}
 & a \left[\frac{|1000\rangle + |1111\rangle}{\sqrt{2}} \cdot \frac{|1000\rangle + |1111\rangle}{\sqrt{2}} \frac{|1000\rangle + |1111\rangle}{\sqrt{2}} \right] \\
 & + b \left[\frac{|1000\rangle - |1111\rangle}{\sqrt{2}} \frac{|1000\rangle - |1111\rangle}{\sqrt{2}} \frac{|1000\rangle - |1111\rangle}{\sqrt{2}} \right]
 \end{aligned}$$

1st qubit is flipped σ_y error.

$$\begin{aligned}
 & a \left[\frac{|1100\rangle \otimes |1011\rangle}{\sqrt{2}} \cdot \frac{|1000\rangle + |1111\rangle}{\sqrt{2}} \frac{|1000\rangle + |1111\rangle}{\sqrt{2}} \right] \\
 & + b \left[\frac{|1100\rangle + |1011\rangle}{\sqrt{2}} \frac{|1000\rangle - |1111\rangle}{\sqrt{2}} \frac{|1000\rangle - |1111\rangle}{\sqrt{2}} \right]
 \end{aligned}$$

$$\begin{aligned}
 & a \left[\frac{|111\rangle - |011\rangle}{\sqrt{2}} \cdot \frac{|1000\rangle + |1100\rangle}{\sqrt{2}} \frac{|1000\rangle + |1100\rangle}{\sqrt{2}} \right] \\
 & + b \left[\frac{|111\rangle + |011\rangle}{\sqrt{2}} \cdot \frac{|1000\rangle - |1100\rangle}{\sqrt{2}} \frac{|1000\rangle - |1100\rangle}{\sqrt{2}} \right]
 \end{aligned}$$

CCNOT →

$$\begin{aligned}
 & a \left[\frac{|1011\rangle - |1111\rangle}{\sqrt{2}} \cdot \frac{|1000\rangle + |1100\rangle}{\sqrt{2}} \frac{|1000\rangle + |1100\rangle}{\sqrt{2}} \right] \\
 & + b \left[\frac{|1011\rangle + |1111\rangle}{\sqrt{2}} \cdot \frac{|1000\rangle - |1100\rangle}{\sqrt{2}} \frac{|1000\rangle - |1100\rangle}{\sqrt{2}} \right] \\
 & = a \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} |11\rangle \frac{|10\rangle + |11\rangle}{\sqrt{2}} |100\rangle \frac{|10\rangle + |11\rangle}{\sqrt{2}} |100\rangle \right] \\
 & + b \left[\frac{|0\rangle + |1\rangle}{\sqrt{2}} |11\rangle \frac{|10\rangle - |11\rangle}{\sqrt{2}} |100\rangle \frac{|10\rangle - |11\rangle}{\sqrt{2}} |100\rangle \right]
 \end{aligned}$$

1st 4th 7th qubits.

- 3 -

After the set of 3 Hadamards.

$$a |1\rangle_1 |0\rangle_4 |0\rangle_7 + b |0\rangle_1 |1\rangle_4 |1\rangle_7$$

CNOTS \rightarrow $a |1\rangle_1 |1\rangle_4 |1\rangle_7 + b |0\rangle_1 |1\rangle_4 |1\rangle_7$

CCNOT \rightarrow $\boxed{a |0\rangle_1 + b |1\rangle_1}$

$$|0\rangle \mapsto (|1000\rangle + |1111\rangle) \otimes |3\rangle$$

$$|1\rangle \mapsto (|1000\rangle - |1111\rangle) \otimes |3\rangle$$

Bit Flip has Occurred.

check parity $(\sigma_{z1} \otimes \sigma_{z2})$

compare 1 & 2 $\begin{cases} +1 \checkmark \\ -1 \cdot \end{cases}$

$(\sigma_{z2} \otimes \sigma_{z3})$

compare 2 & 3 $\begin{cases} +1 \checkmark \\ -1 \cdot \end{cases}$

- + + No error.
- + - error in bit 3

Correct by σ_x

compare $\left\| \begin{array}{l} 1 \ \& \ 2 \\ 2 \ \& \ 3 \end{array} \right.$ $(\sigma_{x_1} \sigma_{x_2} \sigma_{x_3} \dots \sigma_{x_6})$ ⁵

Applying σ_z